Thick-target yields derived from inverse kinematics toward transmutation

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Nuclear transmutation of long-lived fission products (LLFP) into stable and short-lived isotopes is considered as an important technique for reducing nuclear wastes in nuclear power plants1). Thick-target yields (TTY) on radioactive targets are fundamental and key information for establishing such a technique, although experiments on such radioactive targets suffer from high radioactivity. Recent progress in experimental techniques now allow the utilization of beams composed of radioactive isotopes (RI) including LLFP. In fact, an experiment has been performed at RIBF to obtain cross section data relating to 90Sr and 137Cs3). Unstable nuclei of astrophysical interests have also been applied to obtain cross sections deduced from the thick-target method of elastic scattering3). We therefore suggest an estimation method for the TTY on a radioactive target based on inverse kinematics. In the case that the projectile is stopped inside the thick target, the TTY denoted by Y is obtained by the integration value of cross-section σ and density ρ of the target, up to the projectile penetration length. While the energy decrease of the projectile is described by the stopping power \( S(E) = -\frac{dE}{d\rho E} \) at a certain point \( x \), the TTY can be expressed as:

\[
Y(\epsilon_{\text{in}}) = \frac{N_{\Lambda}A_P}{A_T} \int_{\epsilon_{\text{in}}}^{\epsilon_{\text{final}}} \sigma(\epsilon) \frac{1}{S(\epsilon)} d\epsilon, \tag{1}
\]

which leads to:

\[
\frac{dY(\epsilon)}{d\epsilon} = \frac{N_{\Lambda}A_P}{A_T} \sigma(\epsilon) \frac{1}{S(\epsilon)}. \tag{2}
\]

where the Avogadro constant \( N_{\Lambda} \), the mass number of the target \( A_T \), the energy per nucleon \( \epsilon = E/\rho \) with the mass number of the projectile \( A_P \) and its incident energy \( \epsilon_{\text{in}} \) are used. In this report, we call this system the forward kinematics and its TTY Yinv.

The inverse kinematics with an RI beam suggests that by swapping roles of the radioactive target and incident particle, we can obtain the TTY of the inverse kinematics system denoted by Yinv. The ratio of the TTYs between differential yields, \( R(\epsilon) \), at the same energy \( \epsilon \) is given by:

\[
R(\epsilon) \equiv \frac{dY_{\text{inv}}(\epsilon)}{dY_{\text{inv}}(\epsilon)} = \frac{A_T^2}{A_P^2} \frac{S_{\text{inv}}(\epsilon)}{S_{\text{inv}}(\epsilon)}. \tag{3}
\]

Since the \( \sigma(\epsilon) \) of both systems are the same and are canceled out in the ratio, the TTY \( Y_{\text{inv}}(Y_{\text{inv}}) \) can be estimated without cross-section if we know \( Y_{\text{inv}}(Y_{\text{inv}}) \). Note that the stopping powers \( S_{\text{inv}} \) and \( S_{\text{inv}} \) can be computed by SRIM 2008 code4).

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![Fig. 1. Cross-section of \(^{nat}\)Cu(\(^{12}\)C,X)\(^{24}\)Na with experimental data\(^5\) with spline fitting in (a), and the evaluated ratio \( R(\epsilon) \) by SRIM 2008\(^4\) in (b) as functions of \( \epsilon \).]

**Table 1.** TTY \( Y_{\text{inv}}(\epsilon_{\text{in}}) \) at two incident energies evaluated from Eq. (4) taking \( R \approx 1.1 \) and from Eq. (1).

<table>
<thead>
<tr>
<th>( Y_{\text{inv}}(\epsilon_{\text{in}}) )</th>
<th>40 MeV</th>
<th>100 MeV</th>
</tr>
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<tbody>
<tr>
<td>From Eq. (4)</td>
<td>( 0.94 \times 10^{-5} )</td>
<td>( 0.113 \times 10^{-3} )</td>
</tr>
<tr>
<td>From Eq. (1)</td>
<td>( 0.91 \times 10^{-5} )</td>
<td>( 0.114 \times 10^{-3} )</td>
</tr>
</tbody>
</table>

We show the example of \(^{nat}\)Cu(\(^{12}\)C,X)\(^{24}\)Na\(^5\). In order to calculate \( Y_{\text{inv}} \) and \( Y_{\text{inv}} \) using Eq. (1), the cross-section \( \sigma(\epsilon) \) and stopping power \( S(\epsilon) \) should be known (see in Fig. 1). \( R(\epsilon) \) shown in Fig. 1 (b) converges at a constant value at a high energy of more than 50 MeV/A. This simple behavior of \( R(\epsilon) \) at a high energy and small \( \sigma(\epsilon) \) at a low energy allows us to use a more convenient conversion method as:

\[
Y_{\text{inv}}(\epsilon_{\text{in}}) \approx \tilde{R} Y_{\text{inv}}(\epsilon_{\text{in}}), \tag{4}
\]

where \( \tilde{R} \) is a constant value of \( R(\epsilon) \) at the high energy. Indeed, these are in good agreement with values derived from Eq. (1) (Table 1).

This conversion method will be applied to radioactive isotopes, such as \(^{137}\)Cs, and used to search suitable projectiles for transmutation of LLFP.

**References**

2) H. Wang et al.: In this report.