CP-odd gluonic operators in QCD spin physics†

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The explanation of matter-antimatter asymmetry of the universe requires new origins of CP-violation beyond the Standard Model (BSM). One of the interesting CP-violating operators that can be induced in the QCD Lagrangian due to BSM physics is the Weinberg operator.[1)] In this report I point out a novel relation between the hadronic matrix element of the Weinberg operator and a certain twist-four correction in polarized Deep Inelastic Scattering (DIS). Such a relation suggests an exciting possibility that polarized DIS experiments can provide useful information to the physics of the nucleon electric dipole moment (EDM), or more generally, BSM-origins of hadronic CP violations.

The Weinberg operator is a dimension-six purely gluonic operator

$$\mathcal{O}_W = g f_{abc} \tilde{F}^a_{\mu\nu} F^{\mu\alpha}_b F^\nu_{c\alpha}. \tag{1}$$

This operator violates CP and can be induced in the QCD Lagrangian by physics beyond the Standard Model. It is considered as one of the candidate operators to generate a large EDM of the nucleons and nuclei.

The key observation is the following exact operator identity

$$\begin{aligned} \mathcal{O}_W &= -\partial^\mu (\tilde{F}_{\mu\nu} \overleftrightarrow{D}_\alpha F^{\nu\alpha}) - \frac{1}{2} \tilde{F}_{\mu\nu} \overleftrightarrow{D}^2 F^{\mu\nu} \\ &\equiv \mathcal{O}_4 + \mathcal{O}_D, \end{aligned} \tag{2}$$

Eq. (2) shows that one can choose $\mathcal{O}_W$ and $\mathcal{O}_4$ as the independent basis of operators and study their mixing. Due to the equation of motion, one can write

$$\mathcal{O}_4 \approx \partial^\mu (\bar{\psi} g \tilde{F}_{\mu\nu} \gamma^\nu \psi), \tag{3}$$

to linear order in partial derivative ∂^μ. Such mixing is usually neglected in the literature because $\mathcal{O}_4$ is a total derivative and hence does not contribute to the CP-violating effective action $\int d^4x \mathcal{O}_4 = 0$. However, when it comes to hadronic matrix elements, mixing becomes crucial because only the nonforward matrix element is nonvanishing. Specifically, their RG equation takes the form

$$\frac{d}{d \ln \mu^2} \begin{pmatrix} \mathcal{O}_W \\ \mathcal{O}_4 \end{pmatrix} = -\frac{\alpha_s}{4\pi} \begin{pmatrix} \gamma_W & \gamma_{12} \\ 0 & \gamma_4 \end{pmatrix} \begin{pmatrix} \mathcal{O}_W \\ \mathcal{O}_4 \end{pmatrix} \tag{4}$$

where

$$\gamma_W = \frac{7}{3} N_c + \frac{2}{3} n_f \tag{5}$$

is the anomalous dimension of the Weinberg operator.[2)] The anomalous dimension of $\mathcal{O}_4$ is the same as that of the undifferentiated, twist-four operator $\bar{\psi} g \tilde{F}^{\mu\nu} \gamma_\nu \psi$ and is known to be[3)]

$$\gamma_4 = \frac{8}{3} C_F + \frac{2}{3} n_f. \tag{6}$$

To determine the off-diagonal component γ_{12}, I evaluate the following three-point Green's function

$$\langle 0 | \mathrm{T}\{\psi(-k) A^\rho_a(q) \bar{\psi}(p) \mathcal{O}_W\} | 0 \rangle \tag{7}$$

with off-shell momenta and nonzero momentum transfer $\Delta = k - p - q \neq 0$. The result is

$$\gamma_{12} = -3N_c. \tag{8}$$

It immediately follows that the following linear combination is the eigenstate of the RG evolution

$$\mathcal{O}_W + \frac{\gamma_{12}}{\gamma_W - \gamma_4} \mathcal{O}_4 = \mathcal{O}_W - \frac{9N_c^2}{3N_c^2 + 4} \mathcal{O}_4. \tag{9}$$

Since this operator has a rather large anomalous dimension $\gamma_W \sim 10$, in particular larger than γ_4 by a factor of about 2, at high enough renormalization scales μ^2 one has

$$\langle \mathcal{O}_W \rangle \approx \frac{9N_c^2}{3N_c^2 + 4} \langle \mathcal{O}_4 \rangle \approx 2.61 \langle \mathcal{O}_4 \rangle, \tag{10}$$

In terms of the nucleon matrix elements

$$\langle P | \bar{\psi} g \tilde{F}^{\mu\nu} \gamma_\nu \psi | P \rangle = -2 f_0 M^2 S^\mu \tag{11}$$

$$\frac{1}{M^3} \langle P' | g f^{abc} \tilde{F}^a_{\mu\nu} F^{\mu\sigma}_b F^\nu_{c\sigma} | P \rangle = 4E \bar{u}' i\gamma_5 u. \tag{12}$$

I get

$$E \approx \frac{9N_c^2}{2(3N_c^2 + 4)} f_0 \approx 1.3 f_0. \tag{13}$$

Therefore, one can evaluate the matrix element E of the Weinberg operator through the measurement of the f_0 parameter relevant to the twist-four corrections in polarized DIS.[4,5)] f_0 can be extracted from the $g_1(x)$ structure function measured at the future Electron-Ion Collider (EIC) in the U.S. This is a new connection between the EIC and physics beyond the Standard Model. It will demonstrate the EIC's unique capability to address low-energy nucleon observables in a high energy collider.

References
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