Intermediate amplitude collective motion in \(^{52}\text{Ti}\) with TDHFB

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Introduction

Time-dependent mean-field theories

Small amplitude $\rightarrow$ RPA, normal modes

Large amplitude
$\rightarrow$ simulation of nuclear fusion and/or fission
energy transfer:
  relative motion
    ➞ internal collective/nucleonic motions

one-body dissipation
  (2 body collision neglected)

its mechanism ???
  ← candidate: chaotic motion in TDHF (?)
A Numerical Study on the Structure Change of Collective Motions

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Fig. 1. (a) Continuous CHF lines. The lowest line i) passes through the ground state \(|\phi_0\rangle\). The dense calculated points near \(q = 0.32\) and \(q = -0.65\) indicate an abrupt character change occurring in line i). To illustrate a "dissociative line" having no abrupt change, the dashed lines are drawn by hand. (b) The corresponding single-particle energy levels. Solid lines are results from the global configuration dictated (GCD) method, and dashed lines from the local configuration dictated (LCD) method. The filled and open circles represent the occupied and unoccupied states, respectively. (Borrowed from Ref. 4).

Fig. 8. (a) and (b) As in Fig. 6 but for the TDHF trajectory (dashed curve) in Fig. 5. The resulting points are \(\phi_a\), \(\phi_i\) and \(\phi_o\).

Three-level model

Fig. 10. (a) Density oscillations with small (full curve) and large (dashed curve) excitation energies. The resulting points obtained by the TR method are shown by open rhombuses (small amplitude) and pluses (larger amplitude). These are identical to the ground state. The iteration processes (dashed curves) of the TR method are shown for small-amplitude (b) and larger-amplitude (c) TDHF trajectories. The parabola represents the ground state branch in Fig. 8. The open rhombuses on the parabola in (b) and (c) denote the calculated points in the local configuration dictated (LCD) method.

BKN force
From TDHF to TDHFB

☆ make clear the role of pairing correlation in larger amplitude collective motion (within mean-field framework)

☆ invent tools to understand the microscopic (single-particle) dynamics
Equations of motion of matrices $U$ & $V$  

$$i\hbar \frac{\partial}{\partial t} \begin{pmatrix} U(t) \\ V(t) \end{pmatrix} = \mathcal{H} \begin{pmatrix} U(t) \\ V(t) \end{pmatrix}$$

$$i\hbar \frac{\partial}{\partial t} \mathcal{R} = [\mathcal{H}, \mathcal{R}],$$

$$\mathcal{H} = \begin{bmatrix} h_1 & \Delta \\ \Delta^\dagger & -h_1^* \end{bmatrix},$$

$$\mathcal{R} = \begin{bmatrix} \rho & \kappa \\ \kappa^\dagger & 1 - \rho^* \end{bmatrix}, \quad \mathcal{R} = \mathcal{R}^2$$

$$h \equiv t + \Gamma \quad \Gamma_{ij} = \sum_{kl} \bar{v}_{ikjl} \rho_{lk} \quad \Delta_{ij} = \frac{1}{2} \sum_{kl} \bar{v}_{ijkl} \kappa_{kl}$$

$$\rho_{ij} = \langle \Phi | c_j^\dagger c_i | \Phi \rangle = (V^* V^T)_{ij} \quad \kappa_{ij} = \langle \Phi | c_j c_i | \Phi \rangle = (V^* U^T)_{ij}$$
Gogny-D1S

\[ V_{12} = \sum_{i=1}^{2} \exp \left( -\frac{|\vec{r}_1 - \vec{r}_2|^2}{\mu_i^2} \right) \cdot (W_i + B_i \hat{P}_\sigma - H_i \hat{P}_\tau - M_i \hat{P}_\sigma \hat{P}_\tau) + \]

\[ + t_3 (1 + x_0 \hat{P}_\sigma) \delta(\vec{r}_1 - \vec{r}_2) \left[ \rho \left( \frac{\vec{r}_1 + \vec{r}_2}{2} \right) \right]^\gamma + \]

\[ + i W_{LS} (\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \vec{\nabla}_{12} \times \delta(\vec{r}_1 - \vec{r}_2) \vec{\nabla}_{12} + V_{\text{Coul.}} , \]

Coulomb part is NOT included

\[ \mu_1 = 0.7 \text{ fm} \]
\[ W_1 = -1720.3 \text{ MeV} \]
\[ B_1 = 1300 \text{ MeV} \]
\[ H_1 = -1813.53 \text{ MeV} \]
\[ M_1 = 1397.60 \text{ MeV} \]
\[ t_3 = 1390.60 \text{ MeV fm}^{3(1+\gamma)} \]
\[ \gamma = 1/3 \]
\[ \mu_2 = 1.2 \text{ fm} \]
\[ W_2 = 103.639 \text{ MeV} \]
\[ B_2 = -163.483 \text{ MeV} \]
\[ H_2 = 162.812 \text{ MeV} \]
\[ M_2 = -223.934 \text{ MeV} \]
\[ x_0 = 1 \]
\[ W_{LS} = 130 \text{ MeV fm}^5 \]

- basis function: three-dimensional harmonic oscillator wave functions
- space: \[ N_{\text{shell}} = n_x + n_y + n_z \leq 5 \]
initial conditions:
- $Q_{20}$ type impulse on ground state (impulse type)
- constrained state with quadrupole operator (constraint type)

**initial U & V**

$$V_k(t = 0) \rightarrow \exp(i\epsilon Q_0)V_k(t = 0)$$

$$= \sum_{\nu} \frac{i^\nu \epsilon^\nu Q_0^\nu}{\nu!} V_k(t = 0)$$

$$U_k(t = 0) \rightarrow \exp(-i\epsilon Q_0^*)U_k(t = 0)$$

$Q_0$ : matrix representation of multipole operator

$$Q_{20} = 2z^2 - x^2 - y^2 \text{ [fm}^2\text{]}$$
Example-1: $^{20}$O quadrupole oscillation
Example-2. Spherical case: \(^{26}\text{Ne} \ IV\) dipole response

