Are two nucleons bound in lattice QCD for heavy quark masses?†

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The interactions between hadrons are important for understanding the origin of the matter. These interactions are described by quantum chromodynamics. There are two approaches to study two-hadron systems based on lattice QCD. In the direct method, one extracts the eigenenergy of a two-particle system through temporal correlation. In the HAL QCD method, potential is determined by spatial correlation.

These two methods should be equivalent. However, in the previous studies on heavy quarks masses, deuterons and dineutrons have been found to be bound in the direct method and unbound in the HAL QCD method. In a series of papers,[1,2] we reported that the direct method experiences systematic uncertainties resulting from elastic scattering states.

In the direct method, the energy shift is measured using the temporal correlation where the ground state is dominant. For example, inelastic excitation can be neglected around 1 fm. However, in two-particle systems, there are elastic scattering states, whose gap is considerably smaller than that of inelastic scattering states. Typically, ground state saturation requires $\mathcal{O}(10)$ fm. Owing to its slow convergence, one might misidentify the ground state energy around 1.5 fm.

To resolve such a fake measurement problem, we propose a simple check of the energy shift (ΔE_L) using the scattering phase shift, $\delta_0(k)$, which is given by

$$k \cot \delta_0(k) = \frac{1}{\pi L} \sum_{\vec{n} \in \mathbf{Z}^3} \frac{1}{|\vec{n}|^2 - |kL/(2\pi)|^2} \qquad (1)$$

according to Lüscher's finite volume formula, where k is given by $\Delta E_L = 2\sqrt{m_B^2 + k^2} - 2m_B$ in a finite box of volume L^3. The bound state corresponds to the pole of the S-matrix at $k \cot \delta_0(k)|_{k^2=-\kappa_0^2} = -\sqrt{-k^2}|_{k^2=-\kappa_0^2}$, which is determined by the extrapolation of $k \cot \delta_0(k)$ using effective range expansion (ERE), *i.e.*, $k \cot \delta_0(k) \simeq 1/a_0 + (1/2) r_{\mathrm{eff}} k^2 + \cdots$. In addition, the pole satisfies the physical residue condition by

$$\frac{d}{dk^2} [k \cot \delta_0(k)] \Big|_{k^2=-\kappa_0^2} < \frac{d}{dk^2} \left[-\sqrt{-k^2} \right] \Big|_{k^2=-\kappa_0^2}. \qquad (2)$$

However, none of the previous works analyze the bound state correctly based on the finite volume method. We re-examine their results carefully. For example, Fig. 1 shows the scattering phase shift obtained by Yamazaki *et al.*[3] for NN($^1S_0$) and NPLQCD Coll.[4] for NN($^3S_1$). The ERE in the upper panel shows singular behavior, while the EREs in the lower panel are inconsistent with each other. Moreover, ERE (NPL2015) violates the physical pole condition. These EREs suggest that the ground state is not measured correctly. We have concluded that *there are serious uncertainties about the existence of two-nucleon bound states for heavy quark masses in all previous studies.*[1]

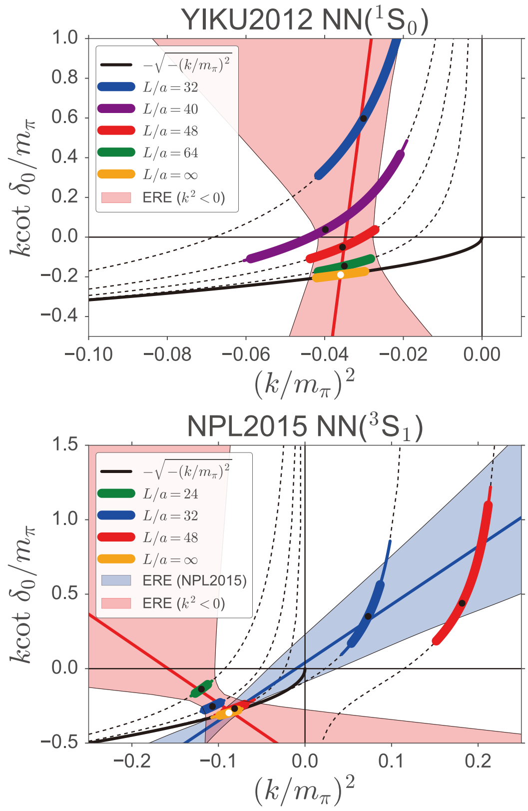

Fig. 1. Scattering phase shift (Upper) NN($^1S_0$) in Yamazaki *et al.*[3] (Lower) NN($^3S_1$) in NPLQCD Coll.[4] The red (blue) band denotes the fitted ERE for $k^2 < 0$ ($k^2 > 0$).

While the temporal correlation in the direct method experiences elastic scattering states, we define potential through spatial correlation in the HAL QCD method, which does not encounter this problem. Furthermore, we show the reliability of the HAL QCD method.[5]

† Condensed from the article in Physical Review D **96**, 034521 (2017)[1]

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References

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