Empirical formulae of the masses of elementary particles

| particle | formula | calculated(c) | measured(m) | |c/m| - |
|----------|---------|---------------|-------------|---------|
| e        | $1/(12\pi^2)\epsilon_0^{1/3}(1+ (1/4)/(1/(6\pi^2)^2))^{-1}M_{pl}$ | 0.511002 MeV | 0.510998946 $\pm$ 0.000000031 MeV | $5.9 \times 10^{-6}$ |
| $\mu$    | $3/2\epsilon_0^{1/3} (1-1/(2\pi) + 3/(4\pi)^2)M_{pl}$ | 105.6594 MeV | 105.6583745 $\pm$ 0.0000024 MeV | $9.6 \times 10^{-6}$ |
| $\tau$   | $9\epsilon_0^{1/3}(1 - 1/(8\pi) + (5/4)/(1/(6\pi^2)^2))^{-1}M_{pl}$ | 1.77684 GeV | 1.77668 $\pm$ 0.12 GeV | $1.9 \times 10^{-5}$ |

The formulae yield the measured mass values well. The mass of a quark is dependent on the mass in terms of the Planck mass $M_{pl}$ and a dimensionless constant $\epsilon_0 = 2 \times (6\pi)^{-1/4}$. There is no adjustable parameter in the formulae. The mass values calculated using the formulae are compared with measured values. For the calculation, the value of Planck mass from CODATA\(^1\) is used:

$$M_{pl} = 1.220910 \pm 0.000029 \times 10^{19} \text{ GeV}.$$ 

The measured values of particle masses are taken from PDG2016\(^2\). The mass of a quark is dependent on the scale and the scheme. In the PDG review, the mass of quarks other than the $t$ quark are given in the $\overline{\text{MS}}$ scheme at $\mu = 2 \text{ GeV}$ for the $u$, $d$, $s$ quarks and at $\mu = m_q$ for the $b$ and $c$ quarks. For the $t$ quark, the measured mass is considered to be the pole mass. The formula for a quark is assumed to yield the $\overline{\text{MS}}$ mass at the $Z$ boson mass $m_Z$. The first-order renormalized group equation (RGE) below is used to correct for the mass value at $m_Z$ to the mass at the scale the PDG uses:

$$\frac{m(t)}{m_0} = \exp \left(-\int \frac{\alpha_s(t)}{\pi} dt \right),$$

where $t = \log \mu^2$ and $\alpha_s(t)$ is the running QCD coupling constant at the scale $\mu$. The value of $\alpha_s(t)$ in the PDG2016 review is used for the calculation.

The formulae, the calculated values ($c$) using the formulae, the measured values ($m$), and difference $|c/m - 1|$ are summarized in the table above. The calculation reproduce the measured mass values well. The agreement is within the uncertainty of the measured mass or the Planck mass ($2.4 \times 10^{-5}$).

There is a pattern in the formulae. The formulae can be summarized as:

$$m_t = \frac{1}{2} N_t (6\pi)^n \epsilon_0^{1/3} (1 + \delta_t)^{-1}M_{pl},$$

$$m_q = 2^n N_q (6\pi)^n \epsilon_0^{1/3} M_{pl},$$

$$m_B = \frac{2^n}{8\pi} \epsilon_0^{1/4} (1 + \delta_B)^{-1}M_{pl},$$

where $m_t$, $m_q$, and $m_B$ are the masses of charged leptons, quarks, and bosons, respectively. $N_t$ and $N_q$ are small positive integers, $n_t$ and $n_q$ are integers ranging from $-2$ to $2$, and $\delta_t$ and $\delta_B$ are small real numbers. $c_b = -1/12$ and $c_q = 0$ for all other quarks, and $c_Z = 0$, $c_W = -1/4$, and $c_H = 1/2$. The pattern suggests the existence of a rule that determines the formulae. Note that all fermion masses are of order $(6\pi)^n \epsilon_0^{1/3}$ with $-2 \leq n_f \leq 2$ and the boson masses are of the order of $1/8\pi^2 \epsilon_0^{1/4}$. Presumably, this part is the main part of the pattern, and $(1 + \delta_t)$ and $(1 + \delta_B)$ are correction factors due to interactions.

Note that the value of $\epsilon_0$ is consistent with the product of the Hubble constant $H_0$ and the Planck time $t_{pl} = 1/M_{pl}$:

$$H_0 \times t_{pl} = (1.211 \pm 0.014) \times 10^{-61}$$

$$\epsilon_0 \equiv 2 \times (6\pi)^{-48} = 1.220608 \times 10^{-61}$$

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Here, the WMAP 9 year value of $H_0$ is used. This suggests that the masses of elementary particles are related to the expansion of space-time.

A theoretical model that can explain these formulae is under development.

References


\(^1\) RIKEN Nishina Center