QST model

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The standard model (SM) successully describes nature, but it has at least 25 free parameters. Simple formulas with no free parameter for 24 SM parameters have been reported.[1)] Moreover, the quarternion-spin-isospin (QST) model that predicts these formulas,[2)] and its implication to gravity and cosmology[3)] have also been reported. This article reports an update of the model.

In QST model, the Planck time $t_{\rm pl} = 5.3912 \times 10^{-44}$ s is the minimum time period in nature; $t_{\rm pl}$ is a fundamental constant in nature, similar to the speed of light in vacuum, c, and the Planck constant, $\hbar$. This implies that the Planck length $l_{pl} = ct_{\rm pl}$ is the minimum length and that a spacetime point has a minimum size, denoted as $d\Omega_E$. In the QST model, it is shown that $d\Omega_E = (6\pi)^{48}c^3 t_{\rm pl}^4$. The QST model is not a continuous quantum field theory (QFT). This does not mean that the QST model is an approximation of an underlying QFT. It is assumed that nature is not continuous at the fundamental level, and that a QFT such as the SM is an approximation.

One can define a spacetime coordinate, assign a coordinate x^μ to a spacetime point, and make x^μ the label of the point. We denote the point as D_{x^μ} and its state as $|x^\mu\rangle$, where $|x^\mu\rangle$ is a multi-dimensional quantity including its spacetime position and its internal state corresponding to the gauge freedom in the SM.

Since D_{x^μ} has a finite size, its coordinate x^μ has an uncertainty δx^μ. The uncertainty in the time, δx^0, and space, δx^k, is larger than $t_{\rm pl}$ and $l_{\rm pl}$, respectively, and the uncertainty of a spacetime volume $\delta x^0 \delta x^1 \delta^2 \delta x^3$ is higher than $d\Omega_E$ ($\delta x^0 \delta x^1 \delta^2 \delta x^3 \geq d\Omega_E$). This can be considered as a generalization of the uncertainty principle ($\delta q \delta p \geq \hbar$).

For a spacetime point D_{x^μ}, there are a finite number of spacetime points $D_{x^\mu - \delta_i^\mu}$ that it is in contact with. The QST model assumes that $|x^\mu\rangle$ is determined as

$$|x^\mu\rangle = \sum (1 + \hat{X}_i) \vee |x^\mu - \delta_i^\mu\rangle,$$

where $\hat{X}_i$ is the action from $D_{x^\mu - \delta_i^\mu}$ to D_{x^μ}. Due to the causality $\hat{X}_i = 0$ unless $D_{x^\mu - \delta_i^\mu}$ is in the the past lightcone of D_{x^μ}. The state $|x^\mu - \delta_i^\mu\rangle$ is again determined from spacetime points of its immediate past. Thus

$$|x^\mu\rangle = \sum_p \sum_{l_p} S_{l_p} \vee |x_p^\mu\rangle = \sum_{l_p} \sum_p S_{l_p} \vee |x_p^\mu\rangle,$$

where $D_{x_p^\mu}$ is a spacetime point of the past lightcone of D_{x^μ}, S_{l_p} is the action along with l_p, and l_p is a path from $D_{x_p^\mu}$ to D_{x^μ}. In the QST model, the form of the action $\hat{X}_k$ from $D_{x_k^\mu}$ to its immediate future point is $\hat{X}_k = \sum \delta_k^p \hat{X}_p^{\rm PA}$ where $\hat{X}_p^{\rm PA}$ is one of the following 64 operators denoted as primordial action (PA): $\Big\{1,\ i,\ I^\mu \sigma^\nu \tau^a,\ 2\varepsilon,\ 2\varepsilon i \tau^3,\ 2\varepsilon \tau^a,\ 2\varepsilon I^c \tau^{[a-c]},\ -3\varepsilon I^c,\ -6\varepsilon i, -2\varepsilon \Big\}$. The operator $\vee$ is a binary product operator among the PAs. Now non-zero 48-fold vee product of PAs

$$d\hat{S}_p^{\rm EA} = \hat{X}_{p_1}^{\rm PA} \vee \hat{X}_{p_2}^{\rm PA} \vee \cdots \vee \hat{X}_{p_{48}}^{\rm PA} \neq 0,$$

is denoted as elementary action. One can show that if $n > 48$ then $\hat{X}_{p_1}^{\rm PA} \vee \cdots \vee \hat{X}_{p_n}^{\rm PA} = 0$ and that there are only 50 independent EAs. The elementray action density (EAD) $\hat{\mathcal{L}}_p^{\rm EAD}$ of $d\hat{S}_p^{\rm EA}$ is defined as $\hat{\mathcal{L}}_p^{\rm EAD} = d\hat{S}_p^{\rm EA}/(6\pi)^{48}$, *i.e.*, $d\hat{S}_p^{\rm EA} = \hat{\mathcal{L}}_p^{\rm EAD} d\Omega_E$.

Consider dividing a long path l_p into 48-step short paths as ($l_p = l_p^1 + \cdots + l_p^n$). Since a vee product of more than 48-folds becomes zero, the action along l_p^k is equivalent to $(1 + d\hat{S}_{l_p^k}^{\rm EA})$ and the action S_{l_p} along l_p is written as

$$\begin{aligned} S_{l_p} \vee &= (1 + d\hat{S}_{l_p^1}^{\rm EA}) \vee (1 + d\hat{S}_{l_p^2}^{\rm EA}) \vee \cdots \vee (1 + d\hat{S}_{l_p^n}^{\rm EA}) \vee \\ &= \exp \sum_k d\hat{S}_{l_p^k}^{\rm EA} \vee . \end{aligned}$$

The state $|x^\mu\rangle$ is written as

$$\begin{aligned} |x^\mu\rangle &= \sum_{l_p} \sum_p S_{l_p} \vee |x_p^\mu\rangle = \sum_{l_p} \exp \sum_{k,p} d\hat{S}_{l_p^k}^{\rm EA} \vee |x_p^\mu\rangle \\ &= \sum_{l_p} \exp \sum_p \sum_k \hat{\mathcal{L}}_{l_p^k}^{\rm EAD} d\Omega_E \vee |x_p^\mu\rangle \end{aligned}$$

The last equation corresponds to the path integral in QFT, where the change of the state vector $|t_i\rangle$ to $|t_f\rangle$ is given as

$$|t_f\rangle = \int \mathcal{D}\phi \exp\Big(i \int \mathcal{L}[\phi(x,t)] d^4x\Big) |t_i\rangle.$$

From the comparison, one can see that $\hat{\mathcal{L}}_p^{\rm EAD}$ and $d\Omega_E$ correspond to a term of the Lagrangian and volume element d^4x, respectively. This means that in the limit $t_{\rm pl} \to 0$ the QST model becomes a QFT, that is, the SM. For each term of the SM Lagrangian there is an EAD corresponding to it. Thus one can calcluate the parameters of the SM Lagragian from corresponding EADs. A full paper of the QST model is in preparation.

References
1) Y. Akiba, RIKEN Accel. Prog. Rep. **54**, 69 (2021).
2) Y. Akiba, RIKEN Accel. Prog. Rep. **54**, 70 (2021).
3) Y. Akiba, RIKEN Accel. Prog. Rep. **54**, 71 (2021).

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