Chiral magnetic superconductivity†

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Materials with charged chiral quasiparticles in external parallel electric and magnetic fields can support an electric current that grows linearly in time, corresponding to diverging DC conductivity. From experimental viewpoint, this “Chiral Magnetic Superconductivity” (CMS) is thus analogous to conventional superconductivity. However the underlying physics is entirely different – the CMS does not require a condensate of Cooper pairs breaking the gauge degeneracy, and is thus not accompanied by Meissner effect. Instead, it owes its existence to the (temperature-independent) quantum chiral anomaly and the conservation of chirality. As a result, this phenomenon can be expected to survive to much higher temperatures. Even though the chirality of quasiparticles is not strictly conserved in real materials, the chiral magnetic superconductivity should still exhibit itself in AC measurements at frequencies larger than the chirality-flipping rate, and in microstructures of Dirac and Weyl semimetals with thickness below the mean chirality-flipping length that is about 1 – 100 μm. In nuclear physics, the CMS should contribute to the charge-dependent elliptic flow in heavy ion collisions.

Let us start with a brief recap of “conventional” superconductivity. Instead of microscopic BCS theory introducing the condensate of Cooper pairs, we will base our discussion on the approach proposed in 1935 by Fritz and Heinz London. The London equations describe electromagnetic response of a superconductor and can be derived from the following bold assumption about the proportionality between the electric current \( J \) and the vector gauge potential \( A \):

\[
J = -\mu^2 A, \tag{1}
\]

where a phenomenological constant \( \mu \) is related to the density of superconducting carriers \( n_s \), electron mass \( m \) and electric charge \( e \) by \( \mu = (e^2 n_s/m)^{1/2} \).

In Coulomb gauge, the electric field is \( E = -\dot{A} \) and the magnetic field is \( B = \nabla \times A \); therefore, the assumption (1) yields the following two equations of superconductor electrodynamics:

\[
\dot{J} = \mu^2 E, \tag{2}
\]

and

\[
\nabla \times J = -\mu^2 B. \tag{3}
\]

The first of these equations (2) tells us that a constant external electric field \( E \) generates inside a superconductor a transient state with an electric current that grows linearly in time. Once the electric field is turned off (or screened away), Eq. (2) tells us that the current \( J \) is not allowed to decay and should persist – this is superconductivity.

To make an analogy with superconductivity, let us consider a chiral material (i.e. a material that hosts massless charged chiral fermions, such as a Dirac or Weyl semimetal, or a quark-gluon plasma) in external parallel magnetic and electric fields. First, let us assume that the chirality of fermions is strictly conserved. The chiral anomaly of quantum electrodynamics then dictates that the electric and magnetic fields generate the chiral current charge density \( \rho_5 \) with the rate given by

\[
\frac{d\rho_5}{dt} = \frac{e^2}{4\pi^2 \hbar c} E \cdot B, \tag{4}
\]

and the chiral current charge density grows linearly in time:

\[
\rho_5 = \frac{e^2}{4\pi^2 \hbar c} E \cdot B \ t. \tag{5}
\]

Let us now relate the density of the chiral charge \( \rho_5 \) and the chiral chemical potential \( \mu_5 \) by introducing \( \chi \equiv \partial \rho_5 / \partial \mu_5 \), so that \( \rho_5 = \chi \mu_5 + ... \) and \( \mu_5 \simeq \chi^{-1} \rho_5 \) for small \( \mu_5 \); note that at late times when \( \mu_5 \) grows larger than other scales in the system, this relation will eventually be violated.

For \( E \| B \) we now get

\[
J = \frac{e^2}{2\pi^2} \mu_5 B = \frac{e^4}{8\pi^2 \hbar^2 c} \chi^{-1} B^2 E \ t \equiv \mu_{CME}^2 \ E \ t. \tag{6}
\]

Taking the time derivative on both sides, we get

\[
\dot{J} = \mu_{CME}^2 E, \tag{7}
\]

which is completely analogous to the corresponding expression for superconductivity (2)! We thus see that the CMS\(^{1,2} \) in an ideal chiral material can be viewed as a new type of superconductivity – we will call it Chiral Magnetic Superconductivity (CMS). In particular, if the electric field is switched off, the CME current according to (7) persists. In real experiments, the current will eventually decay, but on a time scale set by the chirality-flipping transitions that is much longer than a typical transport time.

References