New interpretation of pairing anti-halo effect†

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The pairing anti-halo effect is a phenomenon by which a pairing correlation suppresses the divergence of nuclear radius, which happens for single-particle states with orbital angular momenta of \( l = 0 \) and 1 in the limit of vanishing binding energy. This phenomenon was originally proposed based on the Hartree-Fock-Bogoliubov (HFB) theory. Although the HFB method provides a clear mathematical interpretation of the pairing anti-halo effect, its physical mechanism is less transparent. The aim of this paper is to propose a more intuitive idea on the pairing anti-halo effect, using a three-body model. This model is formulated to include many-body correlations beyond the HFB model, providing a complementary opportunity to clarify the concept based on the HFB method. It can be used to test whether the pairing anti-halo effect is specific only to the mean-field treatment or not.

The Hamiltonian for the three-body model reads\(^1\)

\[
H = \hat{h}(1) + \hat{h}(2) + v_{\text{pair}}(\vec{r}_1, \vec{r}_2) + \frac{\vec{p}_1 \cdot \vec{p}_2}{m_c},
\]

where \( \hat{h} \) is a single-particle (s.p.) Hamiltonian and \( v_{\text{pair}}(\vec{r}_1, \vec{r}_2) \) is the pairing interaction between the two valence neutrons. The last term is the two-body part of the recoil kinetic energy of the core nucleus.

The eigen-functions \( \psi_{nljm}(\vec{r}) \) of \( \hat{h} \) is given by

\[
\psi_{nljm}(\vec{r}) = \phi_{nlj}(r) Y_{jlm}(\vec{r}) = \frac{u_{nlj}(r)}{r} Y_{jlm}(\vec{r}),
\]

where \( \phi_{nlj}(r) \) and \( Y_{jlm}(\vec{r}) \) are the radial and spin angular parts of the s.p. wave function, respectively. Using these eigen-functions, the two-particle wave function for the ground state of the three-body system with spin-parity of \( J^z = 0^+ \) is given as

\[
\Psi(\vec{r}_1, \vec{r}_2) = \sum_{n,n',l,j} C_{nn'l'j} [\psi_{nlj}(\vec{r}_1)\psi_{n'l'j}(\vec{r}_2)]^J_z = 0, \tag{3}
\]

where the coefficients \( C_{nn'l'j} \) are calculated by diagonalizing the three-body Hamiltonian (1). The one-particle density constructed with this two-particle wave function is given by

\[
\rho(\vec{r}) = \int d\vec{r}' |\Psi(\vec{r}, \vec{r}')|^2 = \frac{1}{4\pi} \sum_{k,l,j} \left| \tilde{u}_{klj}(r) \right|^2, \tag{4}
\]

where \( \tilde{u}_{klj}(r) \) is defined as \( \tilde{u}_{klj}(r) \equiv \sum_n C_{nklij} u_{nlj}(r) \).

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Note that this is in a similar form as the one-particle density in the HFB approximation, especially if the quasi-particle wave function is expanded on the Hartree-Fock basis, \( u_{nlj} \).

The solid line in Fig. 1 shows the radial dependence of the quasi-particle wave function for the weakly bound 2s\(_{1/2}\) state; that is, \( \tilde{u}_{klj}(r) \) with \( (klj) = 2s_{1/2} \) in \(^{24}\text{O}\). The dashed and dot-dashed lines show its decomposition into the bound state and continuum state contributions, respectively. They are defined as

\[
\tilde{u}_{klj}(r) = u_{klj}^{(b)}(r) + \tilde{u}_{klj}^{(c)}(r) = \sum_{n=2s_{1/2}} C_{nklij} u_{nlj}(r) + \sum_{n=\text{cont.}} C_{nklij} u_{nlj}(r). \tag{5}
\]

The main feature of this quasi-particle wave function is that the bound state and continuum state contributions largely cancel each other outside the potential while the two components contribute coherently in the inner region. We recognise that the localization due to a coherent superposition of continuum states is the same mechanism as the formation of a localized wave packet. This is an essential ingredient of the pairing anti-halo effect, that is, the formation of a localized wave packet induced by a pairing interaction.

References