Toward realistic nuclear mean fields

H. Nakada (Chiba U.)

@ Japan-France Workshop
(RIKEN; Feb. 26, 2010)

I. Introduction

"MF approx." ≈ "Density Functional Theory (DFT)"

— **good 1st approx. to nuclear structure problems**

- **describing fundamental properties from nucleonic d.o.f.**
  (saturation, shell structure, *etc.*)
  input ··· effective int. (or EDF)
- **covering whole region of nuclear chart (except $A \lesssim 10$?)**
  from light to heavy, spherical to deformed nuclei
  stable to unstable nuclei
  → supernova, neutron star ?
- **good basis for more precise description**
  → GCM, shell model, TDDM, *etc.*

★ **Effective NN int. ?**

fully microscopic ("realistic") int.

$\cdots$ yet insufficient to describe fundamental properties

***e.g.* saturation, LS splitting**

⇒ "semi-realistic" int.

★ **Numerical methods ?**

realistic or semi-realistic int. — quite possibly finite-range

(↔ non-local EDF)

HO bases $\cdots$ unrealistic (or impractical) in unstable nuclei !

⇒ new method desired → Gaussian expension method (GEM)

II. MF & RPA calculations with Gaussian expansion method

'Gaussian expansion method (GEM)'

— **developed by Kamimura *et al.* for few-body calculations**

Ref.: E. Hiyama *et al.*, Prog. Part. Nucl. Phys. 51, 223 ('03)

MF calculations with GEM

Ref.: H.N. & M. Sato, N.P.A 699, 511 ('02); 714, 696 ('03)
H.N., N.P.A 764, 117 ('06); 801, 169 ('08)
H.N., N.P.A 808, 47 ('08)

- **basis**: $\phi_{\nu\ell jm}(\boldsymbol{r}) = R_{\nu\ell j}(r)\,[Y^{(\ell)}(\hat{\boldsymbol{r}})\chi_\sigma]^{(j)}_m\,;\qquad R_{\nu\ell j}(r) = \mathcal{N}_{\nu\ell j}\,r^\ell \exp(-\nu r^2)$

  $\nu \rightarrow$ **complex** $\quad \nu = \nu_\mathrm{r} + i\nu_\mathrm{i}$, ν : **with geometric progression**

$$\left.\begin{matrix} \mathrm{Re}[R_{\nu\ell j}(r)] \\ \mathrm{Im}[R_{\nu\ell j}(r)] \end{matrix}\right\} \propto \; r^\ell \exp(-\nu_\mathrm{r}\,r^2) \left\{\begin{matrix} \cos(\nu_\mathrm{i}\,r^2) \\ \sin(\nu_\mathrm{i}\,r^2) \end{matrix}\right.$$

- **2-body int. matrix elements** $\leftarrow$ **Fourier transform.**

$\Rightarrow$ **solve HF/HFB eq. as generalized eigenvalue problem** $\Rightarrow$ **iteration**

Notes: 1) GEM bases $\cdots$ non-orthogonal
2) ρ-dep. int. ($\cdots$ cannot be stored) $\rightarrow$ zero-range form

Advantages :

- **efficient description of ε-dep. exponential & oscillatory asymptotics**

  $\leftarrow$ **superposition of multi-range Gaussians**

- **applicability to various 2-body interactions**

  $\cdots$ **suitable to studying effective int.**

  central, LS, tensor channels

  function form of r — delta, Gauss, Yukawa, *etc.*

- **basis parameters insensitive to nuclide**

  $\cdots$ **suitable to systematic calculations**

  light & heavy nuclei may be handled with a single basis-set

$$\nu_{\mathrm{r}} = \nu_0\, b^{-2\alpha}\,, \quad \begin{cases} \nu_{\mathrm{i}} = 0 & (\alpha = 0, 1, \cdots, 5) \\ \nu_{\mathrm{i}}/\nu_{\mathrm{r}} = \pm\frac{\pi}{2} & (\alpha = 0, 1, 2) \end{cases} ;$$

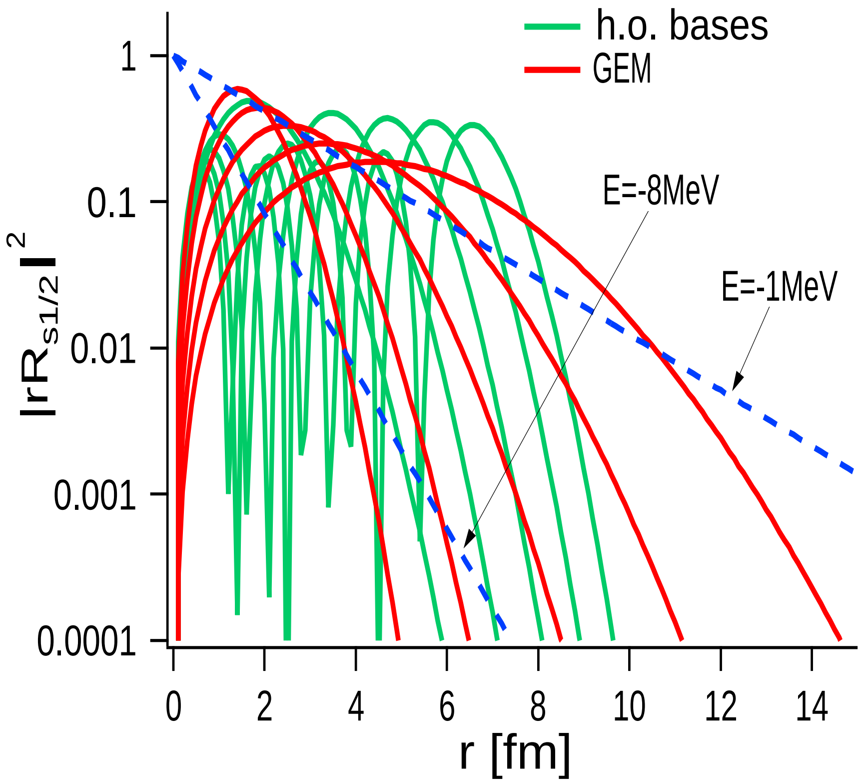
$$\nu_0 = (2.4\,\mathrm{fm})^{-2}\,, \quad b = 1.25$$

