

# 計算核データ構築に向けて

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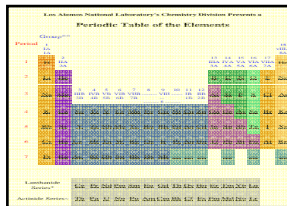
*RIKEN Nishina Center*

•Real-space, real-time approaches

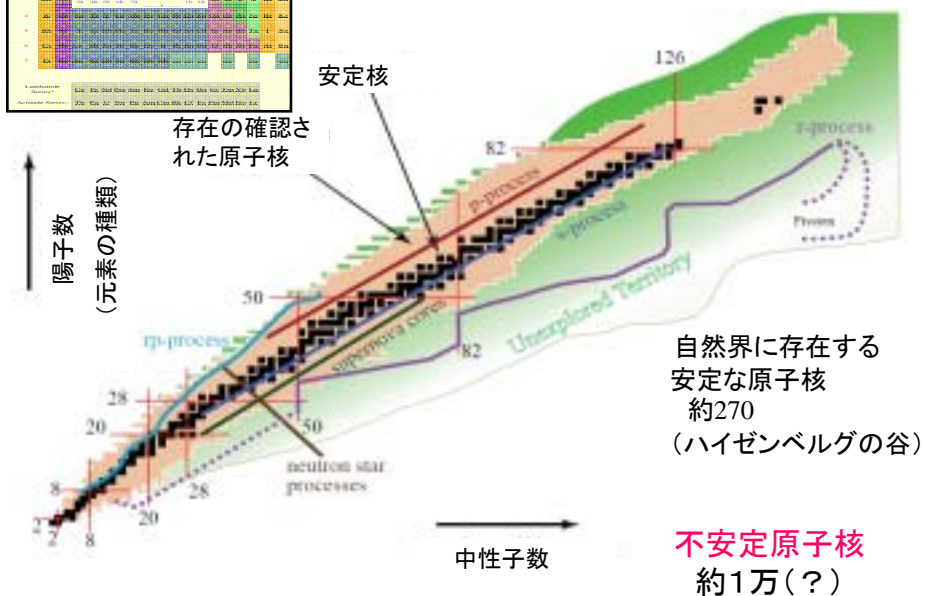
→ DFT, TDDFT ((Q)RPAと相補的)

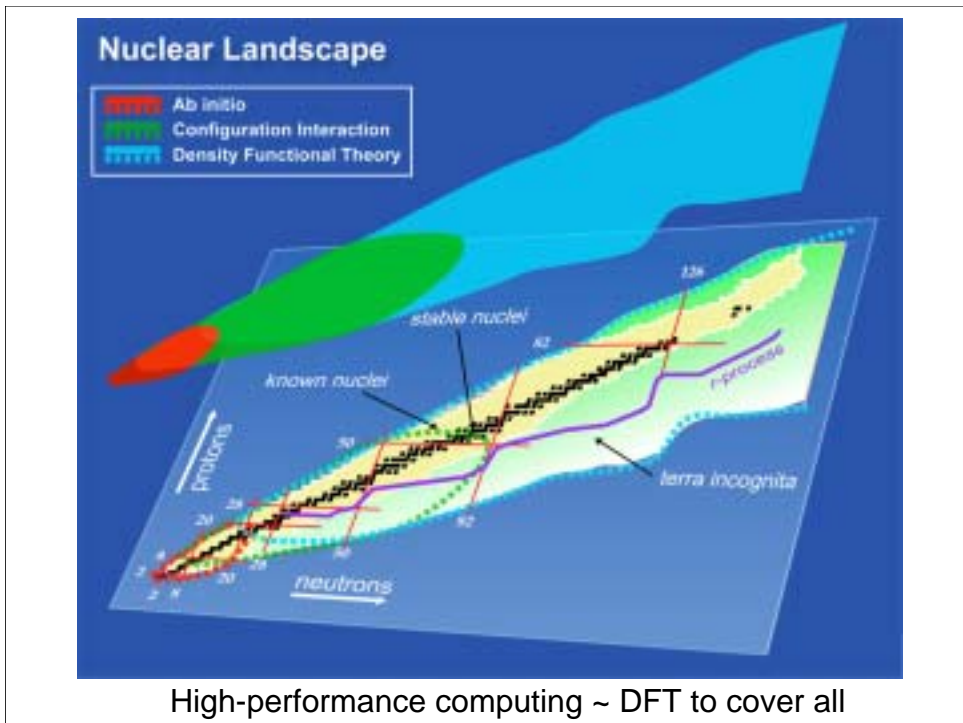
Few-body model (CDCCと相補的)

