Chiral Magnetic and Vortical Effects at Weak Coupling

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Chiral magnetic and vortical effects are parity-odd transport phenomena in the hydrodynamics of a plasma of chiral massless fermions, which are macroscopic manifestations of the underlying microscopic chiral anomaly. For the plasma of a single right-handed Weyl fermion, the chiral magnetic effect dictates a current along an applied external magnetic field $\vec{B}$ as $\vec{J} = \frac{\mu}{4\pi^2}\vec{B}$ where μ is the chemical potential, and in the case of chiral vortical effect, the vorticity of fluid $\vec{\omega} = (1/2)\vec{\nabla} \times \vec{v}$ plays a role similar to the magnetic field, $\vec{J} = (\mu^2 + \pi^2 T^2/3)\vec{\omega}$. These effects are robust due to the topological nature of chiral anomaly, and should persist both in weak and strong coupling limits. Demonstrating the expected universality of these phenomena in weak and strong coupling limits is an interesting and non-trivial test of the topological nature of chiral anomaly. While the strong coupling limit provided by AdS/CFT correspondence has successfully confirmed the universality of these effects, the weak coupling picture, albeit more intuitive, contains more subtleties that need to be carefully taken into accounts.

In general, one can define the chiral magnetic conductivity[1)] $\sigma_\chi(\omega, k)$ by $\vec{J} = \sigma_\chi(\omega, k)\vec{B}(\omega, k)$ where (ω, k) are frequency and longitudinal momentum of the in-homogeneous time-dependent magnetic field $\vec{B}(\omega, k)$. One expects the zero frequency-momentum limit of $\sigma_\chi(\omega, k)$ to reproduce the topologically robust result of chiral magnetic effect, $\lim_{k\to 0}\lim_{\omega\to 0}\sigma_\chi(\omega, k) = \frac{\mu}{4\pi^2}$. In the interaction-free limit, both diagrammatic[1)] and kinetic[2)] approaches give a result for $\sigma_\chi(\omega, k)$ such that $\lim_{k\to 0}\lim_{\omega\to 0}\sigma_\chi(\omega, k) \neq \lim_{\omega\to 0}\lim_{k\to 0}\sigma_\chi(\omega, k) = \frac{1}{3}\cdot\frac{\mu}{4\pi^2}$, while a hydrodynamic argument indicates that there should not be such a difference between the two limits[3)]. Since hydrodynamic regime exists only in an interacting theory, it is natural to study this issue in an interacting theory going beyond the non-interacting limit. In Ref.[3)] we showed in both kinetic and diagrammatic approaches that the above difference between two limits disappears in the presence of relaxation dynamics caused by interactions, confirming the expectation from the hydrodynamics argument.

It is interesting to understand how chiral magnetic and vortical effects arise in the quasi-particle picture of kinetic theory of chiral fermions. The essential ingredient that is responsible for chiral anomaly is the Berry's curvature in momentum space[4,5)], $\vec{b} = \vec{\nabla}_p \times \mathcal{A}_p = \vec{p}/2|\vec{p}|^3$, where $\mathcal{A}_p$ is the Berry's connection of momentum-dependent chiral spinors. We showed in Ref.[6)] that the quasi-particle energy in the presence of magnetic field is dictated to be $\epsilon(\vec{p}) = |\vec{p}| - \vec{B}\cdot\vec{p}/2|\vec{p}|^2$ by Lorentz invariance where the second term is a spin-magnetic moment interaction. This term brings about several interesting consequences. Since the equilibrium distribution is $f^{eq}(\vec{p}) = 1/(\exp[\beta\epsilon(\vec{p})] + 1)$ the distribution becomes distorted by the magnetic field, which causes a net current along the magnetic field. This effect turns out to explain 1/3 of the full chiral magnetic effect. On the other hand, the equation of motions of quasi-particles with the Berry's curvature is given by[5)] $\sqrt{G}\dot{\vec{x}} = \partial\epsilon/\partial\vec{p} + (\vec{b}\cdot\partial\epsilon/\partial\vec{p})\vec{B}$ where $\sqrt{G} = 1 + \vec{b}\cdot\vec{B}$ is the modified phase space density. The second term proportional to $\vec{B}$ induces a net current along the magnetic field even with spherically symmetric equilibrium distribution. This contribution gives the rest 2/3 of the full chiral magnetic effect.

Interestingly, a similar feature also exists in the chiral vortical effect[6)]. With the fluid vorticity $\vec{\omega}$, a detailed valance in the Boltzmann equation with conservation of angular momentum $\vec{L} = \vec{x}\times\vec{p} + (1/2)\vec{p}/|\vec{p}|$, where the second term is the spin angular momentum, dictates the equilibrium distribution to be $f^{eq} = 1/(\exp[\beta(\epsilon - \frac{\vec{\omega}\cdot\vec{p}}{2|\vec{p}|})] + 1)$, which induces a net current proportional to $\vec{\omega}$: the result comprises 1/3 of the full chiral vortical effect. From the spin-magnetic moment interaction term in the energy, $-\vec{B}\cdot\vec{p}/2|\vec{p}|^2$, one can obtain a contribution to the current by the variation of the quasi-particle action with respect to eternal gauge potential, the result of which takes a form $\Delta\vec{J} = \int d^3\vec{p}/(2\pi)^3\,(\vec{\nabla}f \times \vec{p}/(2|\vec{p}|^2))$. With distribution function $f = f^{eq}(\epsilon - \vec{v}\cdot\vec{p})$, the resulting current along $\vec{\omega}$ constitutes the rest 2/3 of the chiral vortical effect.

In space-time dimensions higher than four, there are generalizations of chiral magnetic and vortical effects. A weak coupling computation of them in real-time formalism was performed in Ref.[7)], which successfully confirmed the hydrodynamics prediction in Ref.[8)]

References

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