(→ **12 bases for each** (ℓ, j))

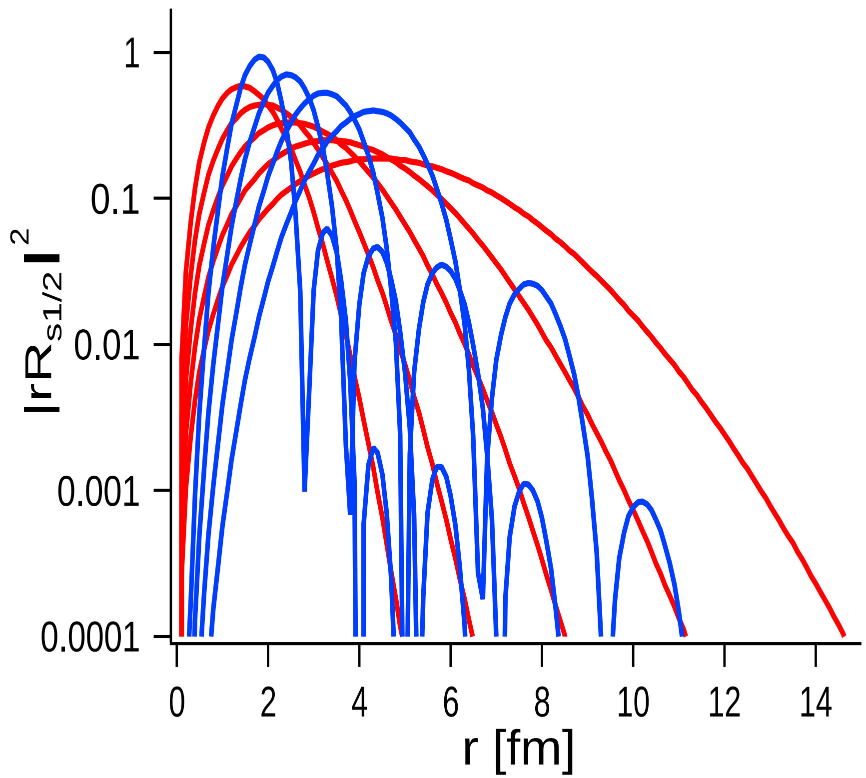
- **exact treatment of Coulomb & c.m. Hamiltonian**

★ **Behavior of GEM bases — $s_{1/2}$ orbits**

real bases :



real & complex bases :



$(\nu_\mathrm{i}/\nu_\mathrm{r} = \pi/2)$

★ **Numerical tests** — **mainly with D1S int.**

- E & $\rho(r)$ **of doubly-magic nuclei in spherical HF (D1S)**

Ref. : H.N., N.P.A 808, 47 ('08)

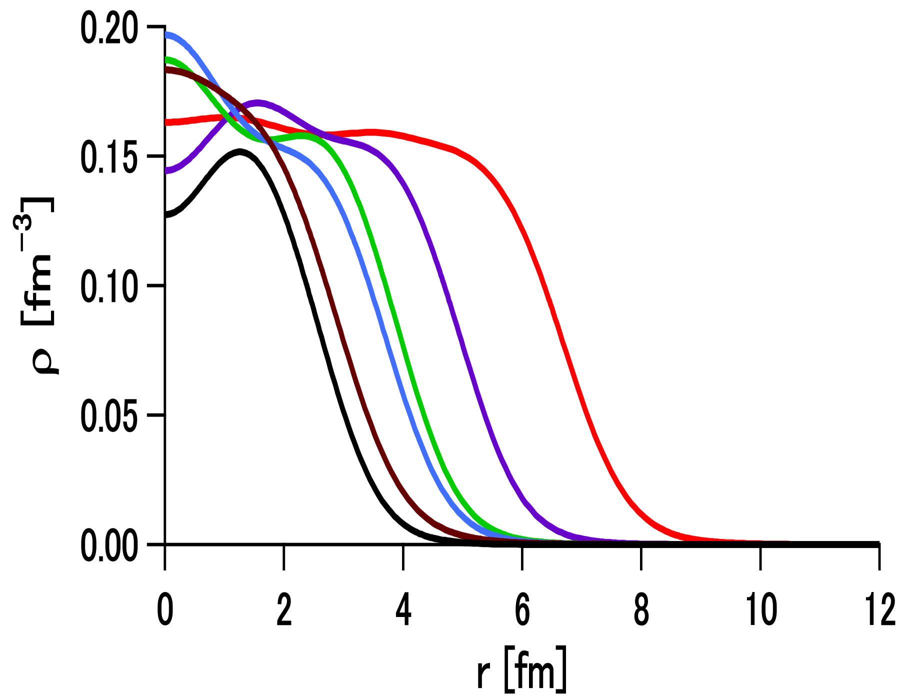
Binding energy $-E\,[\mathrm{MeV}]$ **:**

$\cdots$ **GEM** ***vs.*** **h.o.**

$(N_{\mathrm{osc}} \leq 15,\ \omega_0 = 41.2\,A^{-1/3})$

nuclide	HO	GEM
^{16}O	129.638	129.520
^{24}O	168.573	168.598
^{40}Ca	344.470	344.570
^{48}Ca	416.567	416.764
^{90}Zr	785.126	785.928
^{208}Pb	1638.094	1639.047

