Computing the nucleon charge and axial radii directly at $Q^2 = 0$ in lattice QCD†

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The square of the proton electric charge radius is defined as

$$r_E^2 = -6 \left.\frac{\mathrm{d}G_E}{\mathrm{d}Q^2}\right|_{Q^2=0}, \tag{1}$$

where G_E is the proton electric form factor and $-Q^2 = (p' - p)^2$ is the square of the four-momentum transfer. In view of the proton radius puzzle,[1)] reliable lattice QCD calculations of r_E are needed. The conventional approach involves computing G_E at several discrete values of the momenta allowed by the boundary conditions of the lattice, followed by a fit of the form factor shape that then allows the evaluation of the derivative with respect to Q^2. The fit of the form factor shape may introduce a systematic uncertainty that is difficult to quantify.

In this work, we investigated an alternative approach that allows the lattice computation of r_E^2 and other momentum derivatives of nucleon matrix elements directly at $Q^2 = 0$. In this method, momentum derivatives of quark propagators are taken symbolically at zero momentum[2)] before performing the path integral over the gauge fields. We implemented this method to extract the nucleon isovector magnetic moment and charge radius as well as the isovector induced pseudoscalar form factor at $Q^2 = 0$ and the axial radius. For comparison, we also determined these quantities with the conventional approach, using the z-expansion method[3)] to fit the form factor shape. The calculation was done at the physical pion mass, on a 64^4 lattice with $2 + 1$ flavors of clover fermions.

Some of our results are shown in Fig. 1 and Table 1. The results from the z expansion and from the derivative method are consistent with each other, which provides a valuable cross-check. Compared to the z expansion, the derivative method suffers from larger statistical uncertainties, especially for the radii, which we obtained from second-order derivatives with respect to $\mathbf{p}$.

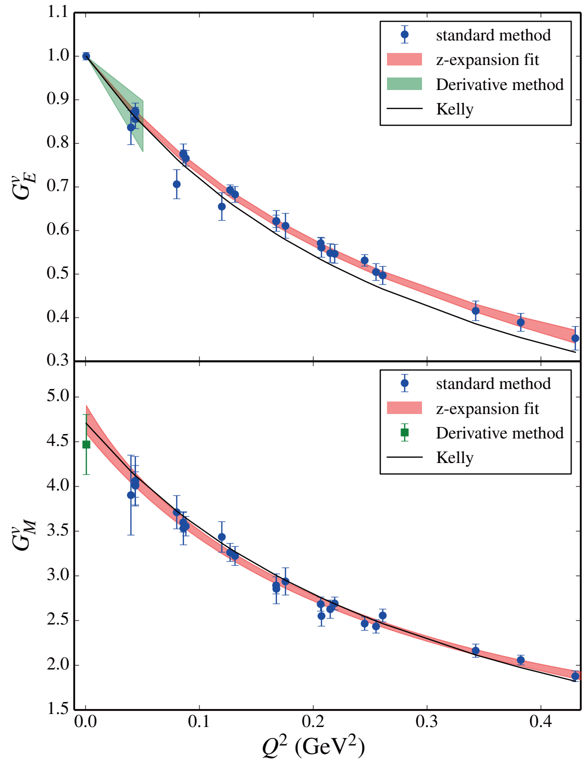

Fig. 1. Results for the nucleon isovector electric and magnetic form factors. The black curve shows a fit to electron-nucleon scattering data.[4)]

Table 1. Summary of our results at $Q^2 = 0$.

Quantity		z-expansion	derivative
μ^v	$T/a = 10$	3.899(38)	3.898(54)
	summation	4.75(15)	4.46(33)
$(r_E^2)^v$ [fm^2]	$T/a = 10$	0.608(15)	0.603(29)
	summation	0.787(87)	0.753(273)
$G_P^v(0)$	$T/a = 10$	75(1)	69(1)
	summation	137(7)	137(15)
$(r_A^2)^v$ [fm^2]	$T/a = 10$	0.249(12)	0.288(61)
	summation	0.295(68)	−0.120(492)

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