Time development of conformal field theories associated with $L_1$ and $L_{-1}$ operators†

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An unconventional time development of the two-dimensional conformal field theory (CFT) induced by the $L_1$ and $L_{-1}$ operators was studied by employing the formalism previously developed in a study of sine-square deformation (SSD). Consequently, we found that the retention of the Virasoro algebra naturally leads to the presence of a cut-off near the fixed points (shown as gray blobs in Fig. 1). The introduction of a scale by the cut-off may appear at odds with the conformal symmetry; however, it is essential to derive the formula for entanglement entropy.2,3

The SSD of two-dimensional CFT has been explained with a formalism developed in Refs. 5, 6) and demonstrate that the entanglement Hamiltonian and other interesting deformations of two-dimensional CFT were studied.

In this study, we visit the following particular time development, which was not studied in Ref. 6):

$$L_1 + L_{-1} + \tilde{L}_1 + \tilde{L}_{-1}, \tag{1}$$

instead of ordinary $L_0 + \tilde{L}_0$. We found that this time-development corresponds to the entanglement Hamiltonian as discussed in Ref. 7).

The significance of the operator (1) can be understood in the following way. The $L_0$, $L_1$, and $L_{-1}$ operators constitute $\text{sl}(2, \mathbb{R})$ algebra. A linear combination of these operators,

$$x^{(0)} L_0 + x^{(1)} L_1 + x^{(-1)} L_{-1}, \tag{2}$$

can be mapped to

$$x^{(0)} L_0 + x^{(1)} L_1 + x^{(-1)} L_{-1}, \tag{3}$$

by the adjoint action of $\text{sl}(2, \mathbb{R})$; however, the following quadratic form of the coefficients, which is known as the quadratic Casimir element, remains the same:

$$c^{(2)} \equiv (x^{(0)})^2 - 4x^{(1)}x^{(-1)}. \tag{4}$$

Using this $c^{(2)}$, the general linear combinations (2) can be categorized into three distinctive groups that are not accessible from each other by the $\text{sl}(2, \mathbb{R})$ action. Typical operators for each group are: $L_0$ represents $c^{(2)} > 0$, $L_0 - \frac{1}{2}(L_1 + L_{-1})$ represents $c^{(2)} = 0$, and $L_1 + L_{-1}$ represents $c^{(2)} < 0$. Thus, we could investigate the $L_1 + L_{-1}$ operator by applying the formalism developed in Refs. 5, 6) and demonstrate that the aforementioned three cases, including $L_1 + L_{-1}$, can be studied in a unified manner.

![Flow of time $t$ can be considered to begin at a section of space with length $L$ (solid line). The remaining space, including infinity, is depicted as a dashed line and denoted by $L'$. The setup corresponds to the entanglement entropy for the segment with length $L$.](https://example.com/figure.png)

References


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