$\rho(r)$ **obtained by GEM from** ^{16}**O to** ^{208}**Pb :**

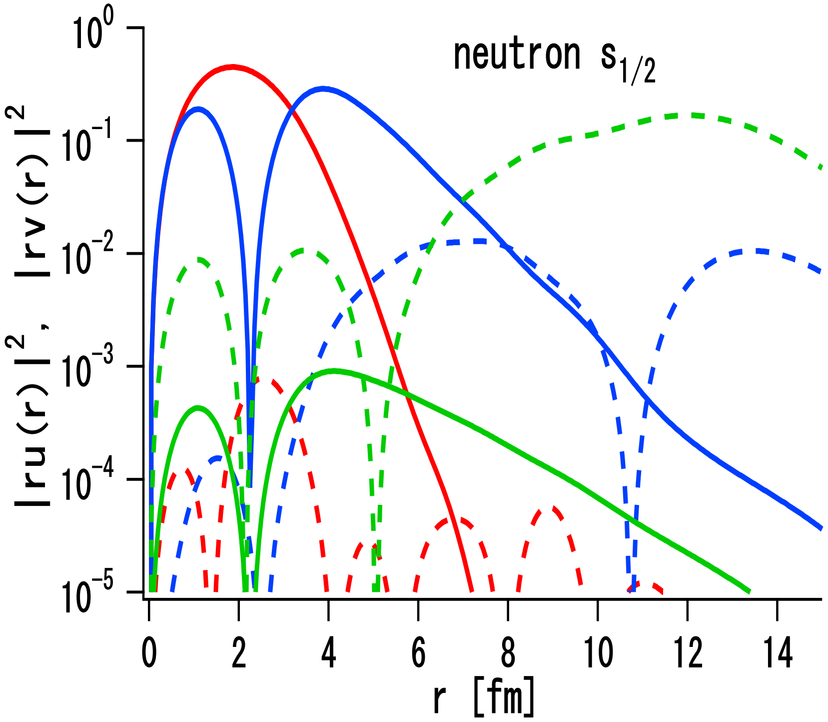


⇒ **wide mass range of nuclei well described by a single GEM basis-set !**

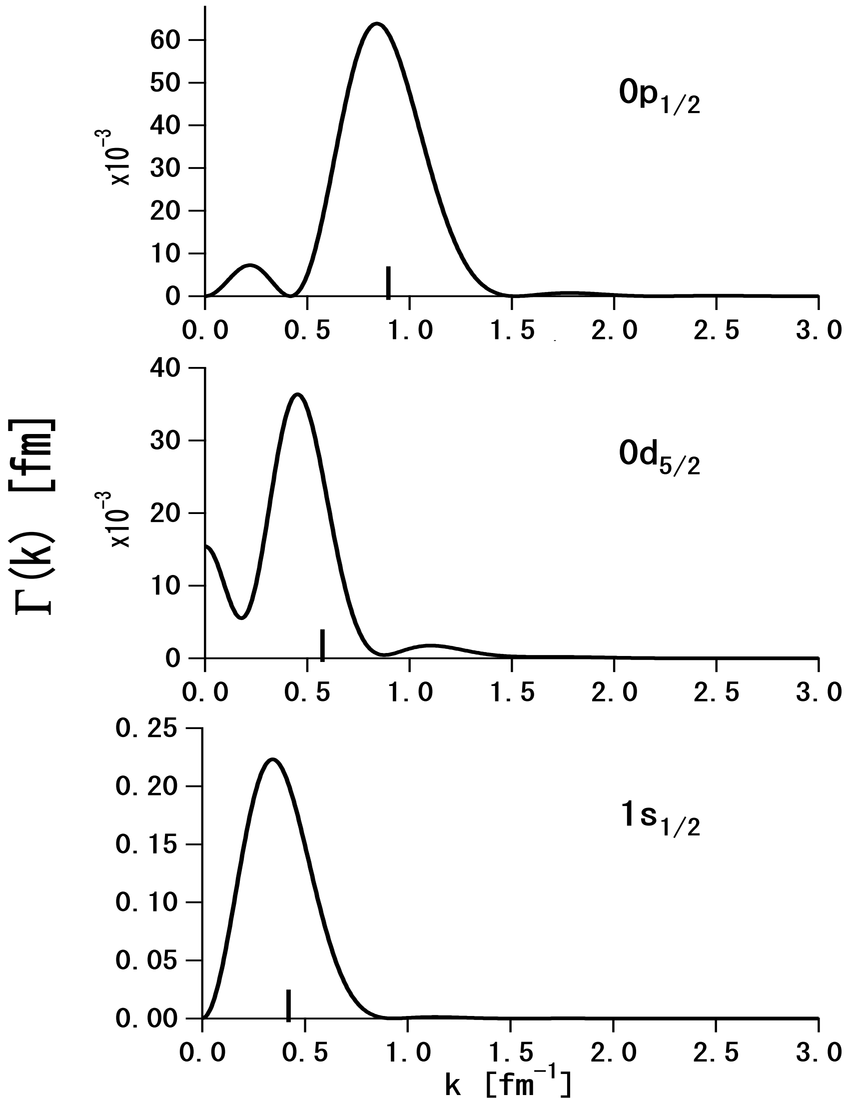
- **S.p. w.f. in spherical HFB (D1S)**

Ref.: H.N., N.P.A 764, 117 ('06); 801, 169 ('08)

neutron $s_{1/2}$ levels in ^{26}O :



Fourier transform of $r\,u_j(r)$:



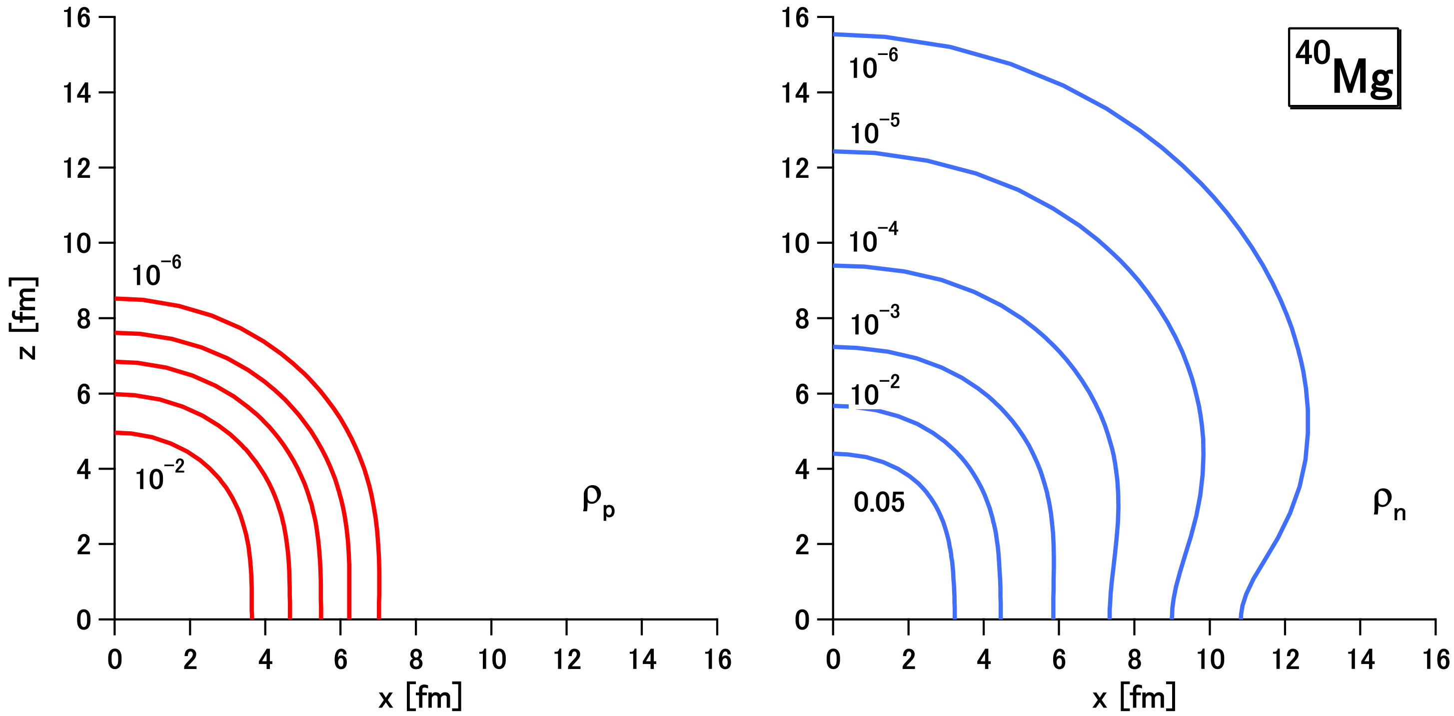
$\cdots$ ε-dep. exp. (& osc.) asymptotics described reasonably well

- **Axial HFB by spherical GEM bases (D1S)** **Ref.: H.N., N.P.A 808, 47 ('08)**

  E $\cdots$ **GEM *vs.* HO** ($N_{\rm osc} \leq 10$ **results** $\leftarrow$ **PLB 474, 15 ('00)**)

Nuclide	HO	GEM	Exp.
^{30}Mg	– 239.30	– 239.48	– 241.63
^{32}Mg	– 248.22	– 248.30	– 249.69
^{34}Mg	−252.82	−254.01	– 256.59