2009.3.25-26 Mini-WS:核データと核理論

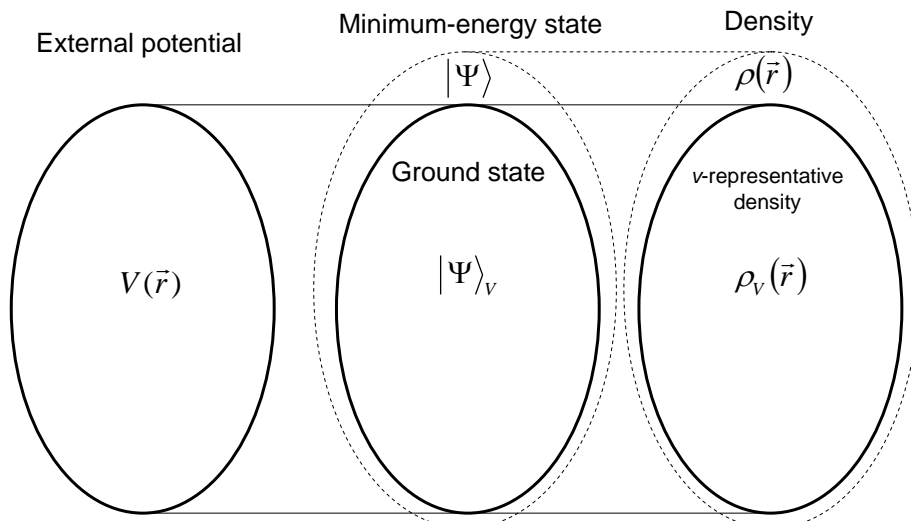


## 核図表 (Nuclear Chart)





## One-to-one Correspondence



The following variation leads to all the ground-state properties.

$$\delta \left\{ F[\rho] + \int \rho(\vec{r}) v(\vec{r}) d\vec{r} - \mu \left( \int \rho(\vec{r}) d\vec{r} - N \right) \right\} = 0$$

In principle, any physical quantity of the ground state should be a functional of density.

Variation with respect to many-body wave functions  $\Psi(\vec{r}_1, \dots, \vec{r}_N)$

↓

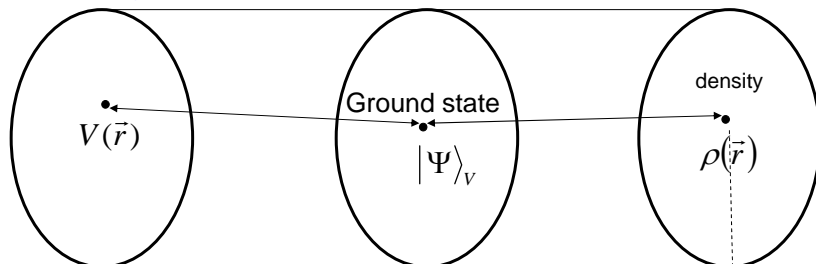
Variation with respect to one-body density  $\rho(\vec{r})$

↓

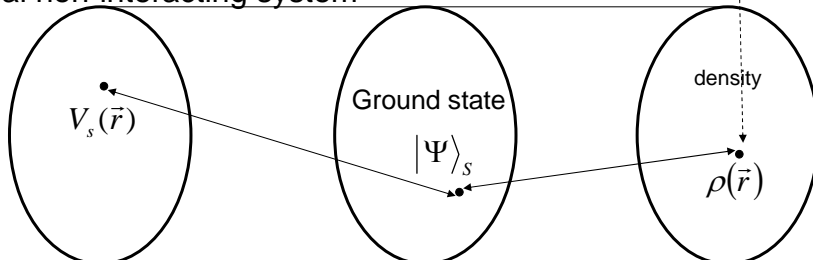
Physical quantity  $A[\rho(\vec{r})] = \langle \Psi[\rho] | \hat{A} | \Psi[\rho] \rangle$

## Kohn-Sham Scheme

Real interacting system



Virtual non-interacting system



# Kohn-Sham scheme

$$\rho(\vec{r}) = \sum_i |\phi_i(\vec{r})|^2 \quad |\Psi\rangle_S = \det\{\phi_i(\vec{r}_j)\}$$

$$-\frac{\hbar^2}{2m} \nabla^2 \phi_i + v_S[\rho] \phi_i = \varepsilon_i \phi_i$$

KS canonical equation

Density functional

$$F[\rho(\vec{r})] = T_S[\rho(\vec{r})] + (F[\rho(\vec{r})] - T_S[\rho(\vec{r})]) \Rightarrow V_{\text{eff}}[\rho(\vec{r})]$$

$$= \sum_i \langle \phi_i | \frac{\vec{p}^2}{2m} | \phi_i \rangle + V_{\text{eff}}[\rho(\vec{r})]$$

Minimization of this density functional leads to

$$v_S[\rho](\vec{r}) = \frac{\delta V_{\text{eff}}}{\delta \rho(\vec{r})}$$

## Nuclear DFT

Global properties, global calculations

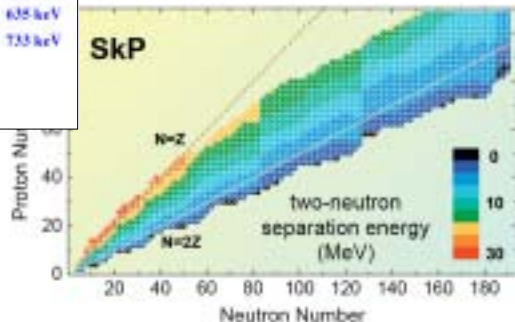
S. Goriely et al., ENAM'04

**HFB models: weapons of mass production**

Recent improvements of HFB mass formulas	Accuracy ( $\sigma_{\text{rms}}$ ) (2004 mass)
HFB-2 masses: $M^*_1=1.04$ , vol. pairing	629 keV
HFB-5 masses: $M^*_1=1.12$ , vol+surf pairing	635 keV
HFB-7 masses: $M^*_1=0.80$ , vol+surf pairing	627 keV
HFB-8 masses: $M^*_1=0.80$ , vol. pairing, PLN (part. ch. prog.)	635 keV
HFB-9 masses: $M^*_1=0.80$ , vol. pairing, PLN, $\delta=30\text{MeV}$	733 keV

