P-odd spectral density of quark-gluon plasma at weak coupling

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Topological fluctuations of QCD is one of the fundamental aspects of QCD dynamics in the plasma of quarks and gluons. They lead to thermal fluctuations of axial charge density that has distinct P- and CP-odd parities, which induces several unique phenomena such as chiral magnetic effect[1)]. At weak coupling regime in high enough temperature, the rate of topological fluctuations is $\Gamma_s \sim \alpha_s^5 \log(1/\alpha_s) T^4$, and this dependence on the coupling constant also governs how fast the axial charge, once created, decays in time. If the time scale one is interested in is faster than this, the axial charge is approximately conserved. The boundary of validity regime of hydrodynamics is $\alpha_s^2 T$, the scale of hard electromagnetic probes such as photon emission rates is $d\Gamma/d^3k \sim \alpha_{EM}\alpha_s T$, the scale of heavy quark momentum diffusion is $\kappa \sim \alpha_s^2 T^3$, and the jet quenching parameter is $\hat{q} \sim \alpha_s^2 T^3$. Therefore, for many interesting dynamics in the quark-gluon plasma at weak coupling regime, the axial charge can be considered as a conserved charge.

Due to its unique P- and CP-odd parities, the axial charge leads to an interesting P-odd structure in the current-current thermal correlation functions of the plasma. The retarded current-current correlation function $G_R^{ij}(k) \equiv (-i)G_{ra}^{ij}(k) = (-i)\langle J_r^i(k) J_a^j(-k)\rangle$ written in the ra-basis of Schwinger-Keldysh formalism, encodes "chiral magnetic conductivity"[2)] $\sigma_\chi(k)$ by $G_R^{ij}(k) \sim i\sigma_\chi(k)\epsilon^{ijl}k^l$, that is responsible for the chiral magnetic effect with arbitrary momentum of the magnetic field: $\vec{J} = \sigma_\chi(k)\vec{B}(k)$. It has been well-established that the zero momentum limit of $\sigma_\chi(k)$ is topologically protected by chiral anomaly coefficient: $\lim_{k\to 0}\sigma_\chi(k) = N_c N_F \mu_A/(2\pi^2)$ where μ_A is axial chemical potential.

To get a better picture of full real-time P-odd correlation functions, one starts with the fluctuation-dissipation relation

$$G_{rr}^{ij}(k) = \left(1/2 + n_B(k^0)\right)\rho^{ij}(k)\,, \tag{1}$$

where $\rho^{ij}(k) \equiv \left(G_{ra}^{ij}(k) - G_{ar}^{ij}(k)\right)$ is the spectral density that governs the strength of thermal fluctuations represented by the left-hand side. From the hermiticity of current operator, one can show that $\rho^{ij}(k)$ is a hermitian matrix in terms of spatial indices i, j: with rotational invariance, it allows to have a P-odd structure $\rho^{ij}(k) \sim i\rho^{\rm odd}(k)\epsilon^{ijl}k^l$ with a real function $\rho^{\rm odd}(k)$. We call $\rho^{\rm odd}(k)$ "P-odd spectral density"[3)]. It is easy to show that it is the imaginary part of the chiral magnetic conductivity: $\rho^{\rm odd}(k) = -2\,{\rm Im}(\sigma_\chi(k))$.

Contrary to the real part of $\sigma_\chi(k)$ in $k \to 0$ limit, the P-odd spectral density is sensitive to real-time thermal dynamics of quark-gluon plasma as it governs the P-odd part of thermal current-current fluctuations, and it shares many common features with the P-even spectral density that is responsible for dissipative transport coefficients in hydrodynamic regime. Like P-even spectral densities, $\rho_{\rm odd}(k)$ is an odd function of $k^0 \equiv \omega$ with a small ω expansion $\rho^{\rm odd}(k) \sim 2\xi_5\omega + \cdots$, where ξ_5 is one of the second order transport coefficients in chiral anomalous hydrodynamics[4)]: $\vec{J} = \sigma_0\vec{B} + \xi_5 d\vec{B}/dt$. Like the ordinary electric conductivity, the ξ_5 turns out to be non-analytic in coupling constant, and the leading log computation requires a re-summation of all ladder diagrams with HTL re-summation of exchanged gluons, that eventually leads to solving second order differential equations in momentum space. For 2-flavor QCD, we have obtained[3)]

$$\xi_5 = -\frac{2.003}{g_s^4\log(1/g_s)}\frac{\mu_A}{T}\,. \tag{2}$$

For other kinematics of momentum k, the P-odd spectral density generally gives P- and CP-odd emission rates of spin-polarized (that is, circular polarized) virtual photons with momentum k. To see this, the unique P- and CP-odd observable in photon emissions is given by the spin polarization[5)]: $\Gamma^{\rm odd} = \Gamma(\epsilon^+) - \Gamma(\epsilon^-)$ where $\epsilon^\pm$ are circular polarization vectors of photons. Using the results in Ref.[5)] that relates $\Gamma^{\rm odd}$ with the P-odd part of current-current correlation functions, one can show that

$$\frac{d\Gamma^{\rm odd}}{d^3k} = -\frac{e^2}{(2\pi)^3}n_B(\omega)\rho^{\rm odd}(k)\Big|_{\omega=|\vec{k}|}\,. \tag{3}$$

We have computed $d\Gamma^{\rm odd}/d^3k$ in complete leading order at weak coupling[6)], by computing 1) hard Compton/pair annihilation rates, 2) soft t- or u-channel exchange contributions with HTL re-summation, and 3) Landau-Pomeranchuk-Migdal re-summation of soft collinear Bremstrahlung and inelastic pair annihilation processes.

References
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