Spin operator and entanglement in quantum field theory†

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Entanglement is studied in the framework of Dyson's S-matrix theory in relativistic quantum field theory, which leads to a natural definition of entangled states of a particle-antiparticle pair and the spin operator from a Noether current. As an explicit example, the decay of a massive pseudo-scalar particle into an electron-positron pair is analyzed. Two spin operators are extracted from the Noether current. The Wigner spin operator characterizes spin states at the rest frame of each fermion and, although not measurable in the laboratory, gives rise to a straightforward generalization of the low energy analysis of entanglement to the ultra-relativistic domain. In contrast, if one adopts a (modified) Dirac spin operator, the entanglement measured using spin correlation becomes maximal near the threshold of the decay, while the entanglement is replaced by the classical correlation for the ultra-relativistic electron-positron pair by analogy to the case of neutrinos, for which a hidden-variables-type description is possible. Entanglement in this sense depends on the energy scale involved. Chiral symmetry which is fundamental in particle physics differentiates the spin angular momentum and the magnetic moment. The use of weak interaction, which can measure helicity, is suggested in the analysis of entanglement at high energies instead of a Stern-Gerlach apparatus, which is known to be useless for the electron. A difference between the electron spin at high energies and the photon linear polarization is also noted.

We formulate the entanglement in the framework of relativistic quantum field theory, or more precisely, in the S-matrix theory defined by Dyson.[1)] In the S-matrix theory, we treat only asymptotic states that contain particles far apart from each other.

We consider the decay of a very massive pseudo-scalar particle P into an electron-positron pair by an interaction Hamiltonian, $H_I(t) = g\int d^3x : P(x)\bar{\psi}(x)i\gamma_5\psi(x) :$ with a coupling constant g. The Dyson formula for the S-matrix gives the state $\Psi = -ig\int d^4x : P(x)\bar{\psi}(x)i\gamma_5\psi(x) : |P(\vec{0})\rangle$, where we assume a small g. We then obtain for the fixed momentum direction of the electron,

$$\Psi(\vec{p}) \equiv \frac{1}{\sqrt{2}}[a^\dagger(\vec{p},s)b^\dagger(-\vec{p},-s) + a^\dagger(\vec{p},-s)b^\dagger(-\vec{p},s)]|0\rangle.$$

This shows a way to prepare a desired state in the framework of local and causal relativistic field theory. All the properties of the asymptotic state are accounted for in the framework which is consistent with locality, causality and the uncertainty principle; in particular, it is important to recognize that we integrate over the entire Minkowski space in defining Ψ; that is, we have no information about when and where the particle decayed.

Spin Operator

The conserved angular momentum operator (Noether charge) of the Dirac action is given by $\hat{\vec{J}} = \int d^3x : \psi^\dagger(x)[\vec{L}+\vec{S}]\psi(x) :$, which is written as

$$\int d^3p \sum_{s,s'} \frac{1}{2}\{\xi(s')^\dagger\vec{\sigma}\xi(s)a^\dagger(\vec{p},s')a(\vec{p},s) - \xi^\dagger(-s)\vec{\sigma}\xi(-s')b^\dagger(\vec{p},s')b(\vec{p},s)\} + \int d^3p \sum_s \{a^\dagger(\vec{p},s)\left(\vec{L}a(\vec{p},s)\right) + b^\dagger(\vec{p},s)\left(\vec{L}b(\vec{p},s)\right)\}.$$

The first term is called the Wigner spin operator, which is not directly measured in the laboratory. We instead define the (modified) Dirac spin operator

$$\begin{aligned}\hat{\vec{S}}(\vec{p}) \equiv \sum_{s,s'}\{&[\frac{1}{2}\frac{m}{E}\xi^\dagger(s')\vec{\sigma}_T\xi(s) + \frac{1}{2}\hat{p}\xi^\dagger(s')(\vec{\sigma}\cdot\hat{p})\xi(s)] \times a^\dagger(\vec{p},s')a(\vec{p},s)\\ &-[\frac{1}{2}\frac{m}{E}\xi^\dagger(-s)\vec{\sigma}_T\xi(-s') + \frac{1}{2}\hat{p}\xi^\dagger(-s)(\vec{\sigma}\cdot\hat{p})\xi(-s')] \times b^\dagger(-\vec{p},s')b(-\vec{p},s)\},\end{aligned}$$

which is close to what is measured in the laboratory. A salient feature of this spin is that it appraoches the helicity state proportional to the momentum direction $\hat{p}$ for $E \to \infty$. That is, the spin correlation at high energies becomes similar to the correlation of neutrinos. The spin correlation of two neutrinos, which can have only two states $h = \pm$, does not define the entanglement in the conventional sense. That is, the high energy electron states behave like classical particles, which show the correlation but not entanglement. This transition to helicity states may be measured using weak interactions.[2)]

References
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2) K. Fujikawa and R. Shrock: Phys. Rev. Lett. **45**, 963 (1980).

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