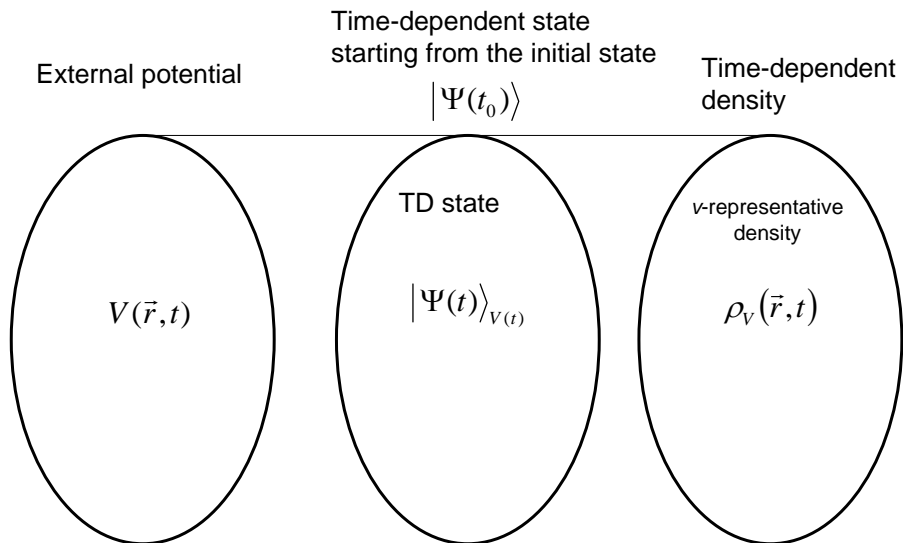
↓  
Constrained on both Stable and Unstable Nuclear Chart for astrophysical applications

M. Stoitsov et al.



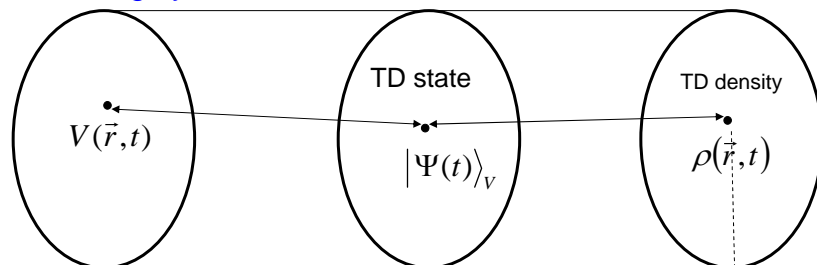
- \* Global DFT mass calculations: HFB mass formula:  $\Delta m \sim 700 \text{ keV}$
- Taking advantage of high-performance computers

# One-to-one Correspondence

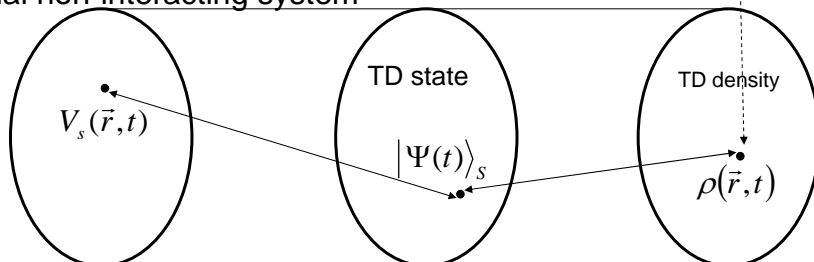


## TD Kohn-Sham Scheme

Real interacting system



Virtual non-interacting system



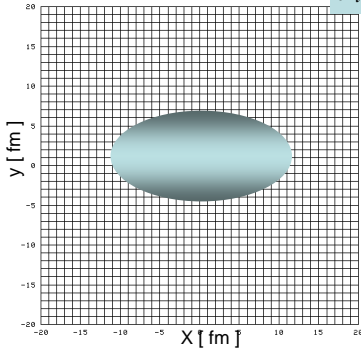
# Skyrme TDDFT in real space

Time-dependent Kohn-Sham equation

$$i \frac{\partial}{\partial t} \psi_i(\mathbf{r} \sigma \tau, t) = \left( h_{\text{HF}}[\rho, \tau, \mathbf{j}, \mathbf{s}, \vec{\mathbf{J}}](t) + V_{\text{ext}}(t) - i \tilde{\eta}(r) \right) \psi_i(\mathbf{r} \sigma \tau, t)$$

3D space is discretized in lattice

Single-particle orbital:  $\varphi_i(\mathbf{r}, t) = \{\varphi_i(\mathbf{r}_k, t_n)\}_{k=1, \dots, Mr}^{n=1, \dots, Mt}$ ,  $i = 1, \dots, N$



$N$ : Number of particles

$Mr$ : Number of mesh points

$Mt$ : Number of time slices

Spatial mesh size is about 1 fm.

