Spatial Wilson loops in high-energy heavy-ion collisions

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Collisions of heavy ions at high energies provide opportunity to study non-linear dynamics of strong QCD color fields. The field of a very dense system of color charges at rapidities far from the source is determined by the classical Yang-Mills equations with a recoilless current along the light cone. It consists of gluons characterized by a transverse momentum $p_T$ on the order of the density of valence charges per unit transverse area $Q_s^2$; this saturation momentum scale separates the regime of non-linear color field interactions at $p_T \lesssim Q_s$ or distances $r \gg \frac{1}{Q_s}$ from the perturbative regime at $p_T \gg Q_s$.

Right after the impact strong longitudinal chromomagnetic fields $B_\perp \sim 1/g$ develop due to the fact that the individual projectile and target fields do not commute. They fluctuate according to the random local color charge densities of the valence sources. Here we show that magnetic loops

$$W_M(R) = \frac{1}{N_c} \left\langle \text{tr} \mathcal{P} \exp \left( \frac{i g}{\pi} \oint \! dx^i A^i \right) \right\rangle$$

effectively exhibit area law scaling, $W_M(R) \sim e^{-\sigma \pi R^2}$, and we compute the magnetic string tension $\sigma$. Furthermore, we argue that at length scales $\sim 1/Q_s$ the field configurations might be viewed as uncorrelated $Z(N)$ vortices. We also compare to the expectation value of the $Z(N_c)$ part of the loop; thus, for two colors we compute

$$W_M^{Z(2)}(R) = \left\langle \text{sgn} \, \text{tr} \mathcal{P} \exp \left( \frac{i g}{\pi} \oint \! dx^i A^i \right) \right\rangle$$

where sgn() denotes the sign function.

The field in the forward light cone immediately after a collision, at proper time $\tau \equiv \sqrt{T^2 - z^2} \to +0$, is given by $A^i = \alpha^i_1 + \alpha^i_2$. In turn, before the collision the individual fields of projectile and target are 2d pure gauges,

$$\alpha^i_m = \frac{i}{g} U_m \frac{\partial}{\partial x} U_m^\dagger, \quad \partial^i \alpha^i_m = g \rho_m,$$

where $m = 1, 2$ labels projectile and target, respectively, and $U_m$ are SU(N) matrices. Note that for a non-Abelian gauge group, the sum $A^i$ of two pure gauges is not a pure gauge, so $W_M \neq 1$.

The large-$x$ valence charge density $\rho$ is a random variable. For a large nucleus, the effective action describing color charge fluctuations is quadratic, $S_{\text{eff}} = \rho^2(x)\rho^2(x)/2\mu^2$. The variance of charge fluctuation determines the saturation scale $Q_s^2 \sim g^4\mu^2$. The brackets in eq. (1) denote an average over the fluctuating color charges $\rho_1(x), \rho_2(x)$ of the two charge sheets corresponding to projectile and target, respectively.

Fig. 1. Expectation value $W_M$ of the magnetic flux loop right after a collision of two nuclei (time $\tau = +0$) as a function of its area $A' \equiv A Q_s^2$. Symbols show numerical results for $SU(2)$ Yang-Mills on a $4096^2$ lattice; the lattice spacing is set by $g^2\mu_L = 0.0661$. The lines represent fits over the range $4 \geq A' \geq 2$.

In fig. 1 we show numerical results for $W_M$ immediately after a collision. It exhibits area law behavior for loops larger than $A \gtrsim 2/Q_s^2$. The corresponding “magnetic string tension” is $\sigma M/Q_s^2 = 0.12(1)$. The area law indicates uncorrelated magnetic flux fluctuations through the Wilson loop and that the area of magnetic vortices is rather small, their radius being on the order of $R_{\text{vtx}} \sim 0.8/Q_s$. We do not observe a breakdown of the area law up to $A \sim 4/Q_s^2$, implying that vortex correlations are small at such distance scales. Also, restricting to the $Z(2)$ part reduces the magnetic flux through small loops but $\sigma M$ is comparable to the full $SU(2)$ result.

References