$R = 12H_0^2$ and its implications to gravity and cosmology

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In the preceding article,[1)] I report a model (QST model) that can yield the formulas of the standard model (SM) parameters. The model implies that the product of the Hubble constant H_0 and the Planck time t_{pl} is $H_0 t_{pl} = 2 \times (6\pi)^{-48} = \epsilon_0$. This relation is derived as follows.

The spacetime metric of the Hubble expansion is

$$ds^2 = -dt^2 + e^{2H_0 t}(dx^2 + dy^2 + dz^2).$$

The Ricci scalar curvature R and $\sqrt{-g}$ of this metric are

$$\begin{aligned} R &= 12H_0^2, \\ \sqrt{-g} &= e^{3H_0 t}. \end{aligned}$$

For $t = t_{\rm pl}$, we have $\sqrt{-g} = e^{3H_0 t} \simeq 1 + 3H_0 t_{\rm pl}$. In the QST model, the correspondence relation is

$$\sqrt{-g} \leftrightarrow 1 + \hat{\mathcal{L}}_{g3D} = 1 + 3\epsilon_0.$$

Thus, we have $H_0 t_{pl} = \epsilon_0$. Because $H_0 = \epsilon_0/t_{pl}$ is a constant, R is a constant. The QST model implies that $R = 12H_0^2$ (constant). We discuss the implication of this equation in general relativity and cosmology below.

We can generalize the metric to allow local changes of the scale with a constraint $R = 12H_0^2$:

$$ds^2 = -e^{2u(x,y,z)}dt^2 + e^{2H_0 t}(dx^2 + dy^2 + dz^2).$$

The R of this metric is

$$\begin{aligned} R &= -(2\Delta u + 3(\nabla u)^2)e^{-2H_0 t} + 12H_0^2 e^{-2u} \\ &= 12H_0^2. \end{aligned}$$

We can show that Coulomb gravity can be derived from this metric. When we take the approximation $H_0 \simeq 0$, we have

$$R = -2\Delta u - 3(\nabla u)^2 = 0.$$

If u is small, we can ignore the $(\nabla u)^2$ term, and we have $\Delta u = 0$. In general relativity, $g_{00} \simeq -1 - 2\phi_G$, where ϕ_G is the gravitational potential. As $e^{2u} = -g_{00}$ and $e^{2u} \simeq 1 + 2u$, $u \simeq \phi_G$. Therefore,

$$\Delta\phi_G \simeq \Delta u = 0.$$

Thus, the relation $R = 0$ implies Coulombic gravitational potential under the weak gravity approximation. If we take into account the fact that $H_0 \neq 0$, the potential equation for gravity becomes non-linear.

Next, we discuss the effect of $R = 12H_0^2$ for the motion of an astronomical object. Consider the following metric (Sp-metric):

$$ds^2 = -dt^2 + e^{2H_0 t}\left(\frac{dr^2}{1 - \frac{2r_s}{r}} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right).$$

One can show that the Ricci tensor $R_{\mu\nu}$ and the Ricci scalar curvature R of this metric are

$$\begin{aligned} R^t_t &= 3H_0^2, R^r_r = 3H_0^2 - \frac{2r_s}{r^3}e^{-2H_0 t}, \\ R^\theta_\theta &= R^\phi_\phi = 3H_0^2 + \frac{r_s}{r^3}e^{-2H_0 t}, \\ R &= R^t_t + R^r_r + R^\theta_\theta + R^\phi_\phi = 12H_0^2. \end{aligned}$$

This metric describes the spacetime metric outside a very large spherical mass, *e.g.*, a star or a planet. It is a metric with spherical symmetry, like the Schwartzchild metric, but with $R = 12H_0^2$. Note that $R_{\mu\nu} = 0$ and $R = 0$ for the Schwartzchild metic.

One can solve Keplerian moton in this metric. The solution implies that the radius r of a circular orbit of an object around the large mass at the origin increases with time as

$$\dot{r}/r = \sqrt{2}H_0 = 1.0104 \times 10^{-10}/\text{yr}.$$

The factor $\sqrt{2}$ arises from the fact that the Keplerian motion is a two dimensional motion. One can show that the scale expansion rate of two-dimensional radius ρ with $R = 12H_0^2$ is $\sqrt{2}H_0$.

This prediction agrees with the observed rate of expansion of the the Moon's orbit radius $r_{\rm Moon}$ around the Earth[2)] within the uncertainty of the data.

$$\begin{aligned} \dot{r}_{\rm Moon} &= 3.82 \pm 0.07\text{cm/yr}, \\ \dot{r}_{\rm Moon}/r_{\rm Moon} &= (0.994 \pm 0.0182) \times 10^{-10}/\text{yr}. \end{aligned}$$

This good agreement indicates that the expansion of the lunar-orbit radius is due to Hubble expansion.

We found a few emprical formulas of cosmological parameters that are similar to those of the SM parameters. For CMB temperature, we found

$$T = \frac{1}{(6\pi)^2}{\epsilon_0}^{1/2}\left(1 + \frac{1}{4\pi} - \frac{1}{(6\pi)^2}\right)^{1/2}.$$

This formula yields $T^{\rm calc}_{\rm CMB} = 2.7249$ K, which agrees with the obsrved value $T_{\rm CMB} = 2.7255 \pm 0.0006$ K within the uncertainty of the data. The QST model can explain the emprical formula.

References

1) Y. Akiba, in this report.
2) B. G. Bills, R. D. Ray, Geophys. Res. Lett. **26**, 3045 (1999).

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