Nuclear moment of inertia in super-normal phase transition region†

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The purpose of this paper is to derive the analytic expression for the angular momentum (I) dependence of the moment of inertia (MoI) from the microscopic many-body theory both for even-even and odd-mass nuclei. The I-dependence of MoI has been proved to be essential in simulating triaxial, strongly deformed (TSD) bands in a series of papers.1–4)

We adapt the approximation developed for the gap (Δ) dependence of the ratio of MoI (J) to the rigid-body value (J'Rig). It assumes that only large matrix elements of single-particle angular momentum of \((j_\lambda)_{\alpha \beta}\) contribute to \(J\) with a common excitation energy of \(\delta (= \varepsilon_\beta - \varepsilon_\alpha)\), where \(\varepsilon_\alpha\) denotes the single-particle energy of the level \(\alpha\). We apply this approximation to the gap equation including the Coriolis anti-pairing (CAP) effect7) through the second-order perturbation to the cranking term.8,9)

When \(\Delta\) is larger than half of the single-particle level distance \(d\), we can apply a definite integral for the gap equation with the CAP effect. When \(\Delta\) is smaller than half of \(d\), we propose the finite sum approximation for the level distribution. In this case, it is proved that \(\Delta\) never tends to zero, and there is no sharp phase transition from the superconducting state to the normal state. Neglecting the higher order in \(2\Delta/\delta\) for the case \(\Delta < d/2\) (finite sum method), we express MoI as an analytic function of \(I\).

In Fig. 1, we compare the approximate solution between even-even and odd-mass nuclei as functions of I measured from the band-head angular momentum \(I_0\). Usually, \(I_0 = 0\) for even-even nuclei, while \(I_0 \neq 0\) for odd-mass nuclei, for example, \(I_0 = 13/2\) for the TSD yrast band in \(^{163}\text{Lu}\).10 We choose the single-particle energy for a valence nucleon as \(\varepsilon_\ell = 0.6\) MeV above the Fermi surface, and the initial pairing gap at \(I=I_0\) for odd mass as 0.6 MeV, smaller than 0.8 MeV for even-even nuclei (blocking effect). The blocking effect reduces the starting value of \(\Delta\) and increases that of the MoI. In odd-mass case, there is a term that correlates the single-particle state of \(\ell\) with \(\alpha\) through \((j_\lambda)_{\alpha \ell}\). The matrix element of \((j_\lambda)_{\alpha \ell}\) is chosen to be 12 for \(\varepsilon_\alpha > \varepsilon_\ell\) and 10 for \(\varepsilon_\alpha < \varepsilon_\ell\). The other parameters are the same as those for the even-even case. We have started both approximate solutions with \(\Delta = 0.15\) MeV corresponding to \(I-I_0 \sim 15\), while \(d = 0.4\) MeV.

As is seen in Fig. 1, the main difference between even-even (dashed line) and odd-mass (solid line) nuclei is from the blocking effect. Then, both curves increase gradually, and approach the value 1. The MoI of odd-mass case is chosen to be slightly larger than that of the even-even case. The curves become convex upward before they reach to rigid-body values. This upward convexity is also necessary for explaining the energy sequence of TSD bands.6) For the case of \(\Delta \geq d/2\) (definite integral), \(J\) goes to \(J'Rig\) around \(I-I_0 \sim 17\) or 18 (sharp phase transition). Even in this case, odd-mass nuclei show an upward convexity before the phase transition at \(I = 17 \pm 18\). Because of larger \(I_0\), the slow phase transition occurs at larger \(I\) for odd-mass nuclei than for even-even nuclei.

References
5) A. Bohr and B. R. Mottelson: Nuclear Structure (Benjamin, Reading, MA, 1975), Vol. II.