Liquid-Gas Phase Transitions in Quantum Field Theories†

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We address the generic problem of liquid-gas phase transitions from a field theory perspective. The general class of field theories we will study is the class of $\mathcal{CK}$-symmetric models obtained from dimensional reduction of a four-dimensional field theory at finite temperature and density. The fermions with mass m of the underlying theory are integrated out, and after dimensional reduction and redefinition of fields, we obtain a Lagrangian of the general form

$$L_{3d} = \frac{1}{2}(\nabla\phi_1)^2 + \frac{1}{2}m_1^2\phi_1^2 + \frac{1}{2}(\nabla\phi_2)^2 + \frac{1}{2}m_2^2\phi_2^2 - F(\phi_1, \phi_2)$$

where ϕ_1 is associated with the attractive force, and ϕ_2 with the repulsive force. Although charge conjugation $\mathcal{C}$ is broken at finite density, there remains a symmetry under $\mathcal{CK}$, where $\mathcal{K}$ is complex conjugation[1]. We consider four models: relativistic fermions, nonrelativistic fermions, static fermions and classical particles, corresponding to four different choices for the function $F(\phi_1, \phi_2)$. For example, F for the static fermions is given as

$$F = \frac{1}{v}\log\left[1 + e^{-\beta m + \beta\mu + \beta^{1/2}g\phi_1 + i\beta^{1/2}e\phi_2}\right]$$

where β is the inverse temperature and v is a parameter with dimensions of volume, which we set it to be one. The coupling constants g and e determine the strength of the attractive and repulsive forces respectively. The thermodynamic behavior is extracted from $\mathcal{CK}$-symmetric complex saddle points of the effective field theory at tree level. The relativistic and static fermions show a liquid-gas transition, manifesting as a first-order line at low temperature and high density, terminated by a critical end point as shown in Fig. 1 and Fig. 2. In the cases of nonrelativistic fermions and classical particles, we find no first-order liquid-gas transitions at tree level.

The mass matrix controlling the behavior of correlation functions is obtained from fluctuations around the saddle points. Due to the $\mathcal{CK}$ symmetry of the models, the eigenvalues of the mass matrix can be complex[2]. This leads to the existence of disorder lines, which mark the boundaries where the eigenvalues go from purely real to complex. The regions where the mass matrix eigenvalues are complex are associated with the critical line. In the case of static fermions, a powerful duality between particles and holes allows for the analytic determination of both the critical line and the disorder lines as shown in Fig. 2. Depending on the values of the parameters, either zero, one or two disorder lines are found. Numerical results for relativistic fermions give a similar picture as shown in Fig. 1.

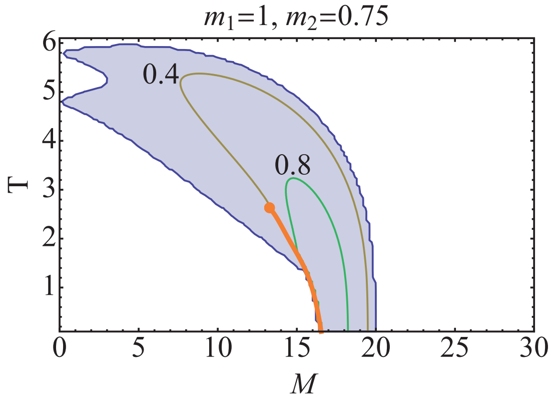

Fig. 1. Phase diagram for relativistic fermions for $m = 20$, $m_1 = 1$, $m_2 = 0.75$ with $g_1 = 1$ and $e = 0.3$. The shaded region indicates where the mass matrix eigenvalues form complex conjugate pairs, and the contour lines refer to the imaginary parts of the mass matrix eigenvalues. The thick line shows a first-order line.

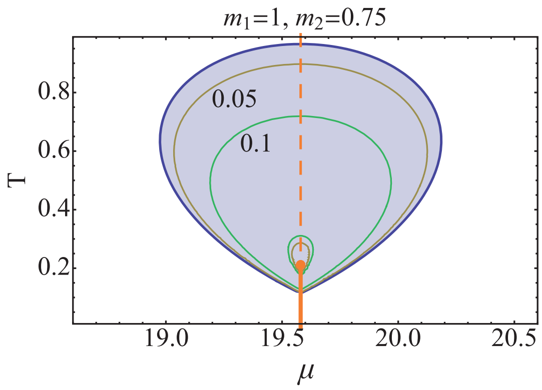

Fig. 2. Phase diagram for static fermions with the same parameter set as the relativistic fermions. The dashed vertical line emerging from the critical end point is the line of particle-hole duality.

References
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2) H. Nishimura, M. C. Ogilvie and K. Pangeni, Phys. Rev. D **91**, no. 5, 054004 (2015).

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