Determination of $|V_{us}|$ from inclusive τ decay with lattice HVPs[†]

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Precise deteminations of the CKM matrix element $|V_{us}|$ are important for flavor physics, which is relevant to the search for new physics beyond the standard model (SM) in particle physics. We propose a new method to determine $|V_{us}|$ from inclusive strange hadronic τ decay data and hadronic vacuum polarization functions (HVPs) computed on the lattice. The basic idea is to consider generalized dispersion relations for the experimentally observed us V+A inclusive distributions. In the SM, the differential distribution of τ decay, $dR_{V/A;us}/ds$, is related to the spectral function $\rho^{(J)}_{us;V/A}(s)$ by

$$\frac{dR_{us;V/A}}{ds} = \frac{12\pi^2|V_{us}|^2 S_{EW}}{m_\tau^2} \times \left[\omega_\tau(s)\rho^{0+1}_{us;V/A}(s) - \omega_L(y_\tau)\rho^0_{us;V/A}(s)\right], \quad (1)$$

where $y_\tau = s/m_\tau^2$, $\omega_\tau(y) = (1-y)^2(1+2y)$, $\omega_L(y) = 2y(1-y)^2$, and S_{EW} is a known short-distance electroweak correction. Using a weight function $\omega_N(s) = \frac{1}{\Pi_{k=1}^N (s+Q_k^2)}$ $(s = -Q_k^2 < 0)$, we obtain the following dispersion relation,

$$\int_0^\infty \rho_{us}(s)\omega_N(s)ds = \sum_k^N \mathrm{Res}\left(\Pi_{us}(-Q_k^2)\omega_N(-Q_k^2)\right) \quad (2)$$

Here, $\Pi_{us}(Q^2)$ is the HVP defined in space-like momentum $-Q^2$. The idea is to use the lattice QCD data for $\Pi_{us}(Q^2)$ on the RHS in Eq. (2) to compare with the LHS, which can be evaluated up to $s = m_\tau^2$ from the experimental $dR_{us;V+A}/ds$ distribution with an unknown $|V_{us}|^2$. The pQCD result is used to approximate the contribution from $s > m_\tau^2$. We use $\omega_N(s)$ with uniformly spaced poles, which is characterized by N, and the center value of Q^2, C in the interval, and the nearest-neighbor spacing is fixed to $\Delta = 0.2/(N-1)$ [GeV^2]. In Fig. 1, we plot some example values for spectrum contributions given in the LHS of Eq. (2). Increasing N or decreasing C can suppress the contributions from high-s modes that have larger errors, so that we can adjust the “inclusiveness”, that is, the impact of the high multiplicity modes.

For the RHS in Eq. (2), we use the lattice HVPs measured near the physical quark masses on the $2+1$ flavor Mobius domain wall fermion ensembles by the RBC and UKQCD collaborations.[a)] Here, $\Pi(Q^2)$ is obtained from the Fourier transformation of the lattice HVPs ($\Pi(t)$) in the coordinate space.

† Condensed from a paper by authors in preparation

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a) We thank RBC-UKQCD collaboration and Kim Maltman for providing lattice HVP and experimental data.

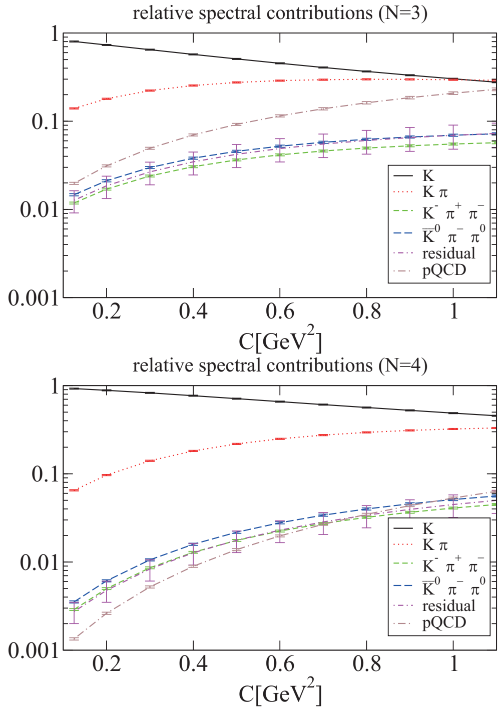

Fig. 1. Ratio of each spectrum integral.

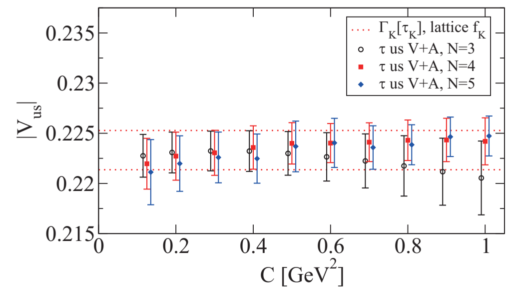

Fig. 2. Result of $|V_{us}|$ as a function of C for $N = 3, 4$, and 5. The result obtained from K is also shown.

Figure 2 shows our result for $|V_{us}|$. Our best value is $|V_{us}| = 0.2241(14)_{exp}(13)_{th}$, from the result with $C = 0.7$ [GeV^2] with $N = 4$. The theoretical error is comparable with the experimental one, and the total error is less than the previous inclusive τ decay determinations.[b)] Moreover, the result is consistent with those obtained from analyses of kaon physics and 3-family CKM unitarities.

Reference

1) P. A. Boyle et al., Int. J. Mod. Phys. Conf. Ser. **35**, 1460441 (2014) doi:10.1142/S2010194514604414 [arXiv:1312.1716 [hep-ph]].

b) For a recent study of the inclusive τ decay by using the finite energy sum rule, see Ref. 1.