Notes on the enhancement of flavor symmetry and 5d superconformal index†

M. Taki*1

Perturbative renormalizability has been a criterion for the predictable quantum field theory. Needless to say, this is because renormalization removes ultraviolet (UV) divergences from Feynman diagram, giving a meaningful finite value to a physical quantity. While an effective theory is permitted to include non-renormalizable interactions, this criterion must be satisfied by a fundamental theory without any cut-off scale, and it excludes many models of the quantum field theory. The renormalizable theories, however, do not exhaust all possibilities.

A quantum theory endowed with a UV fixed point is well defined and valid at the whole energy scale. This possibility is known as the Weinberg asymptotic safety scenario[1)], which perhaps preserves the non-renormalizability of the perturbative quantum gravity. This scenario is also very attractive because a renormalizable but asymptotically non-free theory such as pure QED involves the Landau pole, and the convergence radius of the perturbation becomes zero according to popular opinion. By assuming the existence of the UV fixed point, we can avoid such a theoretical inconsistency included in perturbative quantum field theory.

UV fixed point is a very important notion in the quantum field theory, but it is very difficult in general to determine whether a theory has a UV fixed point. 5d minimal supersymmetric gauge theories are typical and attractive exceptions to circumvent this difficulty. Perturbative five-dimensional gauge theories are non-renormalizable, but Seiberg[2)] showed that perturbative description breaks down at high energy but some of these theories flow up to a strongly coupling, non-Gaussian, UV fixed point. $SU(2)$ gauge theory with $N_f = 0, 1, \cdots, 7$ fundamental flavors provides a concrete example[3,4)]. The flavor symmetry of this gauge theory is $SO(2N_f) \times U(1)_I$, where $U(1)_I$ is associated with the instanton current $J = *\mathrm{Tr}F \wedge F$. The UV fixed point is described by a strongly coupled conformal field theory. At this fixed point, the flavor symmetry is expected to enhance to the larger group E_{N_f+1}: $E_1 = SU(2)$, $E_2 = SU(2)\times U(1)$, $E_3 = SU(3)\times SU(2)$, $E_4 = SU(5)$, $E_5 = SO(10)$, and $E_{6,7,8}$ are the usual exceptional Lie groups.

This enhancement of the flavor symmetries was conjectured by employing superstring theory[2)], and so far it has not been easy to show this enhancement based only on field theory arguments. This is because the UV fixed point theories in question are strongly coupled, and it has prevented us from verifying this conjecture directly. Fortunately, with recent progress in the theories of localization and the superconformal index, we can discuss the strongly coupled fixed point theories quantitatively by evaluating the protected indexes of these theories[5)]. The 5d superconformal index is the following extended version of the Witten index:

$$I_{5\mathrm{d}}(u, z_f, \cdots) = \mathrm{Tr}_{\mathcal{H}_{\frac{1}{8}\mathrm{BPS}}} (-1)^F u^k \prod_f z_f^{H_f} \cdots .$$

Here k is the $U(1)$ charge with respect to the instanton current, and H_f is a Cartan generator of the flavor symmetry. u and z_f are the fugacities for these symmetries. In this paper, we study the detailed structure of the superconformal index, and we provide a justification of the enhancement of the flavor symmetry for $N_f = 0, 1, 2$. We find that the superconformal index satisfies

$$I_{5\mathrm{d}}(u, z_f, t, q) = I_{5\mathrm{d}}(u^{-1}, z_f, t, q).$$

This result indicates that the index is actually invariant under the action of the Weyl group of the expected $SU(2)$ flavor symmetry. We then conclude that the $U(1)$ global symmetry of 5d gauge theory is enhanced to $SU(2)$ at the UV fixed point. This $SU(2)$ flavor symmetry is the core of the full E_{N_f+1} symmetry, and therefore we can expect that extending our result should yield a proof of the full enhancement.

References

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*1 RIKEN Nishina Center