Image reconstruction algorithm for gamma-ray inspection of rotating objects

H. Takeichi,[*1] T. Kambara,[*1] and A. Yoshida[*1]

We develop a new method to determine the spatial distribution of positron-emitting radioisotopes (RIs) on rotating objects and construct a prototype system. The details of the principle and the prototype system based on this method are described elsewhere[1)]. This method is based on the same principle as the medical positron emission tomography (PET) systems in which projection data from all angles are collected. In the PET system, gamma-ray detectors are placed in a circular manner around a stationary object, or the gamma-ray detectors rotate around the object in order to collect projection data. In this method, a pair of gamma-ray detectors are placed in a stationary position and the object being imaged is rotated.

Here, we present the image reconstruction algorithm of the prototype system. The most conventional image reconstruction algorithm in PET is filtered back-projection (FBP) [2)]. Projections from all angles are back-projected onto and overlaid in the image plane using the inverse Radon transform to reconstruct the image. Then, an appropriate image filter is applied to deblur the image.

An alternative to the FBP is the maximum likelihood – expectation maximization (ML–EM) algorithm[3, 4)]. We assume a two-dimensional distribution $\lambda(x,y)$ of RI (image), and the projection data $p(r,\varphi)$ at an angle φ from the y-axis and at a distance r from the center. ML–EM is an iterative method. The iteration starts with an arbitrary image that is updated gradually as

$$\lambda_j^{\,n} = (\lambda_j^{\,n-1} / \sum_i c_{ij})(\sum_i (c_{ij}\, p_i / \sum_k c_{ik}\, \lambda_k^{\,n-1})), \qquad (1)$$

where $\lambda_j^{\,n}$ is the j-th pixel value in the image λ of the n-th iteration, p_i is the value at the i-th position in the projection p, and c_{ij} is the probability that a gamma-ray emitted from the j-th pixel position is counted at the i-th position in the projection (see Fig. 1).

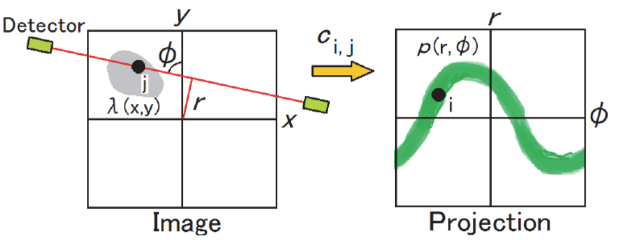

Fig. 1. Schematic illustration of ML–EM

At each iteration, the projection of the current estimate image is calculated and compared with the actual projection. Then, the difference between the estimated and actual projections is back-projected and used to update the current estimate image.

Thus, Equation 1 reads as follows. First, the projection of the current estimate image is calculated ($\Sigma_k\ c_{ik}\ \lambda_k^{\,n-1}$). Second, the ratio of the actual projection to the estimated projection is calculated (p_i / $\Sigma_k\ c_{ik}\ \lambda_k^{\,n-1}$). Third, the ratio is back-projected to the image coordinate (1 / $\Sigma_i\, c_{ij}$)($\Sigma_i\,(c_{ij}\, p_i$ / $\Sigma_k\ c_{ik}\ \lambda_k^{\,n-1}$)). Finally, the back-projected ratio is multiplied by the current estimate image ($\lambda_j^{\,n-1}$ / $\Sigma_i\ c_{ij}$)(Σ_i ($c_{ij}\ p_i$ / $\Sigma_k\ c_{ik}\ \lambda_k^{\,n-1}$)). In the prototype system, the iteration requires 99 steps from the initial uniform image to obtain the current estimate image.

ML–EM is advantageous over FBP for wear diagnosis of mechanical parts in that the image values are all non-negative, the signal to noise ratio is higher, and there are less linear artifacts (see arrows in Fig. 2) around strong RI sources in the image. These advantages are important for the easy detection of weak sources near strong sources. Further, ML–EM is more suitable for quantitative evaluation because the sum of the image values is preserved during the iteration and the gamma-ray attenuation in the machine and collimators can be implemented in c_{ij}.

Figure 2 shows a comparison of the FBP and ML–EM images. The FBP image was obtained using MATLAB *iradon*. The ML-EM image is based on an in-house program.

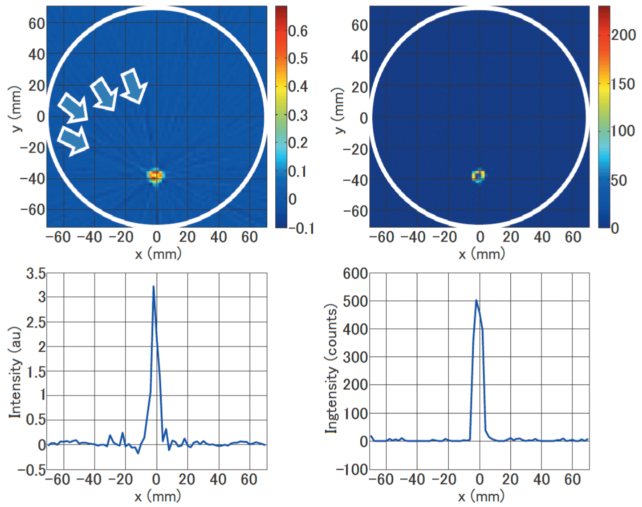

Fig. 2. Comparison of the FBP (left) and ML–EM (right) images (top) and their projections (bottom). The color maps are scaled and optimized for individual images.

References
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*1 RIKEN Nishina Center