Time development of conformal field theories associated with L_1 and L_{-1} operators†

T. Tada[*1,*2]

An unconventional time development of the two-dimensional conformal field theory (CFT) induced by the L_1 and L_{-1} operators was studied by employing the formalism previously developed in a study of sine-square deformation (SSD).[1)] Consequently, we found that the retention of the Virasoro algebra naturally leads to the presence of a cut-off near the fixed points (shown as gray blobs in Fig. 1). The introduction of a scale by the cut-off may appear at odds with the conformal symmetry; however, it is essential to derive the formula for entanglement entropy.[2,3)]

The SSD of two-dimensional CFT[4)] has been explained with a formalism developed in Refs. 5, 6), in which unconventional time developments, other than the radial time development, were utilized to study CFTs. One such time development suited with the study of SSD was named "dipolar quantization." Further generalization was investigated by Wen, Ryu and Ludwig,[7)] where the entanglement Hamiltonian and other interesting deformations of two-dimensional CFT were studied.

In this study, we visit the following particular time development, which was not studied in Ref. 6):

$$L_1 + L_{-1} + \bar{L}_1 + \bar{L}_{-1}, \tag{1}$$

instead of ordinary $L_0 + \bar{L}_0$. We found that this time-development corresponds to the entanglement Hamiltonian as discussed in Ref. 7).

The significance of the operator (1) can be understood in the following way. The L_0, L_1, and L_{-1} operators constitute $sl(2, \mathbf{R})$ algebra. A linear combination of these operators,

$$x^{(0)} L_0 + x^{(1)} L_1 + x^{(-1)} L_{-1}, \tag{2}$$

can be mapped to

$$x'^{(0)} L_0 + x'^{(1)} L_1 + x'^{(-1)} L_{-1}, \tag{3}$$

by the adjoint action of $sl(2, \mathbf{R})$; however, the following quadratic form of the coefficients, which is known as the quadratic Casimir element, remains the same:

$$c^{(2)} \equiv \left(x^{(0)}\right)^2 - 4x^{(1)} x^{(-1)}. \tag{4}$$

Using this $c^{(2)}$, the general linear combinations (2) can be categorized into three distinctive groups that are not accessible from each other by the $sl(2, \mathbf{R})$ action. Typical operators for each group are: L_0 represents $c^{(2)} > 0$, $L_0 - \frac{1}{2}(L_1 + L_{-1})$ represents $c^{(2)} = 0$, and $L_1 + L_{-1}$ represents $c^{(2)} < 0$. Thus, we could investigate the $L_1 + L_{-1}$ operator by applying the formalism developed in Refs. 5, 6) and demonstrate that the aforementioned three cases, including $L_1 + L_{-1}$, can be studied in a unified manner.

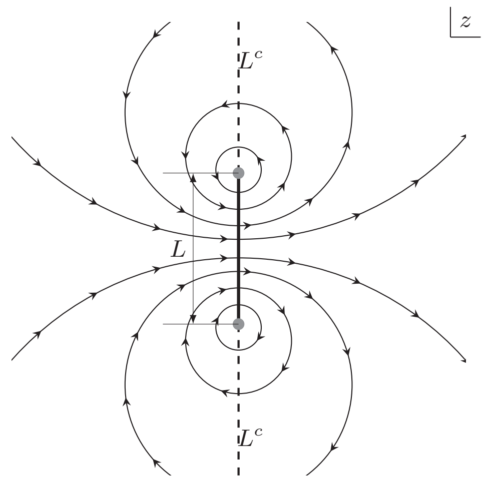

Fig. 1. Flow of time t can be considered to begin at a section of space with length L (solid line). The remaining space, including infinity, is depicted as a dashed line and denoted by L^c. The setup corresponds to the entanglement entropy for the segment with length L.

References
1) A. Gendiar, R. Krcmar, T. Nishino, Prog. Theor. Phys. **122**, 953 (2009), Erratum: [Prog. Theor. Phys. **123**, 393 (2010)].
2) C. Holzhey, F. Larsen, F. Wilczek, Nucl. Phys. B **424**, 443 (1994).
3) H. Casini, M. Huerta, R. C. Myers, J. High Energy Phys. **1105**, 036 (2011).
4) H. Katsura, J. Phys. A **45**, 115003 (2012).
5) N. Ishibashi, T. Tada, J. Phys. A **48**, 315402 (2015).
6) N. Ishibashi, T. Tada, Int. J. Mod. Phys. A **31**, 1650170 (2016).
7) X. Wen, S. Ryu, A. W. W. Ludwig, Phys. Rev. B **93**, 235119 (2016).

† Condensed from the article in J. Phys. A Math Theor. **53**, 255401 (2020)
*1 RIKEN Nishina Center
*2 RIKEN iTHEMS