Columbia plot and 't Hooft loop at imaginary chemical potential[†]

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The investigation of the phase structure in Quantum Chromodynamics (QCD) at finite temperature (T) and real chemical potential (μ_R) is an important subject in the particle and nuclear physics. If we obtain the QCD phase diagram from first-principles calculations, the phase structure would be clear, in principle. First-principles calculations such as the lattice QCD (LQCD) simulation, however, has a sign problem at finite μ_R, and it is therefore not feasible there. Even if we use several methods and approximations, we cannot reach the $\mu_R/T \geq 1$ region. Therefore, several effective models such as the Nambu–Jona-Lasinio model are widely used to investigate QCD phase diagrams. The effective model approach, however, has large ambiguities. Therefore, at the present, we cannot obtain a reliable phase diagram at finite μ_R by using the lattice QCD simulation and effective model approach.

To overcome this problem, we consider the imaginary chemical potential (μ_I). At finite μ_I, there is no sign problem, and thus we can successfully perform the LQCD simulation. In fact, phase structures have already been investigated by lattice QCD simulations; for example, see References[2,3]. In addition, it is possible to prove that the μ_I region has almost all the information of the μ_R region[1]. QCD has some characteristic properties at finite μ_I. One of the characteristic properties is the Roberge-Weiss (RW) periodicity, which is the special $2\pi/3$ periodicity along the μ_I/T axis. This periodicity is a remnant of the Z_3 symmetry in the pure gauge limit. Also, the RW transition and its endpoint which is called the RW endpoint are expected at finite T in the μ_I region. This means that we can obtain some important constraints for model design from these special properties of QCD.

In this study, we consider the heavy-quark mass region, which corresponds to the upper part of the Columbia plot. The Colombia plot is the figure drawn as a function of the light-quark and strange-quark masses and shows the phase boundary. In the paper, we reported the following three results:

(1) 't Hooft loop can be well defined at the RW endpoint.
(2) Model ambiguities can appear largely at the RW endpoint.
(3) Thermodynamics with imaginary chemical potential shows unexpected behavior comparing to standard thermodynamics.

The 't Hooft-loop is related with the deconfinement transition at finite T in the pure-gauge limit. It is known that this quantity cannot be well defined in the system with dynamical quarks[4,5]. In the calculation of the 't Hooft loop, we should set different Z_3 images (charges) at the boundary of the box and consider its surface. If the potential energies in both sides of the surface are different, there is a force that modifies the surface, and thus the 't Hooft loop cannot be well defined. However, we showed that the *effective* Z_3 charges are identical at two of the Z_3 images, and thus the 't Hooft loop can be well defined at the RW endpoint for degenerate Z_3 images because there is no force to modify the surface.

The Columbia plot at the heavy-quark mass region was calculated by using the matrix model for deconfinement and the logarithmic-type Polyakov-loop effective potential for describing the deconfinement transition by the Polyakov loop. Those models are low-energy effective models of QCD. In the case of the matrix model, there is a phase boundary that separates the first-order and second-order transition regions. Conversely, the logarithmic-type Polyakov-loop effective potential does not have any phase boundary down to 1 GeV. Therefore, there exists a large model dependence in the upper part of the Columbia plot at the RW endpoint.

Several thermodynamic quantities, such as the pressure, energy density, quark number density, entropy density, and interaction measure, are calculated by using the matrix model for deconfinement. We observed that the energy density and interaction measure show unexpected behavior near the RW transition line, which is induced by the contribution of the quark number density. This behavior can be exploited to remove the model ambiguities when accurate lattice QCD data will be available in the future.

References
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