PARTICLE TRACKING SIMULATION WITH SPACE CHARGE EFFECTS FOR AN INDUCTION SYNCHROTRON AND PRELIMINARY APPLICATIONTO THE KEK DIGITAL ACCELERATOR

Xingguang Liu

Tokyo Institute of Technology, Nagatsuda, Kanagawaapan

High Energy Accelerator Research Organization (KEK), Tsukuba, Ibaraki, Japan

Abstract

In order to study the beam behaviour of the induction synchrotron which features low energy injection, a dedicated particle tracking simulation code with a 2.5D space charge field solver, which takes into account of the boundary condition, has been developed. The beam dynamics included in this code are discussed and simulation results assuming parameters of the KEK Digital Accelerator are presented. This code will help to understand the various features of the beam behaviour in the present beam commissioning and serve as a tool for the design of the future induction synchrotrons.

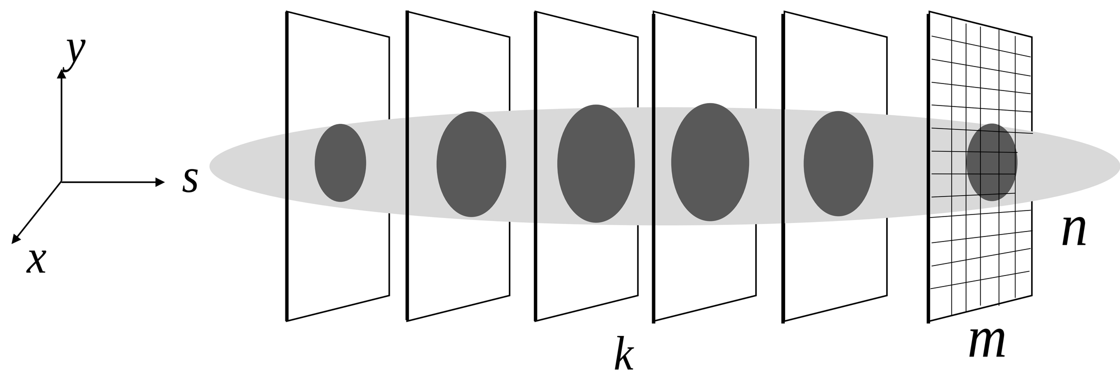
Scheme Design

$$\vec{x} = (x, xp, y, yp, z, dpp)$$

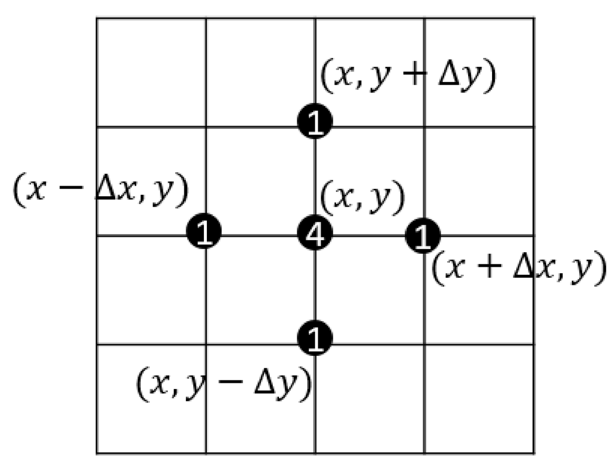
$$x(s) \xrightarrow{K_{sc}} x(s)' \xrightarrow{M} x(s+\Delta s)$$

$$\vec{x}_{sc} = \left(0, \frac{\Delta s \cdot Qe}{\gamma_0^2 p_0 \beta_0 c} E_x, 0, \frac{\Delta s \cdot Qe}{\gamma_0^2 p_0 \beta_0 c} E_y, 0, \frac{\Delta s \cdot Qe}{\beta_0^2 E_{total}} E_z\right)$$

2.5D Space Charge Solver



Finite method for Poisson's equation



$$\nabla \phi = -\frac{\rho}{\varepsilon_0}$$

$$\frac{\partial^2 \phi(x,y)}{\partial x^2} + \frac{\partial^2 \phi(x,y)}{\partial y^2} = -\frac{\rho(x,y)}{\varepsilon_0}$$

$$\begin{cases} \frac{\partial^2 \phi(x,y)}{\partial x^2} = \left(\frac{\phi(x+\Delta x, y) - \phi(x,y)}{\Delta x} - \frac{\phi(x,y) - \phi(x-\Delta x, y)}{\Delta x}\right)/\Delta x \\ \frac{\partial^2 \phi(x,y)}{\partial y^2} = \left(\frac{\phi(x, y+\Delta y) - \phi(x,y)}{\Delta y} - \frac{\phi(x,y) - \phi(x, y-\Delta y)}{\Delta y}\right)/\Delta y \end{cases}$$

