Baryon interaction in lattice QCD†

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Both Lüscher's finite volume method (direct method) and the HAL QCD method are used to study hadron interactions in lattice QCD. Theoretically, these approaches are equivalent. However, both the deuteron and dineutron are considered bound states in the direct method, while both are considered scattering states in the HAL QCD method for heavier pion mass.[2)] To understand the origin of these deviations, we systematically study two-baryon systems using both the HAL QCD and finite volume methods.

In Lüscher's finite volume method, the phase shift $\delta(k)$ is obtained through

$$k \cot \delta(k) = \frac{1}{\pi L} \sum_{\vec{n} \in \mathbf{Z}^3} \frac{1}{|\vec{n}|^2 - |kL/(2\pi)|^2}, \qquad (1)$$

where k is given by the energy shift of two-baryon systems $\Delta E_{\rm BB} = 2\sqrt{m_B^2 + k^2} - 2m_B$ in the finite box L^3. In lattice simulations, $\Delta E_{\rm BB}$ is estimated from the plateau of the effective energy shift

$$\Delta E_{\rm BB}^{\rm eff}(t) \equiv -\frac{1}{a} \log\left(R_{\rm BB}(t+a)/R_{\rm BB}(t)\right), \qquad (2)$$

where $R_{\rm BB}(t) \equiv C_{\rm BB}(t)/\{C_{\rm B}(t)\}^2$ with the two-baryon propagator $C_{\rm BB}(t) \equiv \langle B(t)^2 \bar{B}(0)^2 \rangle$ and the baryon propagator $C_{\rm B}(t) \equiv \langle B(t)\bar{B}(0)\rangle$.

In lattice QCD, the signal-to-noise ratio of the multi-baryon system becomes exponentially worse for A baryons as $S(t)/N(t) \sim \exp[-A(m_B - (3/2)m_M)t]$ with the baryon number A and meson mass m_M. In addition, the energy gap of the elastic scattering states scales as $\mathcal{O}(1/L^2)$, which is about 50 MeV for a typical case. For ground-state saturation, $t \sim \mathcal{O}(10)$ fm is required.

To demonstrate the ground-state saturation problem, we consider the mock-up data as $R(t) = b_1 e^{-\Delta E_{\rm BB} t} + b_2 e^{-(\delta E_{\rm el} + \Delta E_{\rm BB})t} + c_1 e^{-(\delta E_{\rm inel} + \Delta E_{\rm BB})t}$, where $\delta E_{\rm el(inel)}$ is the gap of the (in)elastic state. Fig. 1 shows the effective energy shift of the mock-up data with typical values of the energy gaps $\delta E_{\rm el} = 50$ MeV and $\delta E_{\rm inel} = 500$ MeV. Without the elastic state $b_2/b_1 = 0$, the system converges to the ground state around $t \sim 1$ fm. However, a contamination of only 10% ($b_2/b_1 = \pm 0.1$) creates a fake plateau around $t \sim 1$ fm.

To check such fake plateaux, we study the quark source dependence of the effective energy shift. Fig. 2 shows the effective energy shift using a smeared source and wall source. Both show clear a plateau-like structure around $t \sim 15a \sim 1.5$fm. However, their energy-shift values are different. This fake plateau problem causes serious concerns on previous studies using the direct method.[2,4)]

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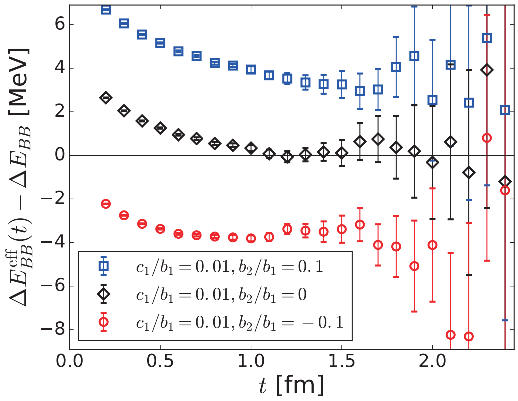

Fig. 1. Mock-up data with fluctuations.

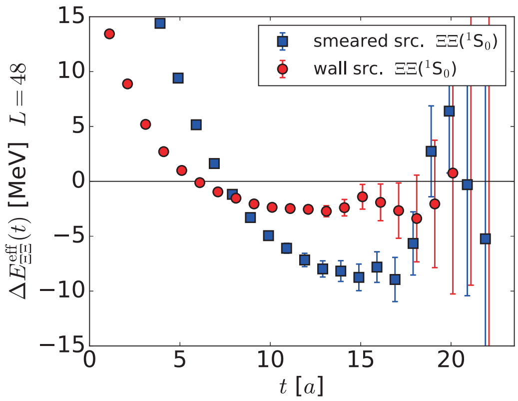

Fig. 2. Example of the effective energy shift $\Delta E^{\rm eff}(t)$.

On the other hand, in the HAL QCD method, we define the potential through the Nambu-Bethe-Salpeter function and study interactions based on this potential. In this approach, the ground-state saturation is not required, and we demonstrate the reliability of the HAL QCD method in the following papers.[3,4)]

References
1) T. Iritani for HAL QCD Collaboration, J. of High Energy Phys. **1610**, 101 (2016).
2) T. Yamazaki, Proceedings of Science (LATTICE2014) 009 (2015), and references therein.
3) T. Iritani for HAL QCD Collaboration, Proceedings of Science (LATTICE2016) 107 (2016).
4) S. Aoki, T. Doi, T. Iritani, Proceedings of Science (LATTICE2016) 109 (2016).