$\rho(\boldsymbol{r})$ **of** ^{40}**Mg:**

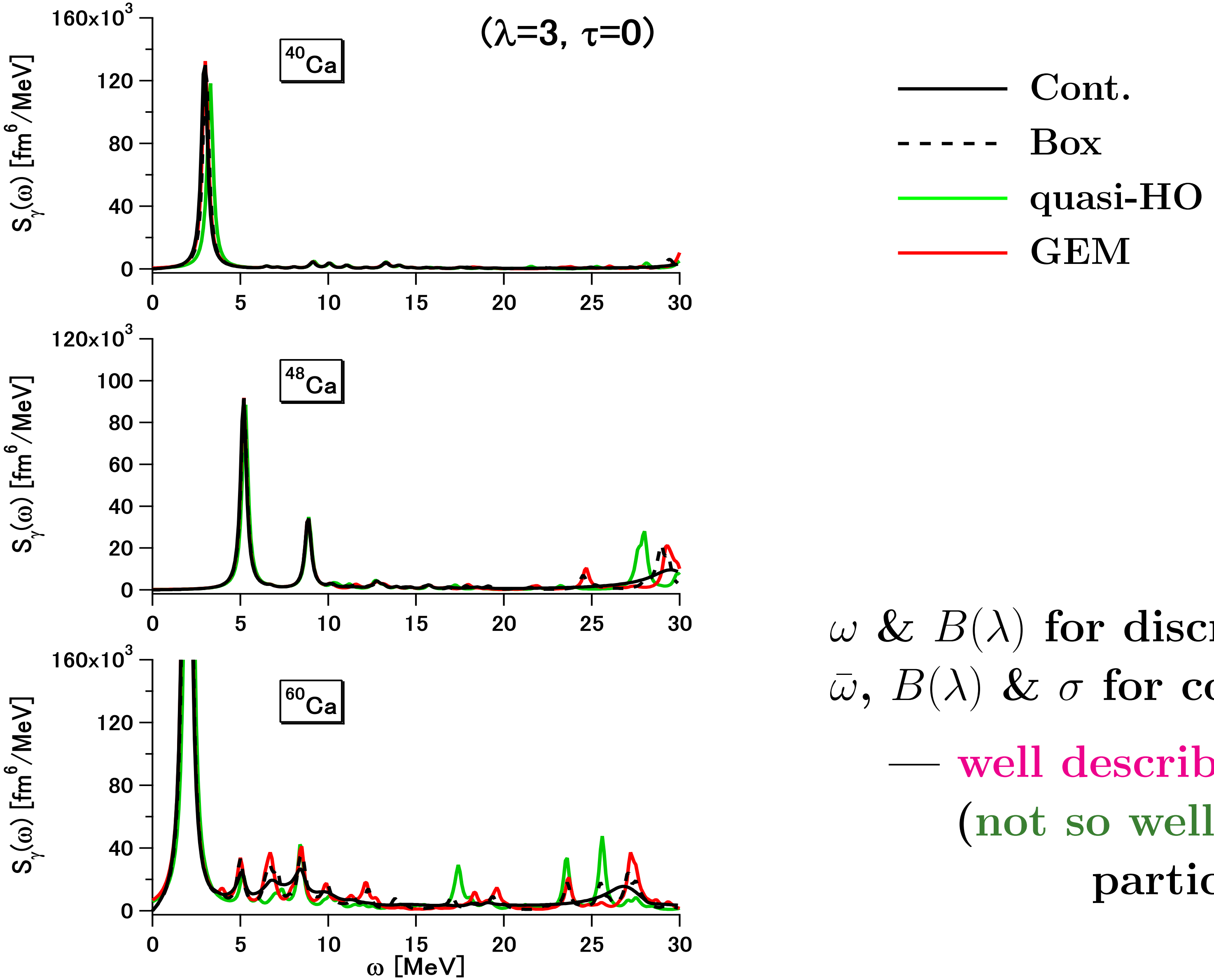


RPA calculations with GEM

Ref.: H.N. *et al.* N.P.A 828, 283 ('09)

★ Numerical tests

Strength function ← WS pot. + Shlomo-Bertsch int.



ω & $B(\lambda)$ for discrete states
$\bar{\omega}$, $B(\lambda)$ & σ for continuum

— well described by GEM
(not so well by quasi-HO, particularly for ^{60}Ca)

HF + RPA (D1S)

spurious c.m. mode:

nuclide	$\omega_s^2\,[\mathrm{MeV}^2]$
^{40}Ca	-5.80×10^{-6}
^{48}Ca	-8.61×10^{-6}
^{60}Ca	-2.67×10^{-6}

EWSR:

$$\mathcal{R}^{(\lambda,\tau)} := \frac{\sum_\alpha \omega_\alpha \left|\langle\alpha|\mathcal{O}^{(\lambda,\tau)}|0\rangle\right|^2}{\frac{1}{2}\langle 0|[\mathcal{O}^{(\lambda,\tau)\dagger},[H,\mathcal{O}^{(\lambda,\tau)}]]|0\rangle} = 1\,\textbf{?}$$

nuclide	$\mathcal{R}^{(\lambda=2,\tau=0)}$	$\mathcal{R}^{(\lambda=3,\tau=0)}$
^{40}Ca	1.005	1.031
^{48}Ca	1.006	1.033
^{60}Ca	1.003	1.010

III. Semi-realistic NN interaction

"**minimal modification**" of realistic force $\leftrightarrow$ saturation, *etc.*

$\Rightarrow$ aiming at $\left\{ \begin{array}{l} \text{higher predictive power than} \\ \text{as wide applicability as} \end{array} \right\}$

conventional MF calculations & their extensions

M3Y int. $\cdots$ Yukawa function $\to$ fit to G-matrix

- OPEP $\to$ longest part of $\hat{v}_{ij}^{(\mathrm{C})}$ $(\equiv \hat{v}_{\mathrm{OPEP}}^{(\mathrm{C})})$
- popular in reaction problems
- no saturation (without modification) $\to$ add $\hat{v}_{ij}^{(\mathrm{DD})}$

'M3Y-P5′' (partly 'M3Y-P5')

Ref.: H.N. P.R.C 78, 054301 ('08); 81, 027301 ('10)

- modifying M3Y-Paris $\left\{ \begin{array}{l} \text{replace short-range part of } \hat{v}^{(\mathrm{C})} \text{ by } \hat{v}^{(\mathrm{DD})} \\ \text{enhance } \hat{v}^{(\mathrm{LS})} \quad (\leftrightarrow \ell s \text{ splitting}) \end{array} \right.$
- keeping $\hat{v}_{\mathrm{OPEP}}^{(\mathrm{C})}$
- no change for $\hat{v}_{ij}^{(\mathrm{TN})}$ from M3Y-Paris (— realistic tensor force)

$\leftrightarrow$ leading order of chiral dynamics

M3Y-type semi-realistic interaction

$$\hat{v}_{ij} = \hat{v}_{ij}^{(\mathrm{C})} + \hat{v}_{ij}^{(\mathrm{LS})} + \hat{v}_{ij}^{(\mathrm{TN})} + \hat{v}_{ij}^{(\mathrm{DD})}\,; \qquad \textbf{(rotational \& translational inv.)}$$

$$\hat{v}_{ij}^{(\mathrm{C})} = \sum_n \left(t_n^{(\mathrm{SE})} P_{\mathrm{SE}} + t_n^{(\mathrm{TE})} P_{\mathrm{TE}} + t_n^{\mathrm{SO}} P_{\mathrm{SO}} + t_n^{(\mathrm{TO})} P_{\mathrm{TO}}\right) f_n^{(\mathrm{C})}(r_{ij})\,,$$

$$\left(\begin{array}{ll} P_{\mathrm{SE}} \equiv \left(\frac{1-P_\sigma}{2}\right)\left(\frac{1+P_\tau}{2}\right), & P_{\mathrm{TE}} \equiv \left(\frac{1+P_\sigma}{2}\right)\left(\frac{1-P_\tau}{2}\right), \\ P_{\mathrm{SO}} \equiv \left(\frac{1-P_\sigma}{2}\right)\left(\frac{1-P_\tau}{2}\right), & P_{\mathrm{TO}} \equiv \left(\frac{1+P_\sigma}{2}\right)\left(\frac{1+P_\tau}{2}\right) \end{array}\right)$$

