Is cosmological constant screened in Liouville gravity with matter?

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Recent observation of dark energy in our universe has led to the conviction that the cosmological constant Λ has an infinitesimal positive value. It means that our space-time is de Sitter (dS) space with the Hubble constant H being $\sqrt{\Lambda}$. It has been proposed that the strong infrared (IR) divergence property of quantum corrections on dS space may explain the smallness of Λ in our current universe (so called cosmological constant problem). The Einstein equation describes the relation between the space-time Ricci tensor $R_{\mu\nu}$ and the energy momentum (EM) tensor $T_{\mu\nu}$ due to the presence of matter. In vacuum, where $T_{\mu\nu}$ is proportional to the metric, the Einstein equation takes the form

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + g_{\mu\nu}\Lambda_{\rm eff} = 0,\ \Lambda_{\rm eff} = \Lambda - \frac{\kappa}{D}T_\rho{}^\rho, \quad (1)$$

where R is the scalar curvature, $\kappa = 8\pi G$ with G being Newton's constant. The vacuum contribution of $T_{\mu\nu}$ is now combined with Λ to define the effective cosmological constant $\Lambda_{\rm eff}$.

In view of this expression, we may wonder if a large value of $T_{\mu\nu}$ cancels the large value of Λ, yielding a very small value of $\Lambda_{\rm eff}$ that we observe today.[1] We study this question in 2-dimensional (2D) Liouville gravity.

We are interested in the dS solution of Liouville gravity, which can describe the interaction between scaler field and gravity. The 2D cosmological constant has two components: the coupling of the Liouville potential and the trace of the EM tensor. By using Weyl transformation to 2D metric $g_{\mu\nu}(g_{\mu\nu} = e^{2\phi}\hat{g}_{\mu\nu})$, we obtain the equation,

$$\begin{aligned} &S_{L+{\rm mat}}[\Phi,\phi] \\ &= -\int d^2x\sqrt{-\hat{g}}\left[\frac{1}{4\pi b^2}\hat{g}^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi + \frac{Q}{4\pi b}\hat{R}\Phi \right. \\ &\quad \left. + \frac{\Lambda}{\kappa}e^{2\Phi} + \frac{1}{2}\hat{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + e^{2\Phi}V(\phi)\right]. \quad (2)\end{aligned}$$

The first three terms are Liouville gravity. At this point, it is important to note that the negative value of Λ corresponds to dS space in Liouville gravity.

As a concrete matter Lagrangian, we have studied a massless scalar field theory with $\lambda\phi^4$ interaction minimally coupled to Liouville gravity.

$$\mathcal{L} = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\sqrt{-g} - \frac{1}{4!}\lambda\phi^4\sqrt{-g} + \Delta\mathcal{L}, \quad (3)$$

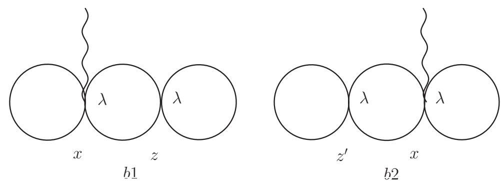

Fig. 1. Part of λ^2-order corrections to the EM tensor $T_{\mu\nu}$.

where $\Delta\mathcal{L}$ consists of counter-terms. In dS space, the massless scalar propagator contains IR divergence in the long wavelength limit and the IR logarithm appears because the cutoff regularization of the IR divergence. Based on the in-in formalism, [2-4] we have computed the VEV of the EM tensor to the order of λ^2.

$$\begin{aligned}\Lambda_{\rm eff} &\sim \Lambda + (\text{Weyl anomaly}) + \frac{\kappa\lambda}{32\pi^2}\ln^2 a(\eta) \\ &\quad + \frac{1}{8\pi}\frac{\kappa\lambda^2}{(4\pi)^2H^2}\ln^4 a(\eta), \quad (4)\end{aligned}$$

where $a = -\frac{1}{H\eta}$ is the scale factor and $\eta = -\frac{1}{H}e^{-Ht}$ is conformal time. The resulting VEV has time dependence through the IR logarithms, and as a consequence, the effective cosmological constant shows the screening effect at late time such that the absolute value decreases with time. This should be in contrast with the situations where $D > 2$, in which the cosmological constant is anti-screened in the $\lambda\phi^4$ theory.[5]

To claim that the observed dS breaking effects are physical, we need to find whether they may be eliminated from the local counter-terms. Here, we discuss rather unfamiliar time-dependent IR counter-terms. This possibility plays a crucial role to understand the (in)equivalence between the Sine-Gordon model and the massive Thirring model in dS space. We are indeed able to recover the dS invariance by adding time-dependent IR counter-terms to the naive perturbative computations using the dS breaking propagator. Within the perturbation theory we have studied, however, a similar mechanism does not seem to be workable in $\lambda\phi^4$ theory. This fact supports the claim that the observed screening mechanism of the cosmological constant should be physical. This sensitive issue will be further discussed in our future publication.

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