Compensation for energy-spread growth in RUNBA

M. Wakasugi,[*1,*2] Y. Abe,[*2] R. Ogawara,[*1,*2] T. Ohnishi,[*2] H. Tongu,[*1] K. Tsukada,[*1] and Y. Yamaguchi[*2]

A small heavy-ion storage ring, RUNBA,[1)] is under construction at E21. It will be used to develop a beam recycling technique, which will be a novel tool for the nuclear reaction study of rare radioactive isotopes (RIs). A few RIs, less than 10 in number, accumulate in RUNBA, which is equipped with an internal target, and hit the target many times until a nuclear reaction occurs. This is the beam recycling technique, which is established by compensating the energy loss U, energy straggling, and angular struggling given at a target. The energy loss can be compensated for by a radio-frequency (RF) cavity. Specially designed devices are, however, required to compensate for, turn by turn, the growth in energy spread and emittance due to the straggling. We will install an energy dispersion corrector (EDC) in RUNBA to compensate for energy-spread growth. Assuming a thin-foil internal target, we form a sinusoidal single-wave signal from the timing signal created when the beam passes through the target. This correction signal is applied to an acceleration gap in EDC in synchronization with the beam arrival. In this report, we analytically show the required characteristics of EDC.

The invariant of the longitudinal particle motion in $(\delta\phi, \delta\epsilon)$ phase space is given as

$$Q(\delta\phi,\delta\epsilon) = \frac{1}{2}\left(\frac{\eta\omega_{rf}}{\beta^2}\right)^2 \delta\epsilon^2 + \frac{qV_{rf}\eta\omega_{rf}}{A\beta^2 ET} \{\cos(\phi_s+\delta\phi)-\cos(\phi_s)+\delta\phi\sin(\phi_s)\}, \quad (1)$$

where E is the energy of ions, $\delta\epsilon = \delta E/E$, ϕ_s a synchronous phase; A the mass number, q the charge, β the velocity, T the revolution time, η the slipping factor, and V_{rf} and ω_{rf} are the amplitude and angular frequency of RF, respectively. The RF phase is shifted by $\Delta\phi_t$ and $\Delta\phi_e$ due to the energy straggling $\Delta\epsilon_t$ at the target and the energy correction $\Delta\epsilon_e$ given at EDC, respectively. Assuming $\Delta\epsilon_e$ is proportional to a time-of-flight difference in the design value from the target to EDC, it is expressed with a proportional coefficient λ_e as

$$\Delta\epsilon_e = K_{te}(\lambda_e)\big(\delta\epsilon + \Delta\epsilon_t\big) + K_\delta(\lambda_e)\delta_t, \quad (2)$$

where $K_{te}(\lambda_e) = \frac{q\lambda_e\eta_{te}T_{te}}{A\beta^2 E}\big(1+\frac{\gamma^2+1}{\gamma^2-1}\frac{U}{E}\big)$; $K_\delta(\lambda_e) = \frac{q\lambda_e}{AE}$; η_{te} and T_{te} are the partial slipping factor and time of fight between the target and EDC, respectively; and δ_t is the uncertainty in timing measurement. Since $\Delta\epsilon_t$ and δ_t are stochastic variables, their probability density distributions are assumed to be Gaussian and denoted by $\psi(\Delta\epsilon_t)$ and $\theta(\delta_t)$, with standard deviations of σ_t and σ_τ, respectively. The averaged time derivative of $Q(\delta\phi, \delta\epsilon)$ is calculated as

$$\left\langle\frac{dQ}{dt}\right\rangle T = \int\big\{Q(\delta\phi+\Delta\phi_t+\Delta\phi_e, \delta\epsilon+\Delta\epsilon_t+\Delta\epsilon_e) - Q(\delta\phi,\delta\epsilon)\big\}\psi(\Delta\epsilon_t)d\Delta\epsilon_t\theta(\delta_t)d\delta_t. \quad (3)$$

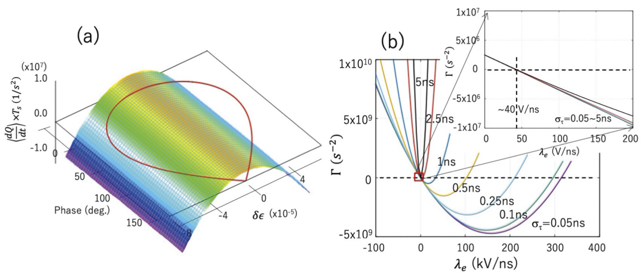

Fig. 1. (a) Time derivative of the invariant Q at $\lambda_e = 40$ V/ns calculated using Eq. (3) and (b) $\Gamma(\lambda_e)$ calculated using Eq. (4) for several cases of uncertainty σ_τ.

Since this is a function of the position in phase space $(\delta\phi, \delta\epsilon)$ and the coefficient λ_e, we take its average within a separatrix as

$$\Gamma(\lambda_e) = \frac{1}{S}\int_S \left\langle\frac{dQ}{dt}\right\rangle T d\delta\phi d\delta\epsilon. \quad (4)$$

Γ is a function of λ_e only. If Γ takes negative values, the invariant $Q(\delta\phi, \delta\epsilon)$ decreases with time on average and, consequently, the beam continues to circulate the ring. Here we omit the explicit expression of Eqs. (3) and (4) because of the limited text space.

We assume in the following calculation that the beam in RUNBA is a 10 MeV/nucleon $^{12}C^{6+}$ beam and the target is a thin ^{12}C foil with a thickness of $10^{18}/cm^2$. Figure 1(a) shows the time-derivative distribution in longitudinal phase space calculated using Eq. (3) at $\lambda_e =$ 40 V/ns. This has a quadric-like surface and is positive in the small $\delta\epsilon$ region but negative in the large $\delta\epsilon$ region. The averaged time derivative calculated using Eq. (4) is shown in Fig. 1(b) for several cases of the uncertainty σ_τ. λ_e has two solutions for each case: one indicates the minimum required correction, and the other indicates the maximum limit to avoid the invariant growth due to overcorrection. Although the λ_e value can be chosen in between the two solutions, the flexibility is reduced as uncertainty increases. Practically, the smallest possible value is the most convenient because we can reduce the signal power for EDC. The inset in Fig. 1(b) shows an expanded view around the lower limit of λ_e. We found that this is insensitive to the uncertainty and the minimum required correction coefficient is $\lambda_e \sim$40 V/ns.

Reference

1) M. Wakasugi *et al.*, RIKEN Accel. Prog. Rep. **54**, 87 (2021).

*1 Institute for Chemical Research, Kyoto University
*2 RIKEN Nishina Center