$$\hat{v}_{ij}^{(\mathrm{LS})} = \sum_n \left(t_n^{(\mathrm{LSE})} P_{\mathrm{TE}} + t_n^{(\mathrm{LSO})} P_{\mathrm{TO}}\right) f_n^{(\mathrm{LS})}(r_{ij})\, \boldsymbol{L}_{ij} \cdot (\boldsymbol{s}_i + \boldsymbol{s}_j)\,,$$

$$\hat{v}_{ij}^{(\mathrm{TN})} = \sum_n \left(t_n^{(\mathrm{TNE})} P_{\mathrm{TE}} + t_n^{(\mathrm{TNO})} P_{\mathrm{TO}}\right) f_n^{(\mathrm{TN})}(r_{ij})\, r_{ij}^2 S_{ij}$$

$$\hat{v}_{ij}^{(\mathrm{DD})} = \left(t_\rho^{(\mathrm{SE})} P_{\mathrm{SE}}\, C_1[\rho(\boldsymbol{r}_i)] + t_\rho^{(\mathrm{TE})} P_{\mathrm{TE}}\, C_0[\rho(\boldsymbol{r}_i)]\right) \delta(\boldsymbol{r}_{ij})\,;$$

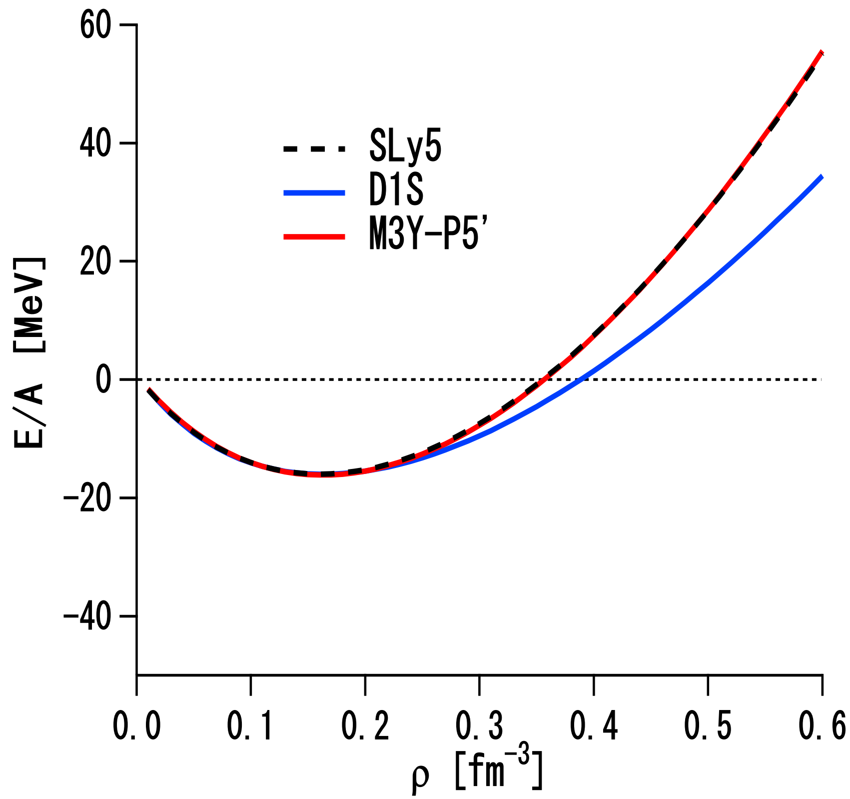
$$\boldsymbol{L}_{ij} \equiv (\boldsymbol{r}_i - \boldsymbol{r}_j) \times \frac{(\boldsymbol{p}_i - \boldsymbol{p}_j)}{2}\,, \qquad S_{ij} \equiv 3(\boldsymbol{\sigma}_i \cdot \hat{\boldsymbol{r}}_{ij})(\boldsymbol{\sigma}_j \cdot \hat{\boldsymbol{r}}_{ij}) - \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j$$

$$f_n(r) = e^{-\mu_n r}/\mu_n r\,, \quad C_T[\rho] = \rho^{\alpha_T} \quad (\alpha_1 = 1\,,\ \alpha_0 = 1/3 \text{ in M3Y-P5}')$$

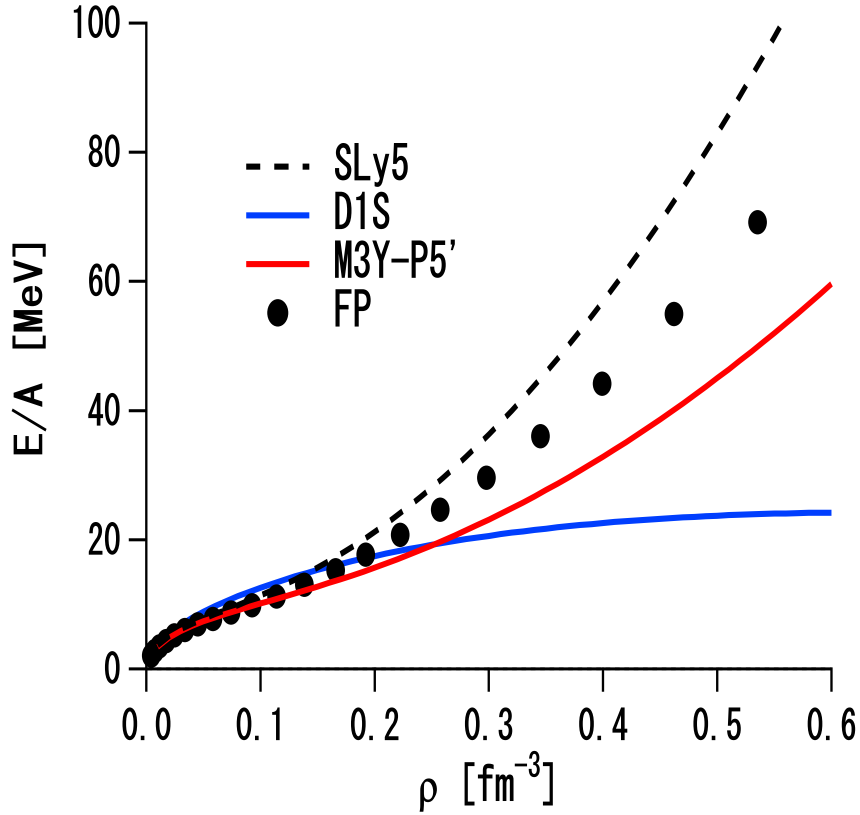
★ Nuclear matter properties

- "Equation of state"

