Improved treatment of blocking effect at finite temperature†

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In odd nuclei with odd numbers of nucleon (neutron or proton), due to the Pauli principle, the odd particle at zero temperature ($T = 0$) completely blocks the level k which it occupies. Consequently, this level k remains unpaired with the occupation number equal to 1. Within the approximation such as the BCS theory, this level is excluded from the BCS equation for the pairing gap, whereas the particle-number equation is solved only for the even core. The resulting approximation is called the "blocked BCS"[1]. At finite temperature $T \neq 0$, the odd particle has the probabilities of occupying all the levels above the Fermi surface. As the result, any observable is then obtained by taking the average of all the above probabilities over the statistical ensembles such as the grand canonical ensemble (GCE) or canonical ensemble (CE). Several approximations have been proposed to describe the properties of odd nuclei at $T \neq 0$. One of the approximations, which is a direct extension of the blocked BCS and is widely used in nuclear physics, was proposed by Maino *et al.*[2]. This approach assumes that at $T \neq 0$, the odd particle can occupy the level k with the unity occupation number ($q_k = 1$) as at $T = 0$, but k can be any level above the Fermi surface, not only $k = k_0$ with k_0 being the lowest level located just above the Fermi level. The approach, called "Maino's" hereafter, then takes the average of any observable as the sum of all its average values taken for each blocked level k with the statistical weight as the ratio of the grand partition function for the blocked level k to the total grand partition function. The assumption of the unity occupation probability of the odd level in the Maino's approach means that temperatures have no effect on it, whereas the occupation numbers of all the levels in the pairing core are obtained by averaging over the GCE, and, they are therefore temperature-dependent. An obvious inconsistency of such assumption is that it fails to reproduce the zero-pairing limit of the equation for the particle number at finite temperature, where all the single particle occupation numbers should follow the Fermi-Dirac distribution of noninteracting fermions.

Present paper proposes an improved treatment of the blocking effect in odd nuclear systems interacting via the constant pairing force. Based on the analysis of the exact solutions of the doubly-degenerate equidistant multilevel pairing model for $N = 9$ particles and $\Omega = 10$, 14, and 20 levels whose the single-particle energies are $\epsilon_k = k$ ($k = 1, 2, ..., \Omega$), we show that the exact occupation number of the blocked level occupied by the odd particle is equal to 1 at $T = 0$ and then decreases with increasing T (See e.g., thick solid lines in Fig. 1). As the result, the conventional assumption of the unity occupation number for this blocked level is no longer valid at $T \neq 0$. Instead, we introduce the temperature-dependent occupation number q_k for the blocked level, which is directly derived from the standard equation for the particle number within the standard finite-temperature BCS [Eqs. (13), (23), and (24) of Ref.[3]]. We also construct the average q_k-blocked BCS [Eqs. (13), (23), and (25) of Ref.[3]], which follows Maino's idea and the results obtained agree well with those obtained within the Mainos approach. However, again by using the exact solutions of the pairing Hamiltonian, we demonstrate that the average procedure over all the levels above the Fermi surface as proposed in the Mainos approach does not correspond to the real situation observed in the exact solutions, where the odd particle actually always remains on the top level $k = k_0$. Based on the analysis of the results obtained in the present paper, we believe that, in the study of the odd systems with pairing, such as atomic nuclei, at finite temperature, if the BCS approach with blocking is ever applied, the q_k-blocked BCS with $k = k_0$ proposed here should be used instead of the blocked BCS, whereas neither the Maino's approach nor the average q_k-blocked BCS is necessary.

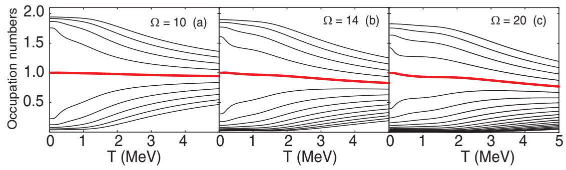

Fig. 1. Exact occupation numbers of the levels as functions of T obtained within the doubly-degenerate equidistant multilevel pairing model for $N = 9$ particles and $\Omega = 10$ (a), 14 (b), and 20 (c) levels. In each panel, the higher-located line is the occupation number of the lower-located level. The thick line, which starts from 1 at $T = 0$, is the occupation number of the level occupied by the odd particle.

References
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† Condensed from the article in Phys. Rev. C 94, 024341 (2016)
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