All existing constrained Hartree-Fock-Bogoliubov (CHFB) calculations neglect the Fock or the exchange terms. Here, the constraint conditions are applicable to the proton number $Z$, neutron number $N$, and the angular momentum along the chosen $x$-axis $\langle I_x \rangle = \sqrt{I(I+1)}$. The numerical calculations start from the spherical single-particle basis and include the residual quadrupole-quadrupole (Q-Q), monopole-pairing (MP), and quadruple-pairing (QP) interactions.\(^1\) In the approximation without the exchange terms, only the terms $Y_2^2$ and $Y_2^2 + Y_2^0$ in the Q-Q interaction are considered but the terms $Y_2^0 \pm Y_2^1$ and $Y_2^0 - Y_2^1$ are not. We have developed the code to include all exchange terms in the residual interactions. The exchange terms of Q-Q contribute to the self-energy $\Gamma$ and the gap $\Delta$, and those of MP and QP to $\Gamma$. Then, the constraint on angular momentum $\langle I_x \rangle$ becomes $\sqrt{I(I+1)} - \langle I_x \rangle$. We chose the signature invariant base that reduces the diagonalization space to half\(2^\dagger\) because the total Hamiltonian with three constraints $H'$ is invariant under the operator $R_x = \exp(-i\pi I_x)$. All input matrix elements are rewritten in this base and the spherical single-particle operator in this base $C_k$ is transformed to quasiparticle operators $\hat{\alpha}_k^+ = \sum_{\nu>0}(C_k^\dagger A_{\nu k} + C_k B_{\nu k})$ and $\hat{\alpha}_k^- = \sum_{\nu>0}(C_k^\dagger A_{\nu k} + C_k B_{\nu k})$, where the notation $\hat{k}$ is the time reversal of $k$. Then, the CHFB equation becomes:

$$\left( h^1 - \omega j_x \Delta^T \begin{array}{cc} \Delta^T & -h^{2s} - \omega j_x \\ \Delta^* & A^* \end{array} \right) \left( \begin{array}{c} \hat{B}^* \\ \hat{A}^* \end{array} \right) = \left( \begin{array}{cc} \hat{B}^* \\ \hat{A}^* \end{array} \right) \begin{array}{cc} \hat{A} & B \\ \hat{A} & \hat{B} \end{array} \right) \begin{array}{c} \hat{A} \\ \hat{B} \end{array},$$

where $h^1$ ($h^2$) includes the spherical single-particle energy and self-energy $\Gamma$ and $\omega$ is the Lagrange multiplier for $I_x = \sum_{\nu>0}(\Delta_{\nu k} A_{\nu k} + C_k B_{\nu k})$. When there is no $\omega j_x$, i.e., without constraint on $\langle I_x \rangle$, $h^2$, $\hat{A}$, $\hat{A}^*$, and $\hat{B}$ are reduced to $h^1$, $\Delta$, $A$, and $B$, respectively. The iteration procedure in the numerical analysis is based on the gradient method.\(^3\)

Figure 1 compares $I$ versus transition energy $\Delta E = E(I) - E(I - 2)$. The parameters are the same spherical single-particle energy as listed in Table 1 in Ref. 1). The strength of MP is $G_{\pi+\pi+} = G_{\pi-\pi-} = 0.22$ MeV and $C_{\nu+\nu+} = C_{\nu-\nu-} = -0.23$ MeV. The strength of QP is 10% of MP, expressed in terms of MeV/$b^4$ and the strength of QQ is $\chi_{++} = -0.030$ MeV/$b^4$, $\chi_{+-} = -0.032$ MeV/$b^4$, and $\chi_{--} = -0.100$ MeV/$b^2$ with an oscillator length $b$. These numerical results are consistent with the experimental data except for the $I = 15/2^-$ state. The constraint on the proton number $Z$ is effective but the value of $Z$ for $+$ and $-$ parity states is mixed owing to $G_{\nu+\nu-}$. For example, $Z_+ = 13.58$ and $Z_- = 17.42$ at $I = 15/2^-$, while $Z_+ = 13.99$ and $Z_- = 17.01$ at $I = 55/2^-$, where $Z = Z_+ + Z_-$ is the proton number outside the magic number 28. For better results, another constraint on $Z$ should be included or the MP and QQ interactions between the $+$ and $-$ parity states should be dropped. This calculation is preliminary and there is room for finding better parameter sets.

References