Improved estimate of neutral B meson mixing in static limit of b quark with AMA technique

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The Cabibbo–Kobayashi–Maskawa (CKM) matrix plays a key role in elementary particle physics, and constraints on the elements V_{ts} and V_{td} can be obtained from $B^0 - \overline{B^0}$ mixing. Treatments of b quark on the lattice QCD are, however, challenging because of a multi-scale problem where a large hierarchy in mass exists between light quarks (u and d) and b quark. A solution to this problem is using Heavy Quark Effective Theory (HQET), in which the theory is expanded by $1/m_b$, where m_b is b quark mass. The leading order of the HQET is a static approximation of the b quark and is the formulation used in this work. While the static approximation is known to have $O(\Lambda_{\mathrm{QCD}}/m_b) \sim 10\%$ uncertainty, it is useful for an interpolation strategy, where the physical b quark mass point (~ 4.2 GeV) is reached by the interpolation between the static limit and simulations in the lower quark mass region (c quark mass region). For this purpose, high precision calculations in the static limit are required.

RBC/UKQCD Collaboration has worked on such calculations for several years [1)2)3)]. While we have successfully improved the estimate, the statistical error remains a major part of the total uncertainty. Recently, an efficient method called All-Mode-Averaging (AMA)[4)] has been proposed for significantly reducing the statistical error. The AMA technique is an operator improvement using symmetries on the action and approximations. The most useful symmetry is translational invariance of space-time. The idea involves locating many source points in the measurement, but using an approximation in obtaining quark propagators to reduce computational cost. As an approximation, we use a sloppy CG, where its stopping condition is relaxed. A crucial point here is the existence of correlation between the original and the approximated operators; thus, we choose the stopping condition so that the good correlation is kept, but the computational cost is reduced significantly. We also note that a low-mode deflation technique helps the cost reduction in addition to enhancement of the correlation in the AMA procedure.

In Fig. 1, we show chiral and continuum extrapolation of the B meson decay constant f_B and the neutral B meson mixing matrix element $\mathcal{M}_B$ without and with AMA. We use gluon ensembles with two lattice spacings, which are depicted as "24c" (coarse) and "32c" (fine). "HYP1" and "HYP2" represent link smearings in the static b action, whose results should coincide each other in the continuum limit. To obtain the AMA results approximately 30% – 40% cost increase is required from the original one ("before AMA" in the figures). A remarkable improvement in statistics is obtained compared with the small increase in the cost. At the physical point of light quark mass and in the continuum limit, the central values only slightly move by using the AMA; the statistical errors are, however, significantly reduced. In addition, the improvement makes a qualitative fact clear: the results with HYP2 smearing have very small scaling violation. We conclude that the AMA technique has potential to enable us to reach statistically decisive results.

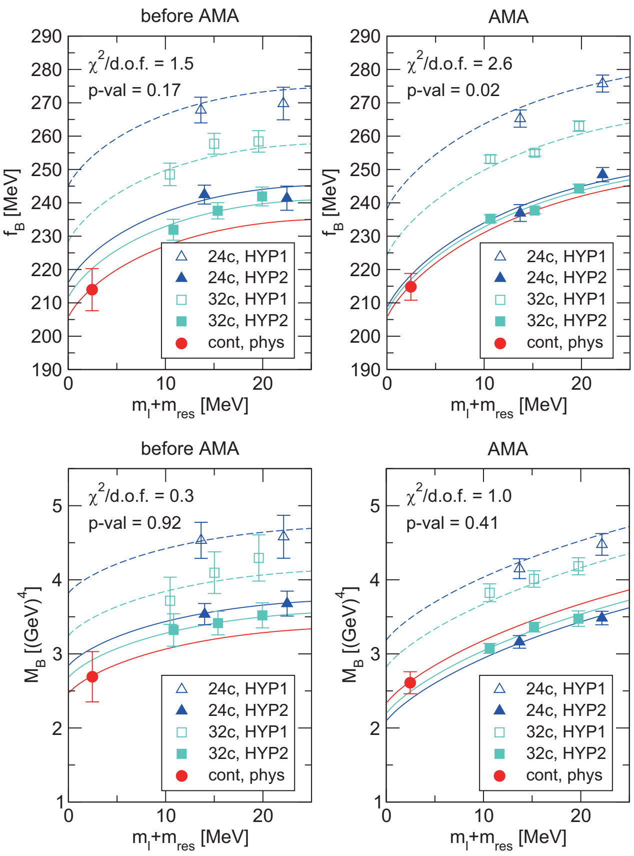

Fig. 1. Chiral and continuum extrapolation of f_B and $\mathcal{M}_B$.

References
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