Quarternion-spin-isospin model for the standard-model parameters

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The standard model (SM) successully describes the nature, but it has at least 25 free parameters. In the preceding article[1)] we reported empirical formulas with no free parameter for 24 SM parameters: 15 particle masses, four Cabbibo-Kobayashi-Masukawa (CKM) quark mixing parameters, three neutrino mixing angles, the fine structure constant, and the strong coupling constant.

The pattern of the formulas suggests that there is an algebraic model underpinning them and that the numbers 6π and $\epsilon_0 = 2 \times (6\pi)^{-48}$ play key roles in this model. The fact that ϵ_0 agrees with the Hubble constant H_0 times the Planck time $t_{\rm pl}$, $\epsilon_0 \simeq H_0 t_{\rm pl}$, suggests that there is a relation between the mass of the SM particles and the metric of spacetime.

In this article, we introduce the quarternion-spin-isospin (QST) model, which explains these formulas and the relation $H_0 t_{pl} = \epsilon_0$. The QST model is so named because it is based on operators that are products of unit quarternions and operators of spin and weak isospin.

The QST model is based on the following 64 operators that are introduced here as normalized primordial action (NPA):

$$\left\{\frac{I^\mu \sigma^\nu \tau^a}{6\pi}, \frac{\varepsilon\tau^a}{6\pi}, \frac{\varepsilon i}{6\pi}, \frac{\varepsilon I^c \tau^c}{6\pi}, \frac{\varepsilon i \tau^3}{6\pi}, \frac{-\varepsilon I^c}{4\pi}, \frac{-\varepsilon i}{2\pi}, -\varepsilon, \varepsilon, i, 1\right\}.$$

The QST model has five primary ansatzes.

(A1) The Planck time $t_{\rm pl} = 5.3912 \times 10^{-44}$ s is the minimum duration of time in nature. The time t is an integer n in the unit of $t_{\rm pl}$, and the physics state at $t = n$ is represented as $|n\rangle$. $t_{\rm pl}$ is a fundamental constant of nature, as are the speed of light in vacuum c and the Planck constant $\hbar$.

(A2) The change of $|n\rangle$ for $t_{\rm pl}$ is represented by a set of 49 operators $\hat{\mathcal{L}}_p^{\rm EAD}$ we name as elementary action density (EAD). An EAD corresponds to a term of the SM Lagrangian.

(A3) An EAD is a sum of the equivalent vee product $P_{48}(S_{48}, \sigma)$ of 48 NPAs,

$$\hat{\mathcal{L}}_p^{\rm EAD} = \sum P_{48}(S_{48}^{NPA}, \sigma) = \sum \hat{\partial}_{\sigma(p_1)} \vee \cdots \vee \hat{\partial}_{\sigma(p_{48})},$$

where $S_{48}^{NPA} = \{\hat{\partial}_{p_1}, \cdots, \hat{\partial}_{p_{48}}\}$ is a set of 48 NPAs and σ is a permutation $(k \to \sigma(k), 1 \le k \le 48)$.

(A4) All 49 EADs are invariant for the following permutations:

$$(I^1, I^2, I^3) \to (I^1, I^2, I^3), (I^2, I^3, I^1), (I^3, I^1, I^2),$$
$$(\sigma^1, \sigma^2, \sigma^3) \to (\sigma^1, \sigma^2, \sigma^3), (\sigma^2, \sigma^3, \sigma^1), (\sigma^3, \sigma^1, \sigma^2).$$

There are 12 EADs that are invariant for permutations:

$$(\tau^1, \tau^2, \tau^3) \to (\tau^1, \tau^2, \tau^3), (\tau^2, \tau^3, \tau^1), (\tau^3, \tau^1, \tau^2). \tag{1}$$

(A5) The SM emerges as a continuous approximation of the QST model as each EAD becomes a term of the SM Lagrangian in the limit of $t_{\rm pl} \to 0$.

We found 49 EADs that correspond to the elementary particles of the SM, which are summarized in Table 1. All of the 24 formulas of the parameters of the standard model can be derived from the 49 EADs in the table. The model also predicts that the CP violation in the neutrino sector is 100%. Thus, all of the 25 free parameters of the standard model are derived from the QST model. In additon, the model predicts that the Hubble constant times the Planck time is $H_0 t_{pl} = 2 \times (6\pi)^{-48}$. We discuss the implications for general relativity and cosmology in the next article.[2)]

Table 1. Values of elementary action densities.

	$(6\pi)^2\epsilon_0 i\tau^3$	$(6\pi)^2\epsilon_0 i$	$(6\pi)\epsilon_0 i\tau^3$	$(6\pi\epsilon_0 i)$	$(6\pi)\epsilon_0$	$\epsilon_0 i\tau^3$	$\epsilon_0 i$	ϵ_0
$\hat{\mathcal{L}}_{g_{3D}}$								3_a^*
$\hat{\mathcal{L}}_{g_{2D}}$								2_a
$\hat{\mathcal{L}}_{GG}$								24_a^*
$\hat{\mathcal{L}}_{uG}$								8_a
$\hat{\mathcal{L}}_{cG}$								24_b^*
$\hat{\mathcal{L}}_{tG}$								8_b
$\hat{\mathcal{L}}_B^m$							18_b	
e	4_a					1		3_b^*
μ	4_a		-12_a			27_a		3_c^*
τ	4_a		-3		3_a^*	$1+4$		
ν_1		12_a		12				2_b
ν_2		-4_a		4				2_b
ν_3		12_a		12	2			
ν_3'		12_a		12	-2			
U_{15}	-12_a					2_a		
U_{17}	$3\times(-4)$		-12_b			2_b		
D	-12_a					1		
u	4_b							-8_a
c	$4_b + 4_c$							-24^*
t	4_c							-8_b
d	-12_a		-12_a		-3^*			
s	-12_b							3_d^*
b	4_d		6		3_b^*	27_b	18_a	
Z		12_b^*		1			1	
H		-4		-6			18_c	
W		12_b^*		-18^*			-27	

References

1) Y. Akiba, in this report.

2) Y. Akiba, in this report.

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