Superconformal indices for gauge duals of $AdS_4 \times SE_7$

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The superconformal index[1,2] of a three-dimensional superconformal field theory can be expressed as the trace over all operators in the theory weighted by their fermion number

$$I(t, z_i) = tr[(-1)^F t^{\epsilon + j_3} z_i^{h_i}]. \quad (1)$$

Here ϵ is the operator dimension, j_3 is the spin of the operator, F is its fermion number, and h_i label the charges of the operator under global symmetries.

In this note we summarise[3] the derivation of the gravity superconformal index for any theory of the form $AdS_4 \times SE_7$. Previously the supergravity index was computed for the homogenous Sasaki-Einstein seven-manifolds using known Kaluza-Klein spectra[4]. However, to match the field theory index and the supergravity index, several of the Kaluza-Klein modes had to be dropped. Since the spectrum has not been well tested, the authors suggested that the Kaluza-Klein spectrum should be revisited. We find that a careful analysis of the Kaluza-Klein modes agrees with known results about field theory index. Our general form of the supergravity index succinctly reproduces previous computations of the gravity index[4]. We find complete agreement with previous large-N computations of the index[4–6].

We construct the Kaluza-Klein multiplets on AdS_4 from various tensors defined on the Sasaki-Einstein manifold following the methodology of[7]. Our analysis focuses on generic Sasaki-Einstein manifolds. Much of our analysis builds upon previous work on Kaluza-Klein spectroscopy for coset manifolds.

Multiplet shortening and the short multiplets contributing to the index can be described using the tangential Cauchy-Riemann operator $\bar{\partial}_b$ and the associated Kohn-Rossi cohomology groups $H^{p,q}_{\bar{\partial}_b}$. In general, the cotangent bundle over a Sasaki-Einstein manifold Y can be decomposed as

$$\Omega_Y = \mathbb{C}\eta \oplus \Omega_Y^{1,0} \oplus \Omega_Y^{0,1}. \quad (2)$$

The operator $\bar{\partial}_b$ is the projection of the exterior derivative on $\Omega_Y^{0,1}$, the cohomology of this complex is $H^{p,q}_{\bar{\partial}_b}$. The Kohn-Rossi cohomology groups are isomorphic to $H^q(X, \wedge^p \Omega'_X)$ defined on the cone, where Ω'_X is the part of the holomorphic cotangent bundle Ω_X perpendicular to the dilatation vector field. Our main result is a formula for the gravity superconformal index as a trace over linear combinations of the groups $H^q(X, \wedge^p \Omega'_X)$.

Table 1 lists the multiplicity of each short multiplet appearing in supergravity solutions of the form $AdS_4 \times SE_7$ and their contribution to the superconformal index. When calculating the index, only states with

$$\{Q, S\} = \epsilon - j_3 - y = 0 \quad (3)$$

contribute, where y is the R-charge. An element f of cohomology has R-charge $\mathcal{L}_D f = 2iDf$. Here $\mathcal{L}_D$ denotes the Lie derivative along the dilation vector field and $2D$ is its corresponding eigenvalue. We normalize each multiplet so that its primary has R-charge y. The R-charge y differs from the R-charge $2D$ of the corresponding cohomology element by a constant shift.

Table 1. Short multiplets and their contribution

Multiplet	(ϵ, j_3, y)	Multiplicity	Index
s. graviton	$(y+2, 1, y)$	$H^0(X, \wedge^3 \Omega'_X)$	$-t^{y+4}$
s. gravitino	$(y+\frac{3}{2}, \frac{1}{2}, y)$	$H^0(X, \Omega'_X)$	t^{y+3}
s. vector Z	$(y+1, 0, y)$	$H^1(X, \Omega'_X)$	$-t^{y+2}$
s. vector A	$(y+1, 0, y)$	$H^0(X, \wedge^2 \Omega'_X)$	$-t^{y+2}$
hyper	$(y, 0, y)$	$H^1(X, \wedge^2 \Omega'_X)$	t^y
hyper	$(y, 0, y)$	$H^2(X, \Omega'_X)$	t^y
hyper	$(y, 0, y)$	$H^0(X, \mathcal{O}_X)$	t^y

Summing the contributions of the short multiplets, we find that the single particle supergravity index is

$$\begin{aligned} 1 + I_{s.t.}(t) = \sum tr[t^{2D} \mid H^0(X, \mathcal{O}_X) \\ \ominus H^0(X, \wedge^2 \Omega'_X) \oplus H^1(X, \wedge^2 \Omega'_X) \\ \oplus t^2 H^0(X, \Omega'_X) \ominus t^2 H^1(X, \Omega'_X) \oplus t^2 H^2(X, \Omega'_X) \\ \ominus t^2 H^0(X, \wedge^3 \Omega'_X)]. \end{aligned} \quad (4)$$

The superconformal index has proven to be a powerful tool in checking proposed dualities. All proposed field theory duals to Saski-Einstein seven manifolds can be tested by computing the field theory index and comparing it with the above gravity index. Currently, there is no general procedure for constructing the field theory dual to a general Sasaki-Einstein seven manifold. One hopes that the superconformal index will help explore new holographic dualities.

References

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