

Di-neutron correlation in asymptotic tail of weakly bound nuclei

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Neutrons penetrating far outside the nuclear surface often exhibit exotic features of neutron-rich nuclei close to the drip-line. An important question is whether and how neutrons are correlated in the external tail region. The di-neutron correlation in two-neutron halo nuclei, such as ^{11}Li , has attracted attention in this context.^{1,2)} However, theoretical analyses using the Hartree-Fock-Bogoliubov (HFB) models suggest that the di-neutron correlation also prevails in heavier mass nuclei, including nuclei close to the stability line.^{3,4)} In the present work, we attempt to clarify the emergence mechanism of the di-neutron correlation by investigating the Cooper pair wave function in the Skyrme HFB model both numerically and analytically, with a focus on its asymptotic behavior at large distances.

We have performed a systematic Skyrme HFB calculation⁵⁾ for even-even Ca, Ni, Zr, and Sn isotopes ranging from the stability line to the neutron drip-line. In order to guarantee convergence at large distances, we solve the HFB equation in the radial coordinate representation, using a very large radial cut-off at 100 fm and the orbital angular momentum cut-off at $l = 72$. We then evaluate the neutron pair condensate (equivalent to the pair density and the pairing tensor) $\tilde{\rho}(\mathbf{R}) \equiv \langle \Phi_0 | \psi(\mathbf{R} \uparrow) \psi(\mathbf{R} \downarrow) | \Phi_0 \rangle = \Psi_{\text{pair}}(\mathbf{R}, \mathbf{R})$. Note that the pair condensate is a part of the neutron Cooper pair wave function, defined by $\Psi_{\text{pair}}(\mathbf{r}_1, \mathbf{r}_2) = \langle \Phi_0 | \psi(\mathbf{r}_1 \uparrow) \psi(\mathbf{r}_2 \downarrow) | \Phi_0 \rangle$.

The asymptotics of the pair condensate is characterized by an exponential behavior $\tilde{\rho}(R) \rightarrow C \exp(-\tilde{\kappa}R)$, and the exponential constant $\tilde{\kappa}$ is extracted by a fitting to the microscopically calculated $\tilde{\rho}(R)$. As shown in Fig. 1, the extracted exponential constants (solid symbols) follow a universal relation $\tilde{\kappa} = \sqrt{8m|\lambda|}/\hbar$, where λ is the Fermi energy and m is the neutron mass. The result is different from the previous estimate^{6,7)} $\tilde{\kappa}_{\text{qp}} = \sqrt{2m(|\lambda| + E_{\text{qp},1})}/\hbar + \sqrt{2m(|\lambda| - E_{\text{qp},1})}/\hbar$ (open symbols), which relies on the asymptotic behavior of the quasiparticle wave function with the lowest quasiparticle energy $E_{\text{qp},1}$.

The universal relation can be interpreted as the penetration of a di-neutron with mass $M = 2m$ and with the binding energy given by the two-neutron separation energy $S_{2n} = 2|\lambda|$, i.e. $\tilde{\kappa} = \sqrt{2MS_{2n}}/\hbar$. We can justify this interpretation via an analytic and general examination of the HFB theory. It should be noted that in the limit $r_1, r_2 \rightarrow \infty$, the following two-particle Schrodinger equation holds for the Cooper pair wave function:

$$(t(1)+t(2)+v(1,2))\Psi_{\text{pair}}(\mathbf{r}_1, \mathbf{r}_2) = 2\lambda\Psi_{\text{pair}}(\mathbf{r}_1, \mathbf{r}_2) \quad (1)$$

where $v(1,2)$ is the nn interaction. As a consequence, the asymptotic form is given in terms of the di-neutron coordinate system $r = |\mathbf{r}_1 - \mathbf{r}_2|$, $R = |(\mathbf{r}_1 + \mathbf{r}_2)/2|$ as

$$\Psi_{\text{pair}}(\mathbf{r}_1, \mathbf{r}_2) \rightarrow C_0^{L=0} \phi_0^{L=0}(r) \exp(-\kappa_d R)/R \quad (2)$$

for small r . Here, $\phi_0^{L=0}(r)$ is the wave function of the S -wave virtual state of the nn system, representing the di-neutron, and the exponential constant $\kappa_d = \sqrt{2M(2|\lambda|)}/\hbar$ arising from the center of mass motion of the di-neutron.

We also found that the di-neutron asymptotics, Eq.(2), dominates in weakly bound neutron-rich nuclei with a small neutron separation energy or small $|\lambda|$. Conversely, single-particle (quasiparticle) components also contribute to the asymptotics of the Cooper pair in nuclei having a larger neutron separation energy, as the single-particle value (open symbol) and the full value (solid one) coincide in these nuclei. As a corollary, the condition for the dominance of the di-neutron correlation is given as $|\lambda| \lesssim \Delta$ or $S_{2n} \lesssim 2\Delta$.

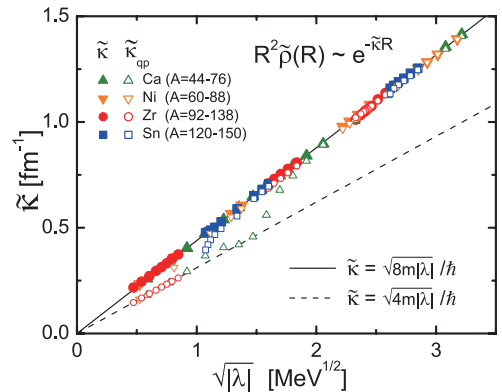


Fig. 1. The asymptotic exponential constant $\tilde{\kappa}$ of the neutron pair condensate $\tilde{\rho}(R)$.

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