

# Isobaric analog state energy in deformed nuclei: A toy model<sup>†</sup>

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The effects of deformation on the energy of the isobaric analog state (IAS) are studied through microscopic deformed Hartree-Fock-Bogolyubov calculations. A simple yet physical toy model is also presented to provide guidance when predicting unknown IAS energies of deformed nuclei. The deformed HFB calculations are performed for several neutron-deficient medium-mass and heavy nuclei to predict the IAS energies.

Isospin is one of the most important (approximate) symmetries in nuclei. The validity of isospin symmetry has been established by the experimental observation of IASs by charge-exchange reactions. Recently, these states have been investigated extensively in connection with the symmetry energy, particularly to determine the so-called slope parameter  $L$ . Thus far, theoretical studies of IAS have mainly focused on spherical nuclei such as  $^{48}\text{Ca}$ ,  $^{90}\text{Zr}$ , and  $^{208}\text{Pb}$ . However, a large number of deformed nuclei exist, but transparent information on the effects of deformation on IAS is still lacking. In this paper, we derive a general formula for the effects of deformation on the Coulomb direct contribution to the energy of the IAS and provide a simple albeit physical model. Then, we study several neutron-deficient medium-mass and heavy nuclei, which are now planned to be studied experimentally in RCNP, Osaka within the LUNESTAR project.

The IAS energy  $E_{\text{IAS}}$  can be defined as the energy difference between the analog state  $|A\rangle$  and the parent state  $|0\rangle$  as follows:

$$E_{\text{IAS}} = \langle A | \mathcal{H} | A \rangle - \langle 0 | \mathcal{H} | 0 \rangle = \frac{\langle 0 | T_+ [\mathcal{H}, T_-] | 0 \rangle}{\langle 0 | T_+ T_- | 0 \rangle}, \quad (1)$$

where  $T_{\pm} = \sum_i^A t_{\pm}(i)$  are the isospin raising/lowering operators. The direct Coulomb term of IAS energy for an axially symmetric nucleus can be evaluated as

$$E_{\text{IAS}}^{\text{Cd}} = E_{\text{IAS}}^{\text{Cd,sph}} \left[ 1 - \frac{\beta_{2n}\beta_{2p}}{4\pi} + \frac{(\beta_{2n} - \beta_{2p})(\beta_{2n} + \beta_{2p})}{4\pi} + \frac{(\beta_{2n} - \beta_{2p})^2}{4\pi} \right], \quad (2)$$

where  $\beta_{2n(2p)}$  is the quadrupole deformations for neutrons (protons). For  $\beta_{2n} = \beta_{2p}$ , this reduces to

$$E_{\text{IAS}}^{\text{Cd}} = E_{\text{IAS}}^{\text{Cd,sph}} \left[ 1 - \frac{\beta_2^2}{4\pi} \right]. \quad (3)$$

From Eq. (3), one should expect that a larger quadrupole deformation corresponds to a smaller IAS energy. For a qualitative understanding of the effect of deformation

Table 1. Results of deformed HFB calculations with SAMi EDF.<sup>1)</sup> All energies are in MeV.  $E_{\text{IAS}}^{\text{HFB}}$  is the sum of the direct Coulomb, exchange Coulomb, and isospin mixing contributions. The deformation effect on IAS energy has been estimated from the HFB calculations,  $\Delta E_{\text{IAS}}^{\text{HFB}} \equiv E_{\text{IAS}}^{\text{HFB}}(\beta_{2n}, \beta_{2p}) - E_{\text{IAS}}^{\text{HFB}}(\beta_{2n} = 0, \beta_{2p} = 0)$ .

Nucl.	$\beta_{2n}$	$\beta_{2p}$	$E_{\text{IAS}}^{\text{HFB}}$	$E_{\text{IAS}}^{\text{exp}2)}$	$\Delta E_{\text{IAS}}^{\text{HFB}}$
$^{102}_{46}\text{Pd}$	0.186	0.174	13.346		-0.162
$^{106}_{48}\text{Cd}$	0.255	0.256	13.810		-0.089
$^{112}_{50}\text{Sn}$	0.192	0.199	14.085	14.019	-0.001
$^{120}_{52}\text{Te}$	0.039	0.042	14.353		-0.001
$^{124}_{54}\text{Xe}$	0.000	0.000	14.749		0.000
$^{130}_{56}\text{Ba}$	0.000	0.000	15.114		0.000
$^{136}_{58}\text{Ce}$	0.126	0.158	15.352		-0.081
$^{138}_{58}\text{Ce}$	0.040	0.047	15.331		-0.010
$^{142}_{60}\text{Nd}$	0.000	0.000	15.509		0.000
$^{144}_{62}\text{Sm}$	0.000	0.000	15.950	16.075	0.000
$^{156}_{66}\text{Dy}$	0.201	0.225	16.755		-0.003
$^{158}_{66}\text{Dy}$	0.197	0.217	16.723		-0.028
$^{162}_{68}\text{Er}$	0.348	0.378	16.975	16.861	-0.112
$^{164}_{68}\text{Er}$	0.346	0.377	16.881	16.778	-0.110
$^{168}_{70}\text{Yb}$	0.385	0.413	17.171		-0.212
$^{174}_{72}\text{Hf}$	0.304	0.319	17.629		-0.091
$^{176}_{72}\text{Hf}$	0.267	0.276	17.538	17.388	-0.086
$^{180}_{74}\text{W}$	0.278	0.299	17.911		-0.101
$^{184}_{76}\text{Os}$	0.335	0.355	18.223		-0.171
$^{190}_{78}\text{Pt}$	-0.147	-0.141	18.649		-0.037
$^{196}_{80}\text{Hg}$	-0.180	-0.187	18.906		-0.070

on the IAS energy, Eq. (3) predicts, for very deformed nuclei with  $\beta_2 \approx 0.8$ , a relative reduction in  $E_{\text{IAS}}$  of approximately 5% with respect to the spherical nucleus.

Table 1 lists the results of deformed HFB calculations for several neutron-deficient nuclei as predicted by SAMi EDF.  $E_{\text{IAS}}^{\text{HFB}}$  is the sum of the direct Coulomb, exchange Coulomb, and isospin mixing contributions. The deformation effect is also shown as  $\Delta E_{\text{IAS}}^{\text{HFB}}$ , and it varies from -212 keV in  $^{168}\text{Yb}$  at the maximum to zero for no deformation cases.

The deformed HFB results reproduce well the experimentally known IAS energies within an error of approximately 100 keV and provide a good guide for the experimental search for new IAS states of these neutron-deficient nuclei. In general, the deformation effect is small, but it might be important for the precise prediction of IAS energy within an accuracy of several tenths of keV.

## References

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