TDHFB (Nshll = 5)

S. Peru, H. Goutte, J.F. Berger,
$^{52}$Ti results

☆ Larger excitation energy

☆ Larger amplitude ⇐ “intermediate amplitude”
energy curve $E$ vs $<Q_{20}>$ ($^{52}$Ti ($Z = 22, N = 30$))
Total Energy
Pairing E (p)
Pairing E (n)

... pairing in neutrons (impulse, $Q_{20} = 140, 230 \sim 240 [fm^2]$)

... NO pairing in neutrons ($Q_{20} = 150 \sim 200 [fm^2]$)

initial conditions

impulse (4.32[MeV])

constraints

Energy [MeV]

$Q_{20} [fm^2]$
... no pairing in neutrons
oscillation around $Q_{20} = 85$ [fm$^2$]

... pairing in neutrons
oscillations around $Q_{20} = 0$ [fm$^2$]
- two types of oscillations after relaxation process
- effects of pairing correlations in oscillating motions

\[ \sum_i V_{ij} V_{ij} \]
Case 1: oscillations around \( Q_{20} = 85 \) [fm\(^2\)] (constraint \( Q_{20} = 170 \) [fm\(^2\)])

- initial condition: \( Q_{20} = 170 \) [fm\(^2\)]
  
  \[ E_x = 2.56 \text{[MeV]} \]
  
  \[ E_{\text{pair(proton)}} = -3.55 \text{[MeV]} \]
  
  \[ E_{\text{pair(neutron)}} = 0 \text{ [MeV]} \]

- time dependence of occupation probability (neutrons)

- no pairing in neutrons
  \( \Rightarrow \) no jump into ground state pocket (\( Q_{20} = 0 \) [fm\(^2\)])
Case 2: oscillations around ground state (Q20 = 0 [fm^2]) (impulse type)

- two main trajectories
- Q_{20} oscillation \leftrightarrow seesaw in occupation probabilities

impulse type
Ex = 4.32[MeV]
E_pair(proton) = -4.79 [MeV]
E_pair(neutron) = -2.75 [MeV]
Case 3: jump into ground state pocket (constraint $Q_{20} = 140 \, [fm^2]$)

- sudden change in occupation probability
  - Q20 oscillation jumps
- characteristic two orbitals
- in phase with Q20 oscillation

**Initial condition:** $Q_{20} = 140 \, [fm^2]$

- $E_x = 1.37 \, [MeV]$
- $E_{\text{pair}}(\text{proton}) = -3.27 \, [MeV]$
- $E_{\text{pair}}(\text{neutron}) = -0.07 \, [MeV]$
variations of occupation probabilities of two main trajectories (constraint $Q_{20} = 140$ [fm$^2$])

(prolate side ... red orbital main)

$Q_{20}$ [fm$^2$]

(prolate)

(obl ate side ... green orbital main)

$(n_x, n_y, n_z) = (0, 3, 0), (0, 0, 3)$

(obl ate)

(prolate)

(obl ate phase ... green orbital main)

$(n_x, n_y, n_z) = (3, 0, 0)$
$^{52}$Ti ($Z = 22, N = 30$) one particle levels (neutrons)

Energy [MeV]

- $p_{3/2}$
- $f_{7/2}$
- $d_{3/2}$
- $s_{1/2}$
- $d_{5/2}$
- $p_{1/2}$
- $p_{3/2}$

$Q_{20}$ [fm$^2$]

Oblate ↔ quadrupole oscillation → prolate

Change of occupation probability
Case 4: slow relaxation (constraint $Q_{20} = 240$ [fm$^2$])

initial condition: $Q_{20} = 240$ [fm$^2$]
Ex = 7.37[MeV]
$E_{\text{pair}}$ (proton) = -4.79 [MeV]
$E_{\text{pair}}$ (neutron) = -0.89 [MeV]

slow relaxation
- occupation probabilities of many orbitals change
  $\rightarrow$ two main orbitals becomes unclear
period $\rightarrow$ OK

unsolved problem: amplitudes
Time [fm]

Energy [MeV]

Q$^2_0$ [fm$^2$]

Initial $Q_20$ = 140
Ex $E$= 1.37[MeV]
Pairing $E$ (p)= -3.27[MeV]
Pairing $E$ (n)= -0.07[MeV]

Initial $Q_20$ = 200
Ex $E$= 4.32 [MeV]
Pairing $E$ (p)= -4.19[MeV]
Pairing $E$ (n)= 0 [MeV]

Initial $Q_20$ = 240
Ex $E$= 7.37[MeV]
Pairing $E$ (p)= -4.79[MeV]
Pairing $E$ (n)= -0.89[MeV]

Impulse type
Ex $E$ = 4.32[MeV]
Pairing $E$ (p)= -4.79[MeV]
Pairing $E$ (n)= -2.75[MeV]
Amplitudes of slow oscillations with excitation energy from 2.72 to 10.85 [MeV] vary very slowly.
Summary

1. Intermediate amplitude collective oscillations are calculated in terms of TDHFB with Gogny interaction.

2. Pairing correlation easily affects the oscillations:
   * center of oscillation
   * period, amplitude
   * relaxation time

3. occupation probability changes continuously
   ←→ quantum numbers are discrete
   Oscillation with long period may easily come out.

4. improvements in future:
   * better basis
     → Gogny TDHFB with GEM, cylindrical HO basis, ....
   * connections with microscopic analysis such as AdSCC.