Time step is about 0.2 fm/c

Nakatsukasa, Yabana, Phys. Rev. C71 (2005) 024301

# Real-time calculation of response functions

1. Weak instantaneous external perturbation

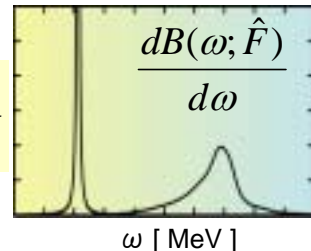
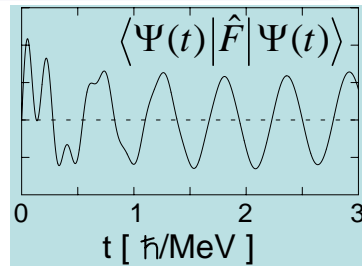
$$V_{\text{ext}}(t) = \hat{F} \delta(t)$$

2. Calculate time evolution of

$$\langle \Psi(t) | \hat{F} | \Psi(t) \rangle$$

3. Fourier transform to energy domain

$$\frac{dB(\omega; \hat{F})}{d\omega} = -\frac{1}{\pi} \text{Im} \int \langle \Psi(t) | \hat{F} | \Psi(t) \rangle e^{i\omega t} dt$$



Neutrons

$^{16}\text{O}$

$$\delta\rho_n(t) = \rho_n(t) - (\rho_0)_n$$

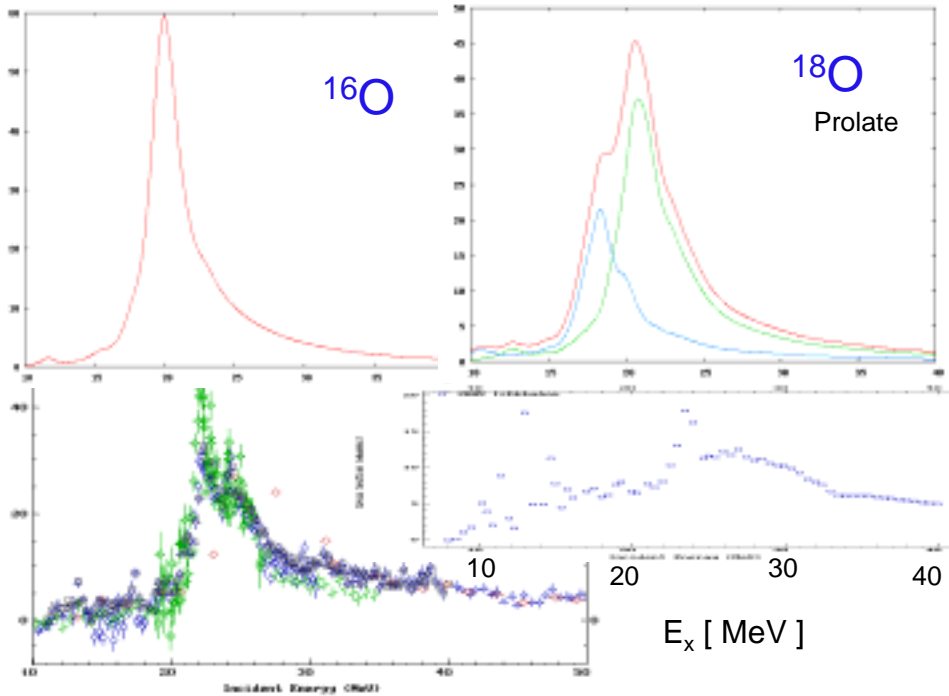
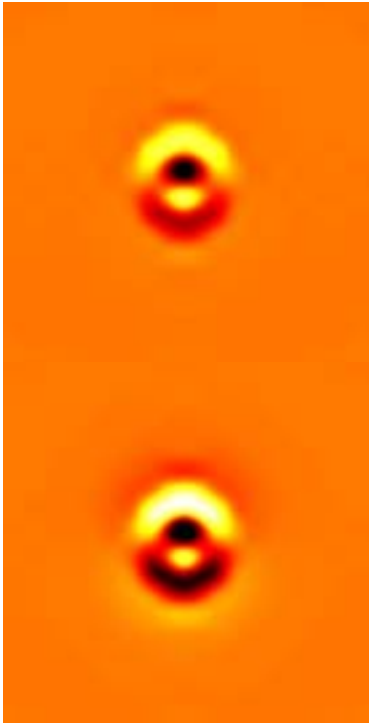
Time-dep. transition density

