Parity-doublet representation of Majorana fermions and neutron oscillation†

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We present a parity-doublet theorem in the representation of the intrinsic parity of Majorana fermions, which is expected to be useful in condensed matter physics as well, and it is illustrated to provide a criterion of neutron-antineutron oscillation in a BCS-like effective theory with $\Delta B = 2$ terms. The CP violation influences the neutron electric dipole moment, but no direct CP violating effects appear in the oscillation itself, as demonstrated by an exact solution. An analogue of Bogoliubov transformation, which preserves P and CP, is crucial in the analysis.

The Majorana fermions are defined by the condition $\psi(x) = C\bar{\psi}^T(x) = \psi^c(x)$ where $C = i\gamma^2\gamma^0$ denotes the charge conjugation matrix. We start with a neutral Dirac fermion $n(x)$ and define the combinations $\psi_\pm(x) = \frac{1}{\sqrt{2}}[n(x) \pm n^c(x)]$, which satisfy $\psi^c_\pm(x) = \pm\psi_\pm(x)$, showing that $\psi_+(x)$ and $\psi_-(x)$ are Majorana fields. We treat the fermion with $\psi^c_-(x) = -\psi_-(x)$ also as a Majorana fermion. The common parity operation, which is called "γ^0-parity" here, is defined by $n(x) \to \gamma^0 n(t, -\vec{x})$ and $n^c(x) \to -\gamma^0 n^c(t, -\vec{x})$ and satisfies $\mathrm{P}^2 = 1$; it leads to a *doublet representation* $\{\psi_+(x), \psi_-(x)\}$, namely $\psi_\pm(x) \to \gamma^0\psi_\mp(t, -\vec{x})$. Only when the two fermions $\psi_\pm(x)$ are degenerate, this doublet representation is consistent with dynamics. The mass splitting in $\psi_\pm(x)$ inevitably breaks the γ^0-parity as a symmetry of the Lagrangian. The parity thus becomes crucial in the analysis of the neutron oscillation[1,2], which is based on the neutron expressed as a superposition of two Majorana-type fermions with different masses.

In the analysis of possible baryon number violation and neutron oscillation, one can study the quadratic effective Hermitian-Lagrangian

$$\begin{aligned}\mathcal{L} &= \overline{n}(x)i\gamma^\mu\partial_\mu n(x) - m\overline{n}(x)n(x)\\ &- \frac{i}{2}\epsilon_1[e^{i\alpha}n^T(x)Cn(x) - e^{-i\alpha}\overline{n}(x)C\overline{n}^T(x)]\\ &- \frac{i}{2}\epsilon_5[n^T(x)C\gamma_5 n(x) + \overline{n}(x)C\gamma_5\overline{n}^T(x)], \qquad (1)\end{aligned}$$

where m, ϵ_1, ϵ_5 and α are real parameters. The first $\Delta B = 2$ term with a real ϵ_1 breaks the γ^0-parity while the second $\Delta B = 2$ term with a real ϵ_5 preserves γ^0-parity. The term with ϵ_5 in (1) preserves C and P, while $\alpha \neq 0$ implies CP violation. An analogy of the Lagrangian in (1) to BCS theory has been emphasized at the early stage of the study of neutron oscillation.[3]

The presence of the neutron oscillation $P(n \to \bar{n}) \propto \sin^2((\Delta M/2)t)$ implies the mass splitting of auxiliary Majorana-type fermions. In connection with this, we have established the γ^0-parity doublet representation $\{\psi_+(x;\epsilon_1), \psi_-(x;-\epsilon_1)\}$ of the solutions of (1),

$$\begin{aligned}&\psi_+(x;\epsilon_1) \to \gamma^0\psi_-(t, -\vec{x}; -\epsilon_1),\\ &\psi_-(x;-\epsilon_1) \to \gamma^0\psi_+(t, -\vec{x}; \epsilon_1), \qquad (2)\end{aligned}$$

which satisfy $\mathrm{P}^2 = 1$. This representation is valid irrespective of whether the γ^0-parity is conserved or not. The γ^0-parity violation by $\epsilon_1 \neq 0$ in (1) is a *necessary condition* of neutron oscillation which requires $M_+ = M(\epsilon_1) \neq M(-\epsilon_1) = M_-$ (*parity-doublet theorem*) .

The solution of this Lagrangian with $\epsilon_1 = 0$ defines the Bogoliubov transformation

$$\begin{pmatrix} n(x) \\ n^c(x) \end{pmatrix} = \begin{pmatrix} \cos\phi N(x) - i\gamma_5 \sin\phi N^c(x) \\ \cos\phi N^c(x) - i\gamma_5\sin\phi N(x)\end{pmatrix}, \qquad (3)$$

with a Dirac fermion $N(x)$, the mass $M = \sqrt{m^2 + \epsilon_5^2}$, and $\sin 2\phi \equiv \epsilon_5/\sqrt{m^2+\epsilon_5^2}$. To solve (1), we first use the Bogoliubov transformation (3) as a change of variables, which preserves C and P. We then obtain the exact mass formula

$$(M \pm \epsilon_1\sqrt{1 - (\tilde{\epsilon}_1/\epsilon_1)^2}) + i\tilde{\epsilon}_1\gamma_5 \equiv M_\pm e^{2i\theta_\pm\gamma_5}, \qquad (4)$$

with $M_\pm = \left([M \pm \epsilon_1\sqrt{1-(\tilde{\epsilon}_1/\epsilon_1)^2}]^2 + (\tilde{\epsilon}_1)^2\right)^{1/2}$ for two majorana fermions, where $\tilde{\epsilon}_1 \equiv \epsilon_1 \sin\alpha \sin 2\phi$.

The oscillation probability is given by ($\theta = \theta_+ - \theta_-$)

$$\begin{aligned}P(n(\vec{p}) \to n^c(\vec{p}); t) &= (1 - \sin^2 2\phi \cos^2\tilde{\alpha})\cos^2\theta\\ &\times \sin^2(\frac{1}{2}\Delta E t), \qquad (5)\end{aligned}$$

with $\sin\tilde{\alpha} = \sin\alpha\cos 2\phi/\sqrt{1 - (\sin\alpha\sin 2\phi)^2}$. Under CP transformation, which is equivalent to $\alpha \to -\alpha$, $\theta \to -\theta$, this oscillation formula with energy difference $\Delta E = \sqrt{\vec{p}^2 + M_+^2} - \sqrt{\vec{p}^2 + M_-^2}$ is invariant. We do not have a manifestation of CP violation in oscillation in vacuum, which is typically expressed by $P(n(\vec{p}) \to n^c(\vec{p}); t) \neq P(n^c(\vec{p}) \to n(\vec{p}); t)$.

References
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