Cutoff effects on lattice nuclear forces†

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Nuclear forces, the interactions among nucleons, serve as the cornerstone in nuclear physics. While they have been traditionally determined through the scattering experiments, their theoretical understanding by using the fundamental theory, quantum chromodynamics (QCD), has not been established yet. Recently, a novel approach was proposed to determine nuclear forces on a lattice[1,2]. In this approach, now called the HAL QCD method, nuclear forces are directly obtained from Nambu-Bethe-Salpeter wave functions calculated on the lattice. The method has been successfully extended to general hadron interactions such as three-nucleon forces[3]. See Ref.[4] for a recent review.

For the quantitative determination of nuclear forces, systematic uncertainties in lattice simulations should be carefully examined, such as the effect of discretization artifacts. There have been, however, no work that performs the continuum extrapolation on nuclear interactions. The aim of this work is to perform the first systematic study for the lattice cutoff dependence of nuclear interactions.

We employ $N_f = 2$ configurations with clover fermion generated by CP-PACS collaboration[5]. The measurements are performed at three lattice spacings, $a = 0.2150, 0.1555, 0.1076$ fm. The physical lattice size is $L^3 \times T \simeq (2.5\,\mathrm{fm})^3 \times 5\,\mathrm{fm}$, and the hadron masses are $(m_\pi, m_N) \simeq (1.1, 2.2)$ GeV. The computational cost in the Wick and color/spinor contractions is reduced by the unified contraction algorithm[6]. For details about the simulation parameters, see Doi (2013)[7].

In Fig. 1, we plot the nuclear central potential in ${}^1S_0$ channel for each lattice cutoff. We observe that cutoff dependence is nonnegligible at short distances, while it is suppressed at long distances. This is a natural consequence of the discretization being an intrinsically short-range effect. It is also interesting that repulsive core is enhanced on a finer lattice, which is consistent with the study on the operator product expansion[8].

Although the lattice cutoff dependence on potentials looks sizable at short distances, such effect is expected to be suppressed in physical observables such as phase shifts and scattering length, because of the phase space factor of $\propto r^2$, as is shown in the inset of Fig. 1.

In order to quantitatively study the cutoff effect on physical observables, we fit the potential and solve the Schrödinger equation in infinite volume. In Fig. 2, we show the preliminary results for the scattering length against the lattice spacing a, with only a statistical error. Because the scattering length represents the low-energy phenomena, the cutoff dependence is found to be negligible compared to the statistical errors. Detailed studies for the systematic uncertainties on phase shifts and scattering length are in progress.

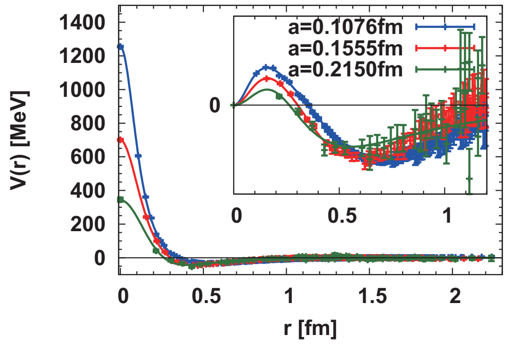

Fig. 1. Central potentials $V_C(r)$ in ${}^1S_0$ channel. Inset shows $r^2V_C(r)$ to include phase space factor. Solid lines correspond the fit for the potentials.

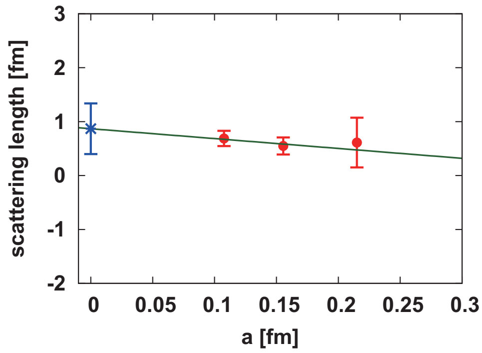

Fig. 2. Scattering length in ${}^1S_0$ channel against lattice spacing a. Blue point corresponds to the result in the continuum limit obtained by linear extrapolation against a.

† Condensed from the article in PoS LAT2013, 226 (2013)

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