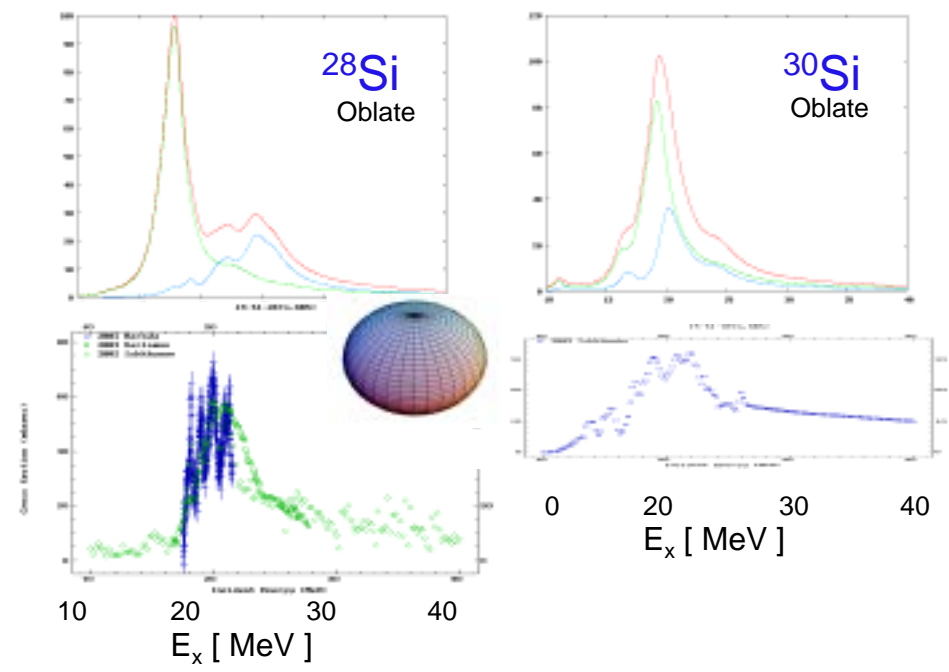
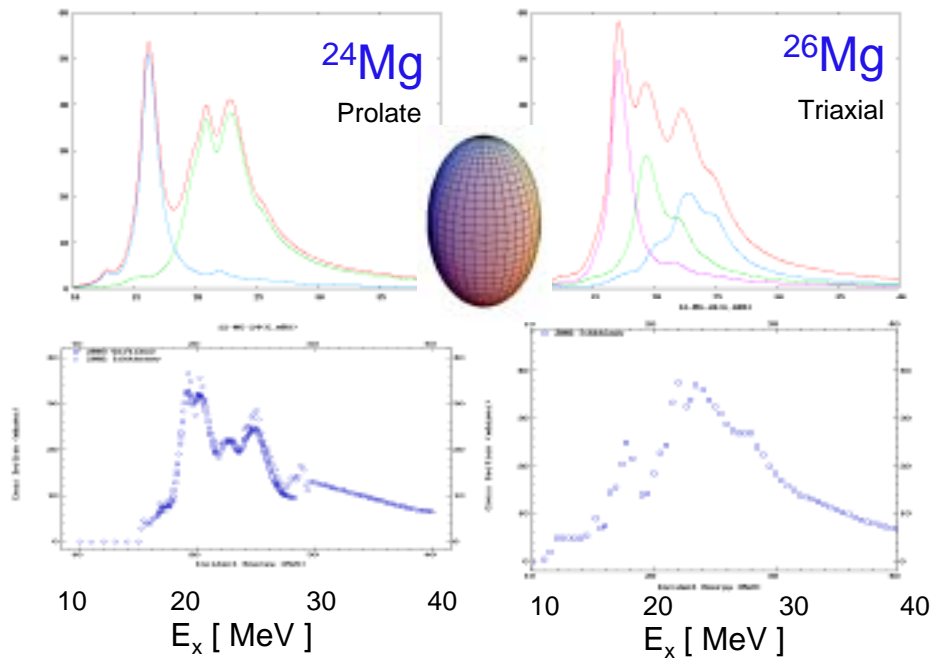
$$\delta\rho > 0$$

$$\delta\rho < 0$$

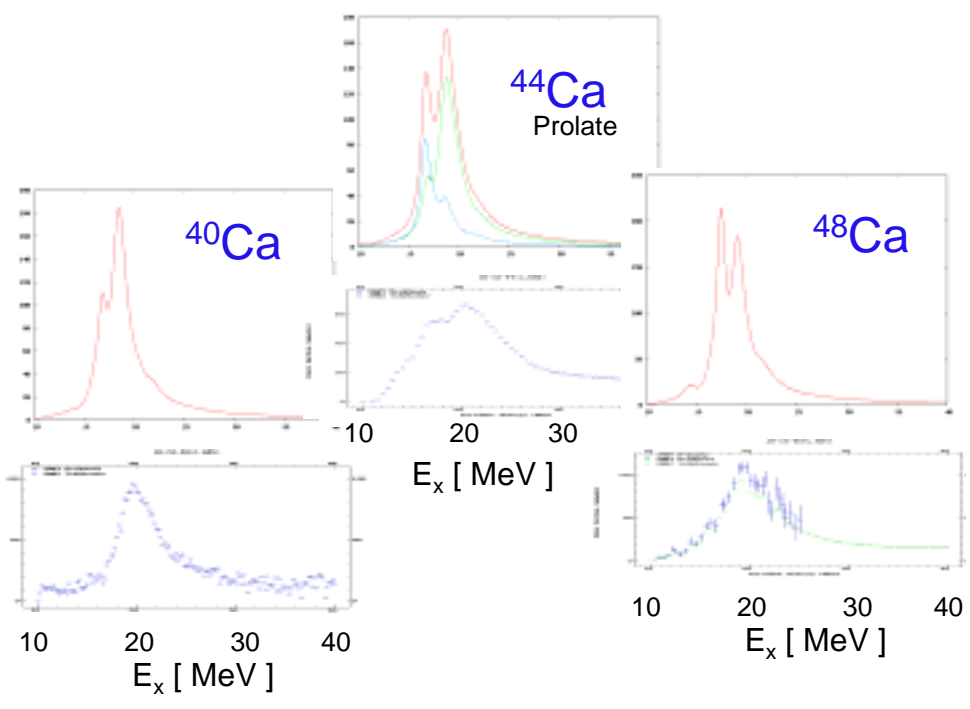
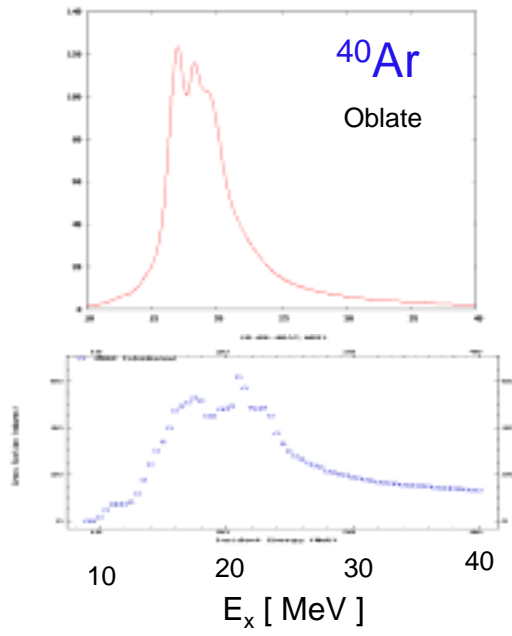
$$\delta\rho_p(t) = \rho_p(t) - (\rho_0)_p$$

