The Application of Gaussian Process Regression to Background Spectrum Modeling at PHENIX

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Often in nuclear and particle physics, we need to estimate the area under a peak which sits over an oddly shaped background (see figure 1). Occasionally we are able to perform an analytical first principles calculation to calculate a shape for the background, but more often we are faced with little or no information about what the shape of the background should be. Faced with this, we typically choose a polynomial and fit this functional form to the background. The choice of this polynomial and its fitting leaves an unquantified uncertainty.

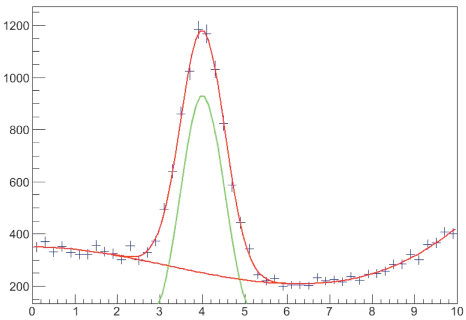

Fig. 1. A sample background (sampled from a known third order polynomial) and Gaussian peak spectrum.

Gaussian processes[1,2] are a mathematical concept that allow for a method of data regression and predictive functional modeling using a minimal set of prior assumptions. A nice feature of this method is that simultaneously with the predictions, uncertainties are provided.

Gaussian processes are a specific type of stochastic process where the variance of each of the random variables comprising the process is Gaussian. An important feature of stochastic processes is that, mathematically, they sample over the space of functions similar to how a random variable samples over a set space of possible outcomes. In defining the variances to be Gaussian, we've narrowed down the space of possible functions and simplified the math needed to specify the process. Specifically, with this requirement of Gaussian uncertainties, the expectation of the process can be defined entirely in terms of a mean function and a covariance function similar to how the Gaussian distribution is defined purely by a mean and a variance.

The specific background spectrum that we are interested in applying this technique to is the background sitting beneath the Jacobian peak, in $W^{\pm}$ production in p+p collisions[4]. This is a steeply falling spectrum that isn't modeled well by using power laws or exponentials. This particular problem requires the extension of the Gaussian process technique to multiscale problems, which required transforming the data[3] and applying the Gaussian process regression in this warped space. Our current application of Gaussian processes to modeling this spectrum can be seen in figure 2.

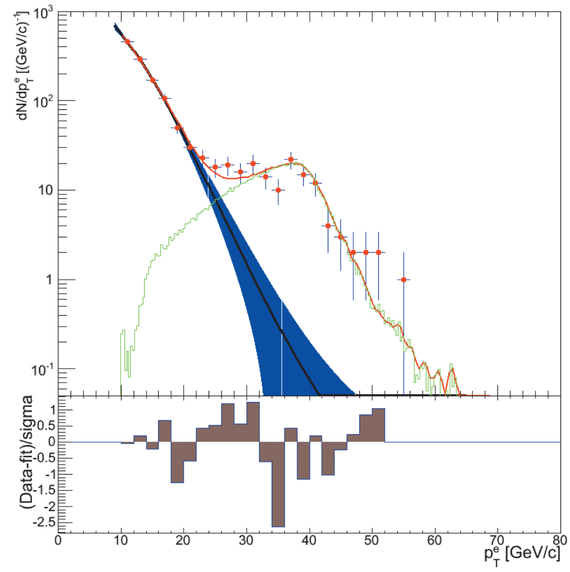

Fig. 2. The W Jacobian peak and background spectra. The black line and blue band represent the Gaussian process best fit and uncertainty band.

The uncertainty band currently encompasses a large space of functions, many of which we don't expect to be physical (e.g. an undulating, falling spectrum). We are currently working to add shape constraints to the Gaussian process modeling by sampling individual, though coarsely grained, functions from the constrained space and then accepting or rejecting those functions based on their individual shapes.

References
1) MacKay, David J. C.: *Information Theory, Inference, and Learning Algorithms* (Cambridge University Press, 2003)
2) Rasmussen, C. E. & Williams, K. I.: *Gaussian Processes for Machine Learning* (The MIT Press, 2006)
3) Snelson, E., Rasmussen, C. E., Ghahramani, Z.: *Warped Gaussian Processes*, NIPS, 2003.
4) Adare, et al. : Phys.Rev.Lett. 106 (2011) 062001

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