symmetric nuclear matter



neutron matter



- **Comparison of nuclear matter properties**

		SLy5	D1S	M3Y-P5′	Exp.
$k_{\mathrm{F}0}$	[fm^{-1}]	1.334	1.342	1.340	$1.32-1.37$
$\mathcal{E}_0$	[MeV]	-15.98	-16.01	-16.14	≈ -16
$\mathcal{K}$	[MeV]	229.9	202.9	239.1	$220-250$
M_0^*/M		0.697	0.697	0.637	$0.6-0.8$
a_t	[MeV]	32.03	31.12	28.42	≈ 30

- **enhancement factor for $E1$ energy-weighted sum:**

$$1+\kappa := \frac{\sum_\alpha \omega_\alpha \left|\langle\alpha|\hat{T}^{(E1)}|0\rangle\right|^2}{(\textbf{TRK sum rule})}$$

- **spin & spin-isospin properties → Landau-Migdal parameters:**

$$\hat{v}_{\rm res} \approx N_0^{-1}\sum_\ell \left[f_\ell + f'_\ell\,(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j) + g_\ell\,(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j) + g'_\ell\,(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j)(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j)\right]\,P_\ell(\hat{\boldsymbol{k}}_i\cdot\hat{\boldsymbol{k}}_j)$$

$$\left(\frac{1}{2}\frac{\partial^2\mathcal{E}}{\partial\eta_s^2}\bigg|_0 = \frac{k_{\rm F0}^2}{6M_0^*}(1+g_0)\,,\quad \frac{1}{2}\frac{\partial^2\mathcal{E}}{\partial\eta_{st}^2}\bigg|_0 = \frac{k_{\rm F0}^2}{6M_0^*}(1+g'_0)\right)$$

	SLy5	**D1S**	**M3Y-P5′**	$(\hat{v}_{\rm OPEP}^{\rm (C)})$	**Exp.**
κ	0.250	0.660	0.884		$\gtrsim$ 0.7(?)
g_0	1.123	0.466	0.216	(0.075)	$\lesssim$ 0.5 ?
g_1	0.253	−0.184	0.255	(0.092)	—
g'_0	−0.141	0.631	1.007	(0.504)	0.8−1.2
g'_1	1.043	0.610	0.146	(−0.031)	—

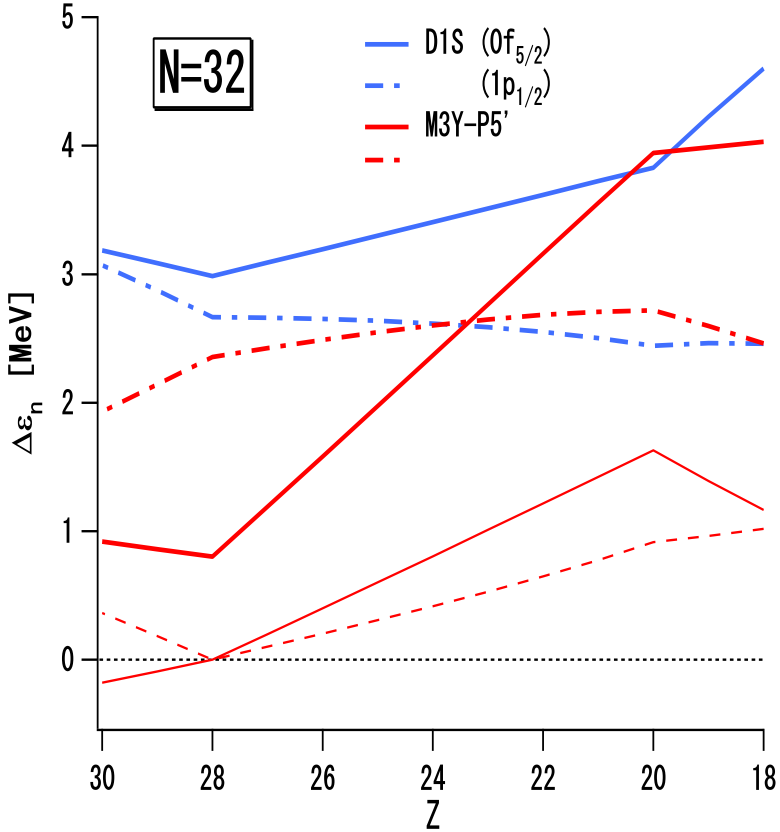
★ **Energies & matter radii of doubly magic nuclei :**

		SLy5	**D1S**	**M3Y-P5′**	**CCSD**	**Exp.**
^{16}**O**	$-E$	128.6	129.5	124.1	107.5	127.6
	$\sqrt{\langle r^2 \rangle}$	2.59	2.61	2.60	—	2.61
^{40}**Ca**	$-E$	344.3	344.6	331.7	308.8	342.1
	$\sqrt{\langle r^2 \rangle}$	3.29	3.37	3.37	—	3.47
^{48}**Ca**	$-E$	416.0	416.8	411.5	355.2	416.0
	$\sqrt{\langle r^2 \rangle}$	3.44	3.51	3.51	—	3.57
^{90}**Zr**	$-E$	782.4	785.9	775.7	—	783.9
	$\sqrt{\langle r^2 \rangle}$	4.22	4.24	4.23	—	4.32
^{208}**Pb**	$-E$	1635.2	1639.0	1635.7	—	1636.4
	$\sqrt{\langle r^2 \rangle}$	5.52	5.51	5.51	—	5.49