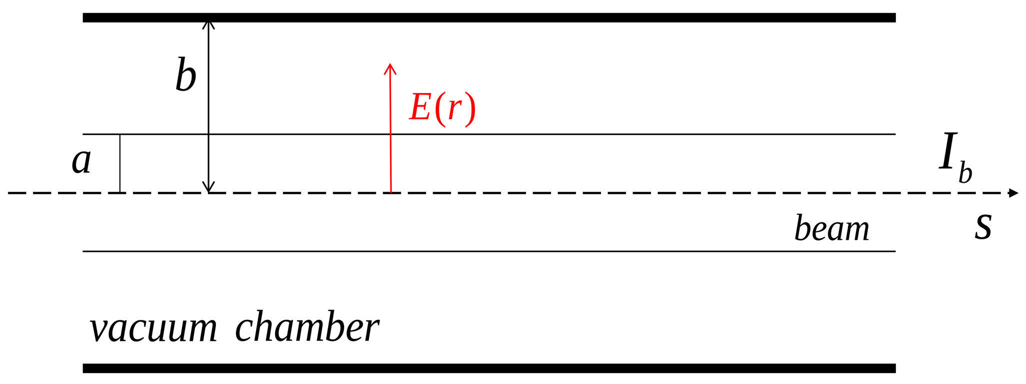
$$\phi(\Delta + x, y) + \phi(x, \Delta + y) - 4\phi(x,y) + \phi(x-\Delta, y) + \phi(x, y-\Delta) = -\frac{\rho(x,y)}{\varepsilon_0}\Delta^2$$

Boundary Matrix

Potential Charge density

$$\begin{pmatrix} -4 & 1 & & & 1 & & & & \\ 1 & -4 & 1 & & & 1 & & \cdots & \\ & 1 & -4 & 1 & & & 1 & & \\ & & 1 & -4 & & & & 1 & \\ & \vdots & & & \ddots & & & \vdots & \\ & & & & & -4 & 1 & & \\ & & & & & 1 & -4 & 1 & \\ & \cdots & & & \cdots & & 1 & -4 & 1 \\ & & & & & & & 1 & -4 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_{n-1} \\ \phi_n \end{pmatrix} = \frac{\Delta^2}{\varepsilon_0} \cdot \begin{pmatrix} \rho_1 \\ \rho_2 \\ \vdots \\ \rho_{n-1} \\ \rho_n \end{pmatrix}$$

