Bulk angular momentum and Hall viscosity in chiral superconductors†

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Chiral superfluids and superconductors (SCs) are exotic states whose time-reversal symmetry is spontaneously broken and Cooper pairs carry nonzero angular momentum (AM). There is a long-standing problem on the AM in chiral ℓ-wave SCs, the so-called intrinsic AM paradox, which is summarized as $L_z = \hbar m N_0/2 \times (\Delta_0/E_{\mathrm{F}})^\gamma$, where $|m| \leq \ell$, N_0, Δ_0, E_{F}, γ are the magnetic quantum number, the total number of electrons, the gap strength, the Fermi energy, and the exponent characterizing the dependence on the SC gap. $\gamma = 0$ is the most natural if all electrons from Cooper pairs with the AM $\ell_z = \hbar m$. On the other hand, $\gamma = 1$ is intuitively plausible if a electrons near the Fermi surface from Cooper pairs.[1] One of the obstacle is that the physical quantities involving the position operator are ill defined in periodic systems, and we have to manage an inevitable divergence in the bulk limit. An interesting clue to the AM is the Hall viscosity (HV), which has been intensively discussed in the context of the quantum Hall effect. The important relation $\eta_{\mathrm{H}} = \hbar N_0 \bar{s}/2$ holds in general gapped systems at zero temperature,[2] in which the orbital spin $\bar{s}$ is equal to $\frac{\ell}{2}$ in chiral ℓ-wave SCs. In this report, we derive the Berry-phase formulas for the AM and the Hall viscosity (HV) to apply to chiral SCs in two and three dimensions, which allow us to deal with the bulk systems.

We examine an angular velocity from the gauge-theoretical viewpoint. Now that the system is rotated, we have to deal with a theory in a curved spacetime. We use the Cartan formalism, consisting of two gauge potentials, a vielbein and a spin connection. Since we are now interested in the orbital AM, we do not consider the spin connection corresponding to the internal AM for simplicity. A vielbein $h^a{}_\mu$ is a gauge potential corresponding to local spacetime translations, while a spin connection is that corresponding to local Lorentz transformations. The spatial component of a vielbein is related to a displacement vector. Since a vielbein is a gauge potential, it induces a field strength called torsion,

$$T^l{}_{j0} = \partial_j h^l{}_0 - \partial_0 h^l{}_j \, , \quad T^l{}_{ij} = \partial_i h^l{}_j - \partial_j h^l{}_i \, . \tag{1}$$

The former is "electric." The first term describes an angular velocity if l and j are antisymmetric, while the second term describes a strain-rate tensor if symmetric. On the other hand, the latter is "magnetic" characterizing edge and screw dislocations in crystals.

Based on this formalism, we derive the momentum polarization, at zero temperature in a gapped fermion system,

$$P_k{}^i = \sum_n^{\mathrm{occ}} \int \frac{d^d\pi}{(2\pi\hbar)^d} \pi_k A^i_{n\vec{\pi}} \, , \tag{2}$$

where $\vec{\pi}$ is the momentum, the summation is taken over the occupied states, the Berry connection is given by $A^i_{n\vec{\pi}} = i\hbar \langle u_{n\vec{\pi}} | \partial_{\pi_i} u_{n\vec{\pi}} \rangle$, and $u_{n\vec{\pi}}$ is the Bloch eigenstate. Then the AM is obtained by the antisymmetric part of the momentum polarization,

$$L_k = \epsilon_{ijk} P^{ji} = \sum_n^{\mathrm{occ}} \int \frac{d^d\pi}{(2\pi\hbar)^d} \epsilon_{ijk} A^i_{n\vec{\pi}} \pi^j \, . \tag{3}$$

Since the Berry connection is regarded as the expectation value of the position operator in the Wannier basis, this Berry-phase formula really indicates $\vec{\ell} = \vec{x} \times \vec{p}$ in the momentum space.

Here we define the nonsymmetric viscosity by $\eta_k{}^i{}_l{}^j = \partial T_k{}^i / \partial \left(-T^l_{j0}\right)$. As well as the AM formula, we obtain

$$\eta_k{}^i{}_l{}^j = \frac{1}{\hbar} \epsilon^{ijm} \sum_n^{\mathrm{occ}} \int \frac{d^d\pi}{(2\pi\hbar)^d} \pi_k \pi_l \Omega_{n\vec{\pi}_m} f_{n\vec{\pi}} \, , \tag{4}$$

where $f_{n\vec{\pi}} = f(\epsilon_{n\vec{\pi}} - \mu)$ is the Fermi distribution function and the Berry curvature is defined by $\Omega_{n\vec{\pi}k} = i\hbar^2 \epsilon_{ijk} \left\langle \partial_{\pi_i} u_{n\vec{\pi}} | \partial_{\pi_j} u_{n\vec{\pi}} \right\rangle$. The proper HV is obtained as its symmetric part. Especially in two dimensions, the antisymmetric part yields

$$\eta_{\mathrm{H}} = \frac{1}{4\hbar} \sum_n \int \frac{d^2\pi}{(2\pi\hbar)^2} \vec{\pi}^2 \Omega_{n\vec{\pi}z} f_{n\vec{\pi}} \, . \tag{5}$$

These expressions are quite analogous to that for the Hall conductivity, corresponding to the charge transport. The integrand just differs in the factor of $\vec{\pi}$.

By applying these formulas to the Bogoliubov–de Gennes system, we obtain the AM for gapped chiral SCs at zero temperature

$$L_z = -\hbar \sum_{\vec{k}} (\vec{A}_{\vec{k}} \times \vec{k})_z = \hbar m N_0/2 \, , \tag{6}$$

which is consistent with $\gamma = 0$, and shows the relation to the HV, $L_z = 2\eta_{\mathrm{H}}$. See, for example, for the recent microscopic studies suggesting $\gamma = 0$.[3]

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