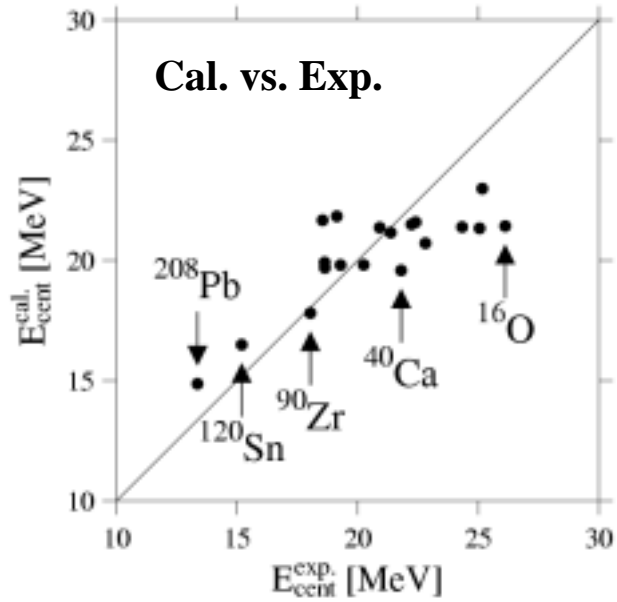
Protons



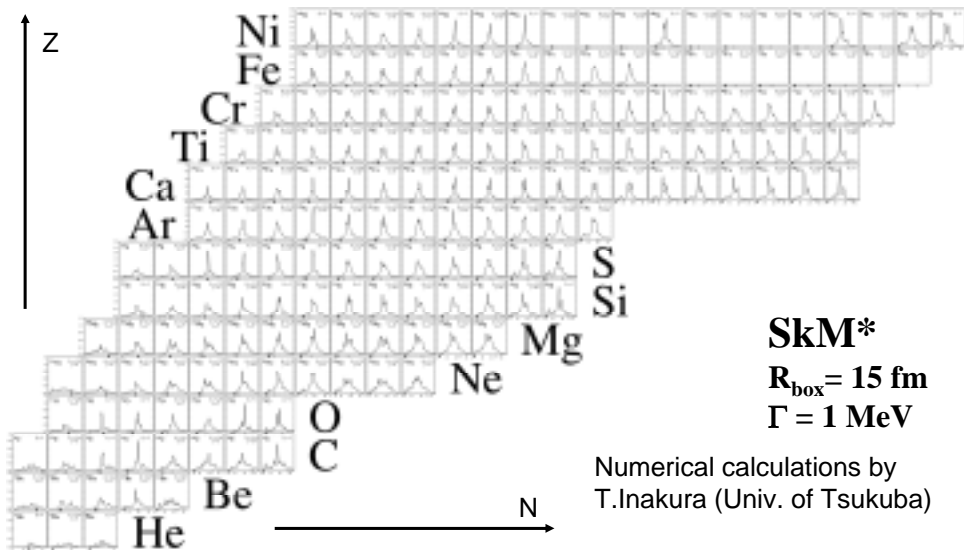








## Electric dipole strengths



# Few-body-model calculation of fusion cross section

- Real-time, real-space approach
- No need for scattering boundary condition
- Alternative method to the CDCC

## Wave packet dynamics of fusion reaction potential scattering with absorption inside a Coulomb barrier

Radial Schroedinger equation for  $l=0$

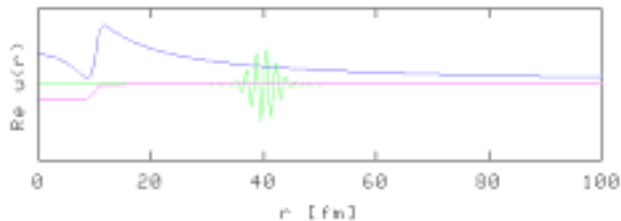
$$i\hbar \frac{\partial}{\partial t} u(r,t) = \left[ -\frac{\hbar^2}{2m} \frac{d^2}{dr^2} + V(r) + iW(r) \right] u(r,t)$$

with incident Gaussian wave packet

$$u(r, t_0) = \exp\left[-ikr - \gamma(r - r_0)^2\right]$$

$^{10}\text{Be}-^{208}\text{Pb}$  (A,Z=10,4 and 208,82)  
 $V_0=50$   $W_0=-10$ ,  $RV=1.26$ ,  $RW=1.215$ ,  $AV=0.44$ ,  $AW=0.45$   
 $E_{\text{inc}}=28$  MeV (+Coulomb at R\_0),  $R_0=40\text{fm}$ ,  $\gamma=0.1\text{fm}^{-2}$   
 $N_r=400$ ,  $dr=0.25$ ,  $N_t=10000$ ,  $dt=0.001$