2D Justification

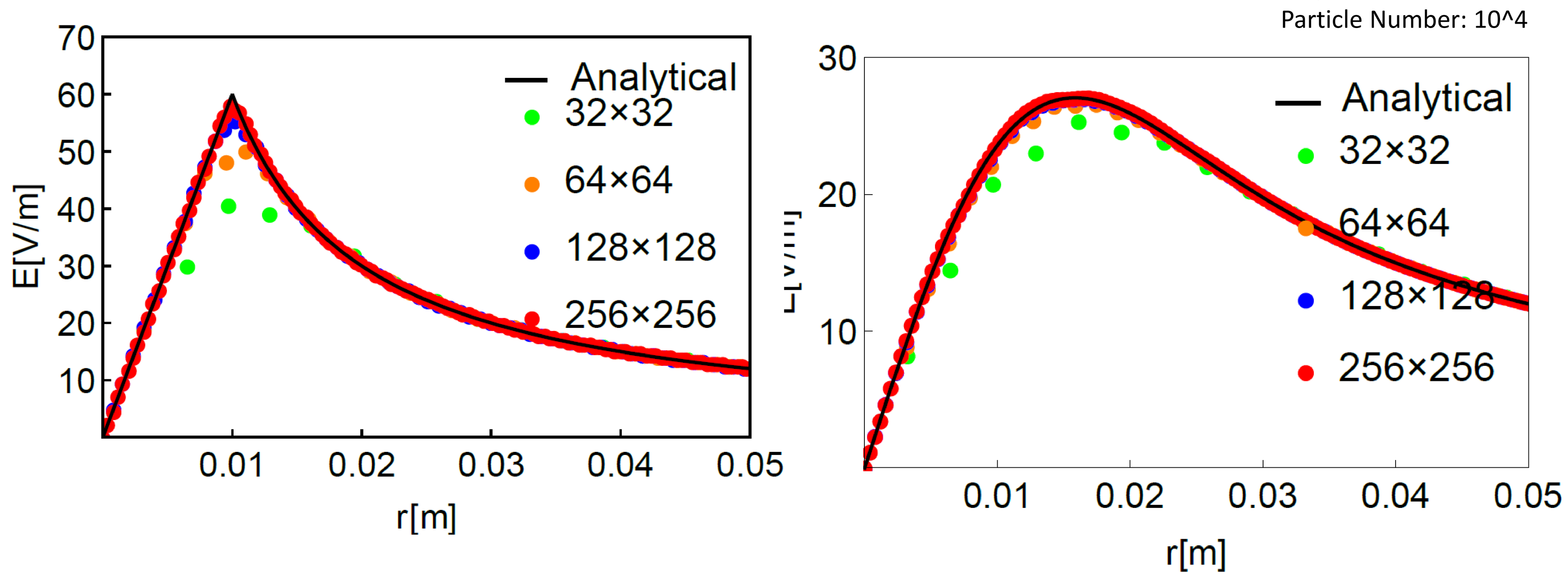


$$\lambda = \frac{I_b}{\beta c}$$

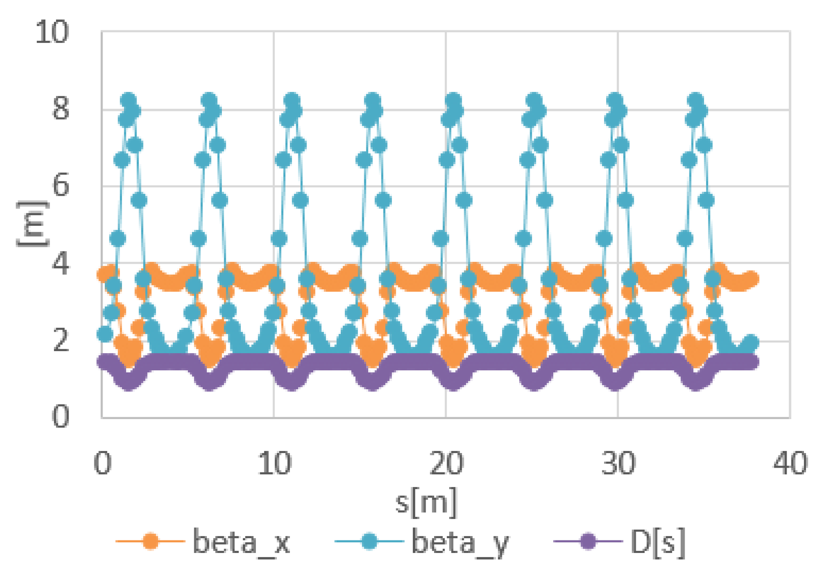
$$E[r] = \begin{cases} \frac{\lambda}{2\pi\varepsilon_0}\frac{r}{a^2}, 0 \le r \le a \\ \frac{\lambda}{2\pi\varepsilon_0}\frac{1}{r}, a \le r \le b \end{cases}$$

$$E[r] = \frac{1 - e^{-\frac{r^2}{2\sigma^2}}}{2\pi r \varepsilon_0}\lambda$$

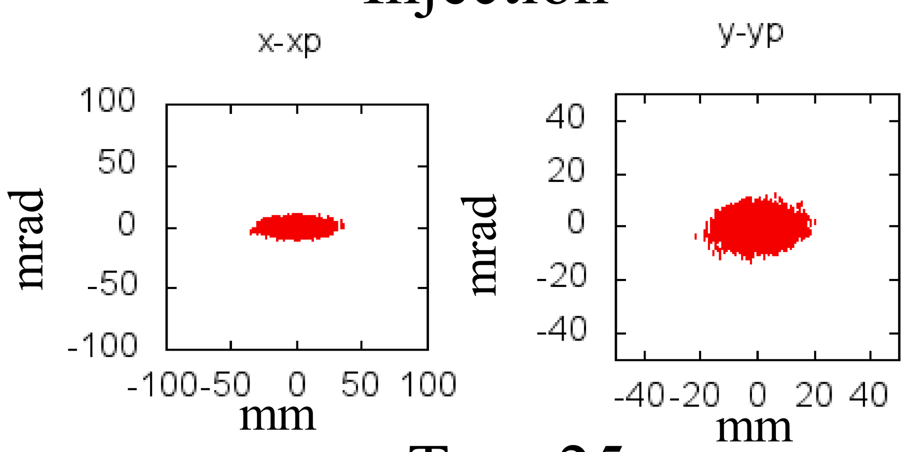
I=100uA
Beta=0.01
a=1cm b=10cm
Step size: 0.2m
Particle Number: 10^4



