Study of Lorentzian sine-square deformed CFT[†]

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The concept of sine-square deformation (SSD) was first introduced into two-dimensional conformal field theory (2D CFT) in Ref. 1). In previous studies, we found that the introduction of SSD in 2D CFT defines a new time translation generated by the SSD Hamiltonian.[1)] The SSD CFT processes the Virasoro algebra with a continuum index.[2)] Further, inspired by the discovery in SSD CFT, we generalized this study to CFT using more general modular Hamiltonians, and found three different types of Virasoro algebra in the Euclidean CFT.[3)] In this study, we extend our analysis to Lorentzian CFT. To consider CFT in the Minkowski spacetime, the universal covering space of the Minkowski spacetime must be introduced. Generally, a spacetime is mapped to a Penrose diamond on the cylinder, which is the universal covering of the Penrose diamond. The time translation is defined on the universal covering using the Luscher-Mack Hamiltonian $\frac{\hat{P}_0+\hat{K}_0}{2}$.[4)] Consider the worldline for a particle in a Penrose diamond; if we perform a special conformal transformation, the worldline may cross the boundary of the Penrose diamond and move on to the next patch. This problem can be solved if the time translation for the CFT is defined to be confined in a single Penrose diamond, implying that the CFT becomes effectively non-compact under such time translations. This can be achieved by applying SSD in Lorentzian CFT; the non-compactness of CFT can be observed from the continuum index of the Virasoro Algebra.[5)] We performed similar analysis as that in the Euclidean signature.[2,3)] Next, we investigate the three different cases with Hamiltonians having plus, minus, and zero values in the quadratic Casimir element. We select three examples from the three cases:

- The Luscher Mack Hamiltonian $\frac{\hat{P}_0+\hat{K}_0}{2}$,
- The Rindler Hamiltonian $\hat{M}_{01}$,
- The Naive Hamiltonian $\hat{P}_0$.

We define different mode decompositions of the energy-stress tensor with 0th modes corresponding to each Hamiltonian in the three cases. By applying the commutation relation of the energy-stress tensor, we can obtain three types of Virasoro Algebra:

- Luscher Mack type: The Virasoro generators in this type must bear discrete indices; otherwise, ambiguities are observed in the generators.
- Rindler type: The Virasoro generators in this type must bear discrete indices to remove the ambiguities and preserve the local conformal symmetry. There is UV divergence in the central extensional term in the Virasoro Algebra.
- The Naive type: The Virasoro generators can have continuum indices. This corresponds to the SSD case, where the theory is effectively non-compact. Thus, the time translation is confined inside a single Penrose diagram. See Fig. 1.

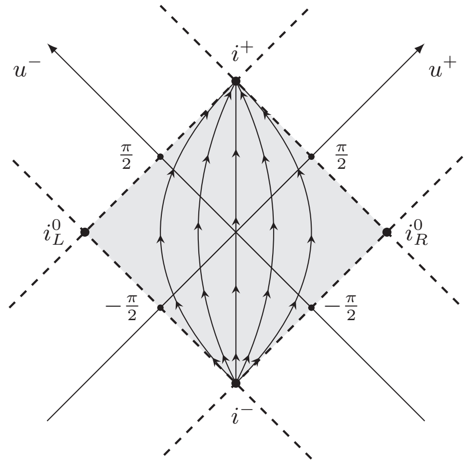

Fig. 1. The universal covering space of the Penrose diamond (shaded region) is charted by u^+ and u^-. While the conformal symmetry is represented using the entire covering space, the time flow by P_0 (arrowed line) is confined within a single Penrose diamond.

The conformal invariance requires the introduction of the universal covering space; moreover, occasionally, the Lorentzian CFT is considered to be unphysical owing to the existence of a closed time-like curve. In this study, our analysis ensured that there is no closed time-like curve if the Naive Hamiltonian is selected instead of the Luscher-Mack Hamiltonian.[5)]

References

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