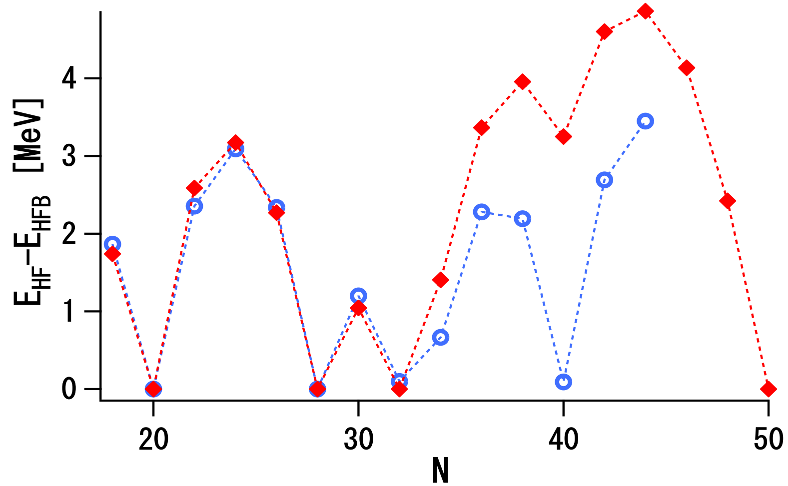
CCSD ··· **G. Hagen *et al.*, PRL 101, 092502 ('08)**

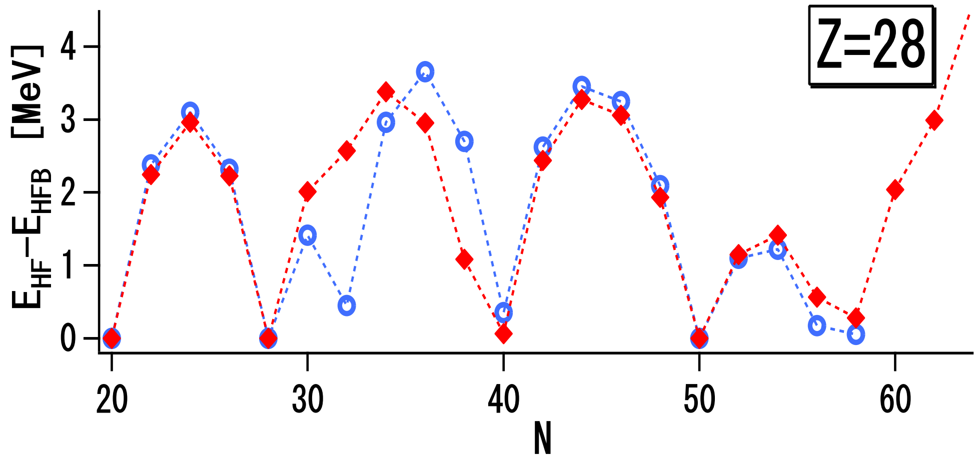
(chiral N^{3}LO without 3NF)

★ S.p. energies (“shell evolution”) — n shell structure in Ca to Ni region



$$\Delta\varepsilon_n(j) = \varepsilon_n(j) - \varepsilon_n(1p_{3/2})$$
$$(j = 0f_{5/2},\, 1p_{1/2})$$





- $N=32$ $\cdots$ $\approx$ **open at ^{60}Ni, $\approx$ closed at ^{52}Ca** $\leftrightarrow$ $\hat{v}^{(\mathrm{TN})}$ & $\hat{v}^{(\mathrm{C})}_{\mathrm{OPEP}}$

  ($N=34$ $\cdots$ $\approx$ **open at ^{54}Ca**)
- $N=40$ $\cdots$ $\approx$ **closed at ^{68}Ni, could be open at ^{60}Ca**
- $N=58$ $\cdots$ $\approx$ **closed at ^{86}Ni ($1d_{5/2}$ & $2s_{1/2}$ occupied)**

★ HF + RPA — $M1$ excitations in ^{208}Pb

			M3Y-P5		Exp.
			$\hat{v} - \hat{v}^{(\mathrm{TN})}$	$\hat{v}$	
"IS"	E_x	(MeV)	6.87	5.85	5.85
	$B(M1)\uparrow$	(μ_N^2)	4.7	2.4	2.0
"IV"	E_x	(MeV)	9.2 – 10.9	9.2 – 10.9	7.1 – 8.7
	$(\bar{E}_x)$		(9.9)	(9.6)	
	$\sum B(M1)\uparrow$	(μ_N^2)	16.3	19.4	16.3 **or** 18.2

($\hat{T}^{(M1)}$ ··· **including corrections from $2p$-$2h$ & MEC**)

··· **role of tensor force reconfirmed**

Ref. : T. Shizuma *et al.*, P.R.C 78, 061303(R) ('08)

(effects of 2-body correlations ? — { IS ··· weak; IV ··· strong)

V. Summary

★ **Application of Gaussian expansion method** $\cdots$ **useful !**

Advantages of the method

- **(i)** **efficient description of ε-dep. asymptotics**
- **(ii)** **applicability to various 2-body interactions**
- **(iii)** **basis parameters insensitive to nuclide**
- **(iv)** **exact treatment of Coulomb & c.m. Hamiltonian**

★ **Semi-realistic interaction**

MF & RPA calculations $\rightarrow$ promising

gross properties
detailed structure (to some degree) } **of nuclei are well described**

(wide applicability & predictive power ?)

$\cdots$ **steps toward 'realistic nuclear mean fields'**

Future prospect

- **2-body correlations** → **GCM, shell model, TDDM, *etc.***
- **more applications (*e.g.* to unstable nuclei)**
- **MF calculations with fully microscopic int.?** ··· **not yet practical**

  ↔ **precise microscopic understanding of saturation**