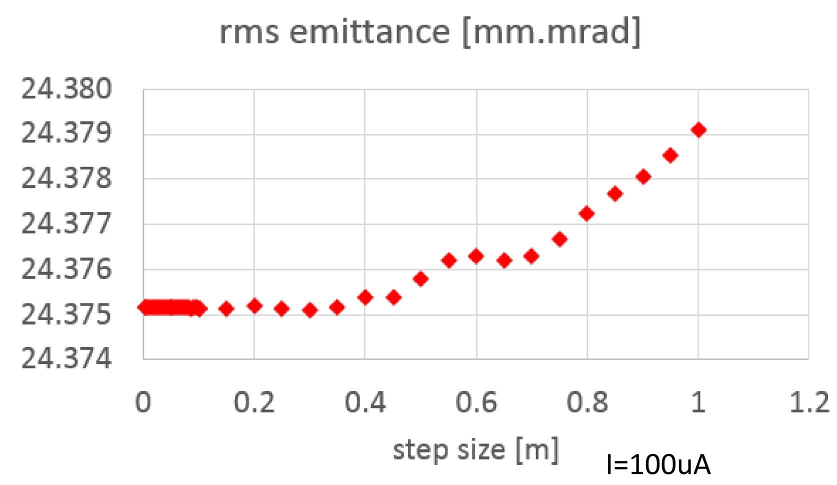
Preliminary Application



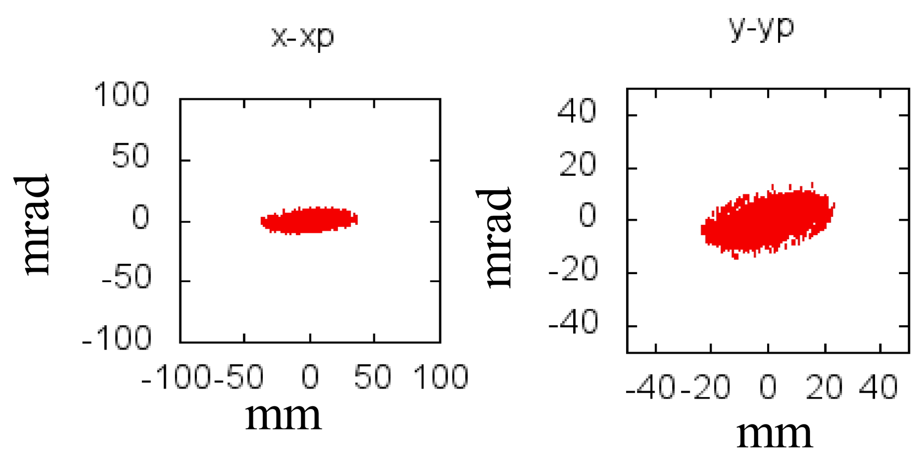
Injection



Turn 25



I=100uA
64*64
Step size: *
Particle Number: 10^4
One turn



I=100uA
64*64
Step size: 0.2m
Particle Number: 10^4

Summary & Future work

- ✓ A general simulation code considering the beam induced space charge fields with boundary conditions for the induction synchrotron.
- ✓ A 2.5D "slice-by-slice" scheme and space charge solver
- ✓ Preliminary application to the KEK-DA.
- ➢ Further study with the simulation to understand the beam behaviour in the KEK-DA ring will be continued and this simulation code can also be applied to the future induction synchrotron's design and study.

高エネルギー加速器研究機構(KEK) HIGH ENERGY ACCELERATOR RESEARCH ORGANIZATION 東京工業大学 TOKYO INSTITUTE OF TECHNOLOGY