Empirical formulas for the standard-model parameters

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We report empirical formulas for the parameters of the standard-model. Table 1 lists the formulas for the mass of the charged leptons (e, μ, τ), three neutrinos (ν_1, ν_2, ν_3), six quarks (u, c, t, d, s, b), and gauge bosons (W, Z), and Higgs boson (H). The formulas yield the masses in terms of the Planck mass

$$M_{pl} = 1.220910 \pm 0.000029 \times 10^{19}\ \text{GeV}.$$

The last column of the table presents the relative difference $|m_p^c/m_p^m - 1|$ of the calculated value m_p^c and the measured value m_p^m for particle p. Table 2 compares the mass values from the formulas and neutrino oscillation data. Table 3 lists the formulas for the Cabibbo-Kobayashi-Maskawa quark mixing parameters. Table 4 lists the formulas for the neutrino-mixing angles. Table 5 lists the formulas for the fine structure constant α and the strong coupling constant α_s. These formulas yield 24 of 25 free parameters of the standard-model. The remaining one, the neutrino CP violation angle δ_{CP}, has not been measured. The values calculated from the formulas are in good agreement with the data. The one common constant in the mass formulas, $\epsilon_0 = 2 \times (6\pi)^{-48}$, which agrees with the Hubble constant H_0 times the Planck time $t_{\rm pl}$ ($\epsilon_0 \simeq H_0 \times t_{\rm pl}$) within the accuracy of H_0, suggests that the particle masses are related to the expansion of the universe. A model to explain these formulae is reported in the next article,[1)] and implications to gravity and cosmology are reported in the article appearing after that.[2)]

Table 1. Formulas for the masses of the SM particles.

p	formula ($\mu_p = m_p/M_{\rm pl}$)	$\lvert m_p^c/m_p^m - 1\rvert$
e	$\frac{1}{12\pi^2}\epsilon_0^{1/3}\left(1+\frac{1}{4}\frac{1}{(6\pi)^2}\right)^{-1}$	5.9×10^{-6}
μ	$\frac{3}{2}\epsilon_0^{1/3}\left(1-\frac{3}{6\pi}+\frac{27}{4}\frac{1}{(6\pi)^2}\right)^{-1}$	5.2×10^{-5}
τ	$9\pi\epsilon_0^{1/3}\left(1-\frac{3}{4}\frac{1}{6\pi}+\frac{5}{4}\frac{1}{(6\pi)^2}\right)^{-1}$	1.6×10^{-5}
ν_1	$\frac{2}{3}\epsilon_0^{1/2}\left(1+\frac{1}{6\pi}\right)^{-1}$	See Table 2
ν_2	$2\epsilon_0^{1/2}\left(1-\frac{1}{6\pi}\right)^{-1}$	See Table 2
ν_3	$4\pi\epsilon_0^{1/2}\left(1+\frac{1}{6\pi}\right)^{-1}$	See Table 2
t	$8(6\pi)^2\epsilon_0^{1/3}$	5.5×10^{-3}
c	$12\epsilon_0^{1/3}$	3.8×10^{-2}
u	$8(6\pi)^{-2}\epsilon_0^{1/3}$	3.9×10^{-3}
b	$3(6\pi)\epsilon_0^{1/3}\left(1+\frac{3}{2}\frac{1}{6\pi}+\frac{27}{4}\frac{1}{(6\pi)^2}\right)^{-1}$	5.6×10^{-3}
s	$\epsilon_0^{1/3}$	2.2×10^{-2}
d	$(6\pi)^{-1}\epsilon_0^{1/3}\left(1+\frac{1}{6\pi}\right)^{-1}$	1.6×10^{-2}
Z	$\frac{1}{(8\pi^2)}\epsilon_0^{1/4}\left(1+\frac{1}{12}\frac{1}{6\pi}+\frac{1}{12}\frac{1}{(6\pi)^2}\right)^{-1/2}$	1.5×10^{-5}
W	$\frac{2^{-1/4}}{(8\pi^2)}\epsilon_0^{1/4}\left(1-\frac{3}{2}\frac{1}{6\pi}-\frac{9}{4}\frac{1}{(6\pi)^2}\right)^{-1/2}$	1.0×10^{-4}
H	$\frac{2^{1/2}}{8\pi^2}\epsilon_0^{1/4}\left(1+\frac{3}{2}\frac{1}{6\pi}-\frac{9}{2}\frac{1}{(6\pi)^2}\right)^{-1/2}$	3.4×10^{-4}

Table 2. Comparison of the calculated masses of neutrinos with the neutrino oscillation data.

Quantity	Calculated	Measured
$m_2^2 - m_1^2$	7.39×10^{-5} eV^2	$7.37^{+0.20}_{-0.15} \times 10^{-5}$ eV^2
$m_3^2 - m_1^2$	2.58×10^{-3} eV^2	$2.56 \pm 0.04 \times 10^{-3}$ eV^2

Table 3. Formulas of the CKM matrix elements.

	formula	calculated	measured
V_{us}	$\left(\frac{1}{6\pi}\left(1+\frac{1}{6\pi}\right)^{-1}\right)^{1/2}$	0.22445	0.22452 ± 0.0044
V_{cb}	$\left(\frac{2}{3}\right)^{1/2}\frac{1}{6\pi}$	0.04332	0.04214 ± 0.00076
V_{ub}	$\frac{4}{3}\frac{1}{(6\pi)^2}$	0.003753	0.00365 ± 0.00012
$\bar{\eta}$	$\left(1+\frac{3}{2}\frac{1}{6\pi}+\frac{27}{4}\frac{1}{(6\pi)^2}\right)\frac{1}{\pi}$	0.3497	$0.355^{+0.012}_{-0.011}$

Table 4. Formulas of the neutrino-mixing matrix.

	formula	calculated	measured
s_{12}	$\left(\frac{1}{3}\left(1-\frac{1}{6\pi}\right)\left(1+\frac{1}{6\pi}\right)^{-1}\right)^{1/2}$	0.547	0.545 ± 0.016
s_{23}	$\left(\frac{3}{2\pi}\left(1+\frac{1}{6\pi}\right)\left(1-\frac{1}{6\pi}\right)^{-1}\right)^{1/2}$	0.729	0.714 ± 0.053
s_{13}	$\left(\frac{1}{12\pi}\right)^{1/2}\left(1-\frac{1}{6\pi}\right)\left(1+\frac{1}{6\pi}\right)^{-1}$	0.146	0.147 ± 0.003

Table 5. Formulas of the coupling constants α and $\alpha_s(M_Z)$

	formula	calculated	rel. error
α^{-1}	$44\pi\left(1+\frac{1}{3}\frac{1}{6\pi}\right)^{-1/2}$	137.0238	8.9×10^{-5}
$\alpha_s(M_Z)$	$\frac{\sqrt{2}}{4\pi}\left(1+\frac{1}{6\pi}\right)$	0.11851	3.5×10^{-3}

References
1) Y. Akiba, in this report.
2) Y. Akiba, in this report.

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