$^{10}\text{Be} - ^{208}\text{Pb}$

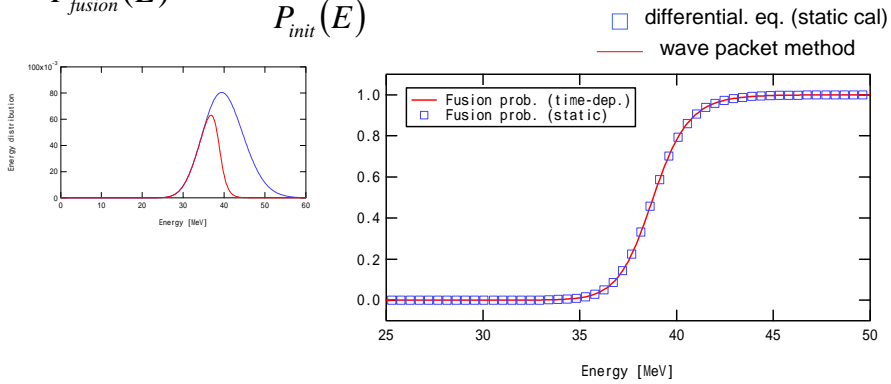


Flux absorbed by  $W(r)$   
represents fusion.

Wave packet dynamics include scattering information for wide energy region.  
Then, how to extract reaction information for a fixed energy?

## Fusion probability

$$P_{fusion}(E) = \frac{P_{init}(E) - P_{final}(E)}{P_{init}(E)}$$



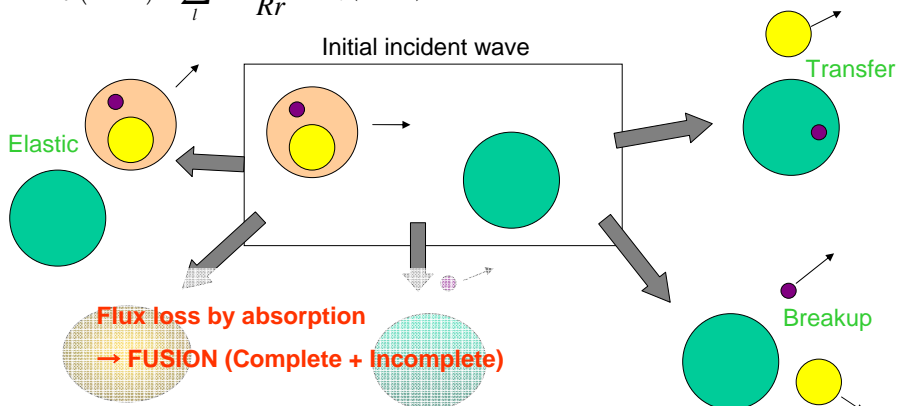
Fusion probability for whole barrier region from single wave-packet calculation. No boundary condition required in the wave packet calculation.

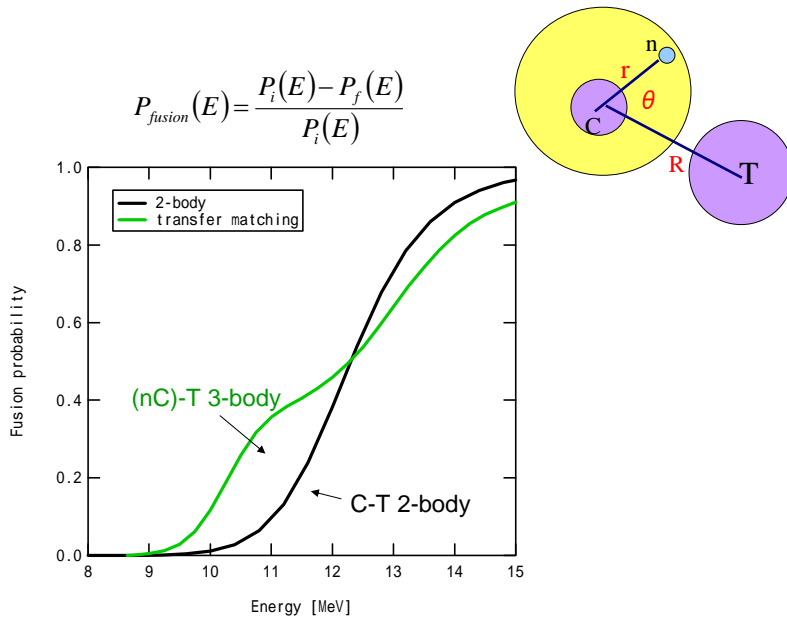
## Fusion probability of three-body reaction

$$i\hbar \frac{\partial}{\partial t} \psi(\vec{R}, \vec{r}, t) = \left( -\frac{\hbar^2}{2\mu} \nabla_R^2 - \frac{\hbar^2}{2m} \nabla_r^2 + V_{nC}(r_{nC}) + V_{CT}(r_{CT}) + V_{nT}(r_{nT}) \right) \psi(\vec{R}, \vec{r}, t)$$

$$\psi_j(\vec{R}, \vec{r}, t) = \sum_l \frac{u_l^j(R, r, t)}{Rr} P_l(\cos \theta)$$

