Stability of the wobbling motion in an odd-A nucleus†

K. Tanabe*1,1,2 and K. Sugawara-Tanabe*1,3

Recently, the transverse wobbling mode was proposed in the yrast band near the ground state before the first backbending in 135Pr1). The transverse wobbling mode is the wobbling motion around the middle moment of inertia (MoI)2), which does not exist in the pure rotor as discussed in the context of classical mechanics3) and by Bohr-Mottelson4) quantum mechanically. Regarding the particle-rotor model, as long as the rigid MoI is adopted, there is no chance to find transverse wobbling, because the single-particle oscillator strength \( \omega_k \) and the rigid MoI are derived from a common radius, and their magnitudes increase or decrease in the same direction as functions of \( \gamma \) periodically with a span of \( 2\pi/3 \). On the other hand, the hydrodynamical (hyd) MoI changes its role in every span of \( \pi/3 \) in an opposite direction to \( \omega_k \). Thus, there remains a possibility to find the transverse wobbling mode for the particle-rotor model with hyd MoI.

We extend the Holstein-Primakoff (HP) boson expansion method to the odd-A case5-8) by introducing two kinds of bosons for the total angular momentum \( \sum \) and the single-particle angular momentum \( j \). We can identify the nature of each band by referring to two kinds of quantum numbers \( (n_\alpha, n_\beta) \) which indicate the wobbling of \( \sum \) and the precession of \( j \), respectively. In this paper we extend this method to the particle-rotor model with hyd MoI. We choose a representation in which \( I_\alpha \) and \( j_\beta \) are diagonal because the hyd MoI is maximum around the \( y \)-axis with the relation \( J^\text{hyd}_y \geq J^\text{hyd}_x \geq J^\text{hyd}_z \) in the range of \( 0 \leq \gamma \leq \pi/6 \), while \( \omega_k \) favors \( j \) to align along the \( x \)-axis in the same range of \( \gamma \). We notice that, if we choose the diagonal representation for \( \sum \) and \( j \), in the same direction, we cannot find any stable physical solution for common values of \( \gamma \) and \( V \) (the strength of the single-particle potential5-8)) in the range of \( 11/2 \leq I \leq 33/2 \). We solve the energy-eigenvalue equation to obtain two real solutions, i.e., \( \omega_+ \), which is the higher energy, and \( \omega_- \), which is the smaller one. In the symmetric limit of \( \gamma = 30^\circ \) and \( V = 0 \), \( \omega_+ \) corresponds to the wobbling motion around the \( y \)-axis with the maximum MoI, while \( \omega_- \) corresponds to the precession of \( j \) around the \( x \)-axis.

We adopt \( j = 11/2 \), \( J_0 = 25 \text{ MeV}^{-1} \) (\( J^\text{hyd}_y = \frac{3}{2} J_0 \sin^2 (\gamma + \frac{2}{3} \pi \kappa) \)), \( \beta = 0.18 \) and \( \gamma = 26^\circ \) (proposed by Ref1)), and \( V = 1.6 \text{ MeV} \) (related to the single-particle strength with these \( \beta, \gamma \) and \( j \) by Wigner-Eckart theorem8)). In Fig. 1 we compare the energies labeled by \( (n_\alpha, n_\beta) = (0,1) \) and \( (0,0) \) with the exact ones obtained by diagonalizing the same Hamiltonian. The exact levels are reproduced by the approximate ones labeled \( (0,1) \), the precession mode of \( j \), irrespective of \( I - j \). Both \( \omega_+ \) and \( \omega_- \) monotonically increase with \( I \), and never decrease.

In conclusion, there is no transverse wobbling mode within the framework of the particle-rotor model even with the hyd MoI.

References
4) A. Bohr, B. R. Mottelson: Nuclear Structure (Benjamin, Reading, MA, 1975), Vol. II.

† Condensed from the article at the autumn meeting of Japan Physical Society, Osaka City University, Sept. 27th (2015)