P-wave $\pi\pi$ scattering and the ρ resonance from lattice QCD[†]

C. Alexandrou,[*1,*2] L. Leskovec,[*3] S. Meinel,[*3,*4] J. Negele,[*5] S. Paul,[*2] M. Petschlies,[*6] A. Pochinsky,[*5] G. Rendon,[*3] and S. Syritsyn[*7,*4]

The study of hadron-hadron scattering and resonance properties using lattice QCD is a rapidly growing field.[1)] There are now many calculations of $I = 1$, P-wave $\pi\pi$ scattering, in which the ρ resonance appears, but a number of questions remain open. For example: Which models best describe the energy-dependence of the phase shift? How large is the nonresonant contribution? How exactly do m_ρ and $g_{\rho\pi\pi}$ depend on the quark masses?

In this work, we have begun to address some of these questions using a high-statistics calculation with $2+1$ flavors of clover fermions at a pion mass of approximately 320 MeV. The lattice size was $32^3 \times 96$ with a lattice spacing of $a \approx 0.114$ fm; we constructed the relevant hadronic correlation functions using a method based on forward, sequential, and stochastic quark propagators, which has a favorable volume scaling compared with the distillation method introduced in Ref. 2). We extracted the lowest two or three energy levels in eight different irreducible representations with total momenta up to $(1,1,1)\frac{2\pi}{L}$, carefully studying systematic uncertainties associated with the choice of fit method and fit range in Euclidean time. The $\pi\pi$ scattering phase shift values obtained from the lattice energy levels using Lüscher's method are shown in Fig. 1.

We performed fits of several different models for the $\sqrt{s}$-dependence of the phase shift: two purely resonant Breit-Wigner models (without or with a Blatt-Weisskopf barrier factor), as well as the combination of these models with three different parameterizations of a nonresonant contribution. The fit results for the nonresonant contribution were consistent with zero. We found that the minimal Breit-Wigner model

$$\delta_\ell(s) = \arctan\frac{\sqrt{s}\,\Gamma(s)}{m_\rho^2 - s}, \qquad \Gamma(s) = \frac{g_{\rho\pi\pi}^2}{6\pi}\frac{k^3}{s}, \tag{1}$$

which depends only on the two parameters m_ρ and $g_{\rho\pi\pi}$, was sufficient to describe our data. The curve corresponding to this model is also shown in Fig. 1.

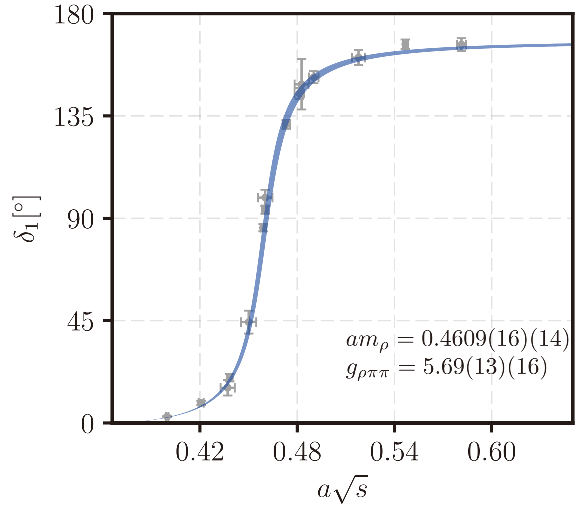

Fig. 1. Lattice QCD results for the $I = 1$, P-wave $\pi\pi$ scattering phase shift as a function of the $\pi\pi$ invariant mass. The data points were obtained by applying Lüscher's method individually to each energy level, while the curve was obtained by fitting the parameters of the minimal Breit-Wigner model directly to the whole energy spectrum.

A comparison of lattice results for m_ρ at several different heavier-than-physical pion masses revealed substantial scale setting ambiguities. It is therefore better to compare dimensionless ratios such as m_ρ/m_N and m_π/m_N, where m_N is the lattice result for the nucleon mass from the same ensemble of gauge configurations. Our calculation gives

$$\frac{m_\rho}{m_N} = 0.7476(38)(23) \quad \text{at} \quad \frac{m_\pi}{m_N} = 0.2968(13), \tag{2}$$

and $g_{\rho\pi\pi} = 5.69(13)(16)$. The most recent lattice results obtained with $2+1$ flavors of clover fermions (Refs. 3–5) along with this work) are consistent with a linear dependence of m_ρ/m_N on m_π/m_N, with m_ρ/m_N reaching the experimental value at the physical pion mass.

† Condensed from the article in Phys. Rev. D **96**, 034525 (2017)
*1 Department of Physics, University of Cyprus
*2 Computation-based Science and Technology Research Center, Cyprus Institute
*3 Department of Physics, University of Arizona
*4 RIKEN Nishina Center
*5 Center for Theoretical Physics, Massachusetts Institute of Technology
*6 Helmholtz-Institut für Strahlen- und Kernphysik, University of Bonn
*7 Department of Physics and Astronomy, Stony Brook University

References

1) R. A. Briceno, J. J. Dudek, R. D. Young, arXiv:1706.06223.
2) M. Peardon *et al.*, (Hadron Spectrum Collaboration), Phys. Rev. D **80**, 054506 (2009).
3) J. J. Dudek *et al.*, (Hadron Spectrum Collaboration), Phys. Rev. D **87**, 034505 (2013).
4) D. J. Wilson, R. A. Briceno, J. J. Dudek, R. G. Edwards, C. E. Thomas, Phys. Rev. D **92**, 094502 (2015).
5) J. Bulava, B. Fahy, B. Hörz, K. J. Juge, C. Morningstar, C. H. Wong, Nucl. Phys. **910**, 842 (2016).