Coulomb + Nuclear potential  
Absorption => C-T fusion



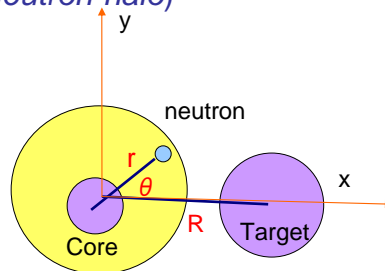


Enhancement of fusion probability at sub-barrier energies

Case (2): Weakly-bound projectile (*Neutron-halo*)

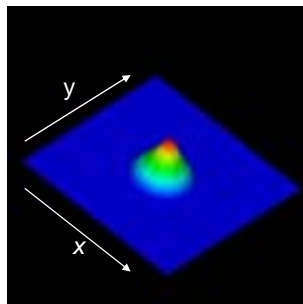
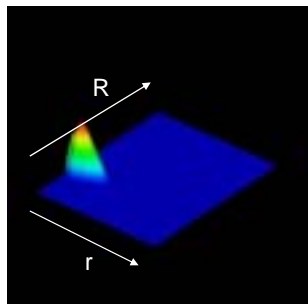
• n-C orbital energy: -0.6 MeV (Halo)

$^{11}\text{Be}(n+^{10}\text{Be})-^{208}\text{Pb}$   
head-on collision ( $J=0$ )

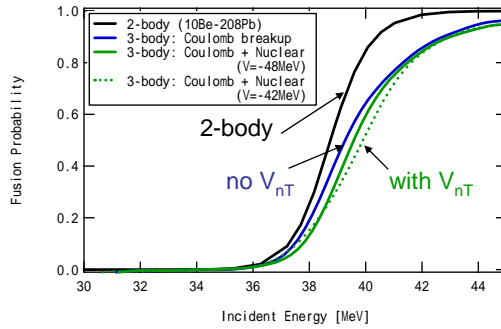


$$\rho(R, r, t) = \int d(\cos\theta) |\psi(R, r, \theta, t)|^2$$

$$\rho(r, \theta, t) = \int dR |\psi(R, r, \theta, t)|^2$$

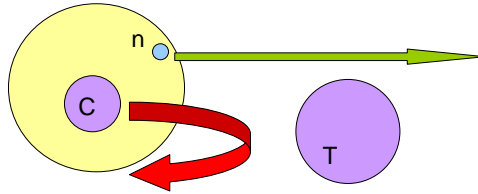


# Fusion probability of neutron-halo nuclei is suppressed

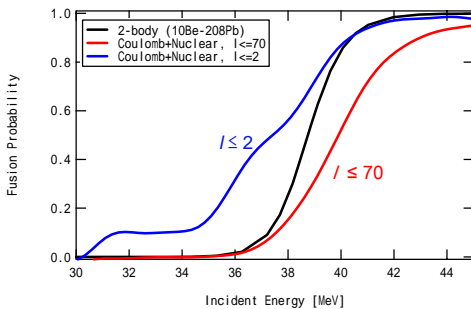


Core incident energy decreases effectively by neutron breakup

$$E_{core} \approx \frac{M_{core}}{M_{core} + M_n} E_{projectile}$$

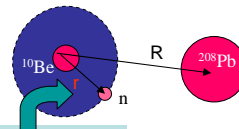


## Why different from other studies?



### Conclusions of other studies

- Quantum calculations have been done using the discretized continuum channels.
  - Hagino et al, PRC61 (2000) 037602
  - Diaz-Torres & Thompson, PRC65 (2002) 024606
- Fusion was enhanced with a weakly-bound neutron at sub-barrier energies
- Nuclear coupling was important for the fusion enhancement



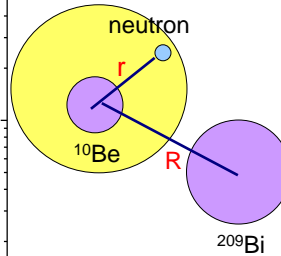
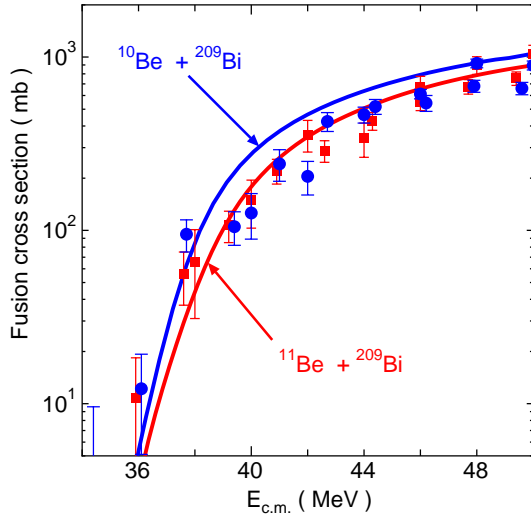
We need to include high-partial waves for n-<sup>10</sup>Be motions.

The low-partial-wave truncation leads to an opposite conclusion!

## Fusion Cross Section of $^{11}\text{Be}$

Three body full calculation of  $^{11}\text{Be} + ^{209}\text{Bi}$

Fusion probability is hindered by the presence of the halo neutron



Experiment

C. Signorini et.al, Nucl. Phys. 735 (2004) 329.

Theory

M. Ito, M. Ueda, T. Nakatsukasa, K. Yabana,  
Phys. Lett. B 637, 53(2006)

## Summary

- DFT/TDDFT
  - Systematic calculations for all nuclei including those far from the stability line
  - Description of large amplitude dynamics, such as fission
- Real-time, real-space approach to few-body models
  - Accurate few-body scattering dynamics
  - An alternative approach to CDCC