Sum rule study for Double Gamow-Teller state[†]

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Double charge exchange excitations (DCX) induced by heavy ion beams at intermediate energies[1,2] attract a lot of interest with respect to new collective excitations such as the double Gamow-Teller giant resonance (DGTR) and double beta decay matrix elements. A new research program based on a new DCX reaction (^{12}C, ^{12}Be(0_2^+)) is planned at the RIKEN RIBF facility with high intensity heavy ion beams at the optimal energy of E_{lab} =250MeV/u to excite the spin-isospin response. One notable advantage of this reaction is based on the fact that it is a $(2p, 2n)$ type DCX reaction, and one can use a neutron-rich target to excite DGT strength. Although many theoretical efforts have been made to study double beta decays, DGT strengths corresponding to the double beta decays are still too small to be identified in the reaction experiments. Minimum-biased theoretical prediction based on sum rules will provide a robust and global view of the DGTR and can be a good guideline for future experimental studies. In this paper, we present useful formulas for analyzing DGT strength using several sum rules for the double spin-isospin operator $(\sigma t_-)^2$. We also study DIAS excited through the double Fermi transition operator $(t_-)^2$ and double GT operator $(\sigma t_-)^2$.

The sum rule for the single GT transitions is well known and proportional to the neutron excess,

$$
\begin{aligned}
&S_- - S_+ \\
&= \sum_f |\langle f|\hat{O}_-(\mathrm{GT})|i\rangle|^2 - \sum_f |\langle f|\hat{O}_+(\mathrm{GT})|i\rangle|^2, \\
&= 3(N-Z), \qquad (1)
\end{aligned}
$$

where the GT transition operators read

$$
\hat{O}_\pm(\mathrm{GT}) = \sum_\alpha \sigma(\alpha)\mathrm{t}_\pm(\alpha). \qquad (2)
$$

The DGT transition operator $\hat{O}_\pm(\mathrm{GT})^2$ can be projected to good multipole states to be

$$
[\hat{O}_\pm(\mathrm{GT}) \times \hat{O}_\pm(\mathrm{GT})]^J_\mu, J = 0, 2. \qquad (3)
$$

The sum rule strength is expressed by the reduced matrix element

$$
D^J_\pm = \frac{1}{2J_i + 1} \sum_{J_f} |\langle J_f||[\hat{O}_\pm \times \hat{O}_\pm]^J||J_i\rangle|^2, \qquad (4)
$$

where J_i and J_f are the angular momenta of the initial and final states, respectively. Hereafter, we denote the initial state $|J_iM_i\rangle$ by the simple notation $|i\rangle$. The sum rule value for $J = 0$ excitations is evaluated as

$$
\begin{aligned}
&D_-^{(J=0)} - D_+^{(J=0)} \\
&= \langle i| \left[[\hat{O}_+ \times \hat{O}_+]^{(J=0)}, [\hat{O}_- \times \hat{O}_-]^{(J=0)} \right] |i\rangle, \\
&= 2(N-Z)(N-Z+1) \\
&+ \frac{4}{3}\left[(N-Z)S_+ - \langle i|[i\hat{\Sigma}\cdot(\hat{O}_- \times \hat{O}_+) + \hat{\Sigma}\cdot\hat{\Sigma}]|i\rangle \right], \\
&\qquad (5)
\end{aligned}
$$

where $\hat{\Sigma} = \sum_\alpha \sigma(\alpha)$ and $(\hat{O}_- \times \hat{O}_+)$ is the vector product of two operators. The sum rule for the $J^\pi = 2^+$ final states can be obtained in a similar way to that for the $J = 0$ states:

$$
\begin{aligned}
&D_-^{(J=2)} - D_+^{(J=2)} = 10(N-Z)(N-Z-2) \\
&+ \frac{10}{3}\left[2(N-Z)S_+ + \langle i|[i\hat{\Sigma}\cdot(\hat{O}_- \times \hat{O}_+) + \hat{\Sigma}\cdot\hat{\Sigma}]|i\rangle \right]. \\
&\qquad (6)
\end{aligned}
$$

Sum rule values for DGT transitions to $J^\pi = 0^+$ and 2^+ in Eqs. (5) and (6) are calculated for 10 nuclei, ^{6}He, ^{8}He, ^{14}C, ^{18}O , ^{20}O, ^{42}Ca, ^{44}Ca, ^{46}Ca, ^{48}Ca, and ^{90}Zr, together with those for the DIAS states by the Fermi and GT operators. In $N >> Z$ nuclei, the sum rule values of DGT transitions are approximately proportional to a factor (2J+1), i.e., the value for $J = 2$ is five times larger than that for $J = 0$ in the same nucleus. However, this proportionality is significantly modified in $N \sim Z$ nuclei. In the extreme, the sum rule for $J = 2$ transitions is smaller than that for $J = 0$ in the nuclei ^{6}He, ^{14}C, and ^{18}O with $N = Z + 2$, and the two values are almost equal in ^{42}Ca. In nuclei $N > Z+2$, the $J^\pi = 2^+$ excitations dominate the DGT strength because of the multipole factor $(2J+1)$, more than 0^+ excitations. However, in nuclei with $N \sim Z$, the 0^+ excitations become substantially strong, even larger than 2^+ excitations in light nuclei with $T = 1$ such as ^{14}C and ^{18}O. The excitation to DIAS is also studied through $(\sigma t_-)^2$ and t_-^2 operators to investigate the possibility of extracting the unit cross sections for DGT strength. We point out that the strength of DIAS excitations by the $(\sigma t_-)^2$ operator is competitive with the DGT strength in the light $T = 1$ nuclei. This characteristic feature may give a good opportunity to extract the unit cross section for DGT strength.

References

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[†] Condensed from the article in Phys. Rev. C **94**, 064325(2016)

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