

# QCD-based charge symmetry breaking (CSB) interaction and Okamoto-Nolen-Schiffer anomaly<sup>†</sup>

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An anomaly in the energy differences of mirror nuclei, which is not yet well understood from a microscopic point of view, was discovered more than 50 years ago and is called the Okamoto-Nolen-Schiffer (ONS) anomaly. A systematic study from light to heavy nuclei within the framework of the independent-particle model finds that theoretical values of the energy difference underestimate the experimental values by 3–9%.

The aim of this study is to drive a QCD-based CSB interaction by making a quantitative link between the Skyrme-type CSB interactions and the CSB effect due to the  $u$ - $d$  quark mass difference and the chiral condensate in QCD.

We split the mass difference between mirror nuclei  $\Delta E = E(Z+1, N) - E(Z, N+1)$  into the Coulomb contribution  $\Delta E_C$  and the ONS anomaly  $\delta_{\text{ONS}}$  as

$$\Delta E = \Delta E_C + \delta_{\text{ONS}}. \quad (1)$$

The CSB effect to  $\delta_{\text{ONS}}$  from the partial restoration of chiral symmetry in the uniform symmetric nuclear matter (denoted as  $\delta_{\text{chiral}}$ ) can be estimated as<sup>1)</sup>

$$\delta_{\text{chiral}} \equiv \Delta_{np}(0) - \Delta_{np}(\rho) = C_1 [1 - G(\rho)]. \quad (2)$$

where  $\Delta_{np}(\rho)$  is the binding-energy difference between the neutron and the proton in the symmetric nuclear matter with the baryon density  $\rho$ . In the leading order of the  $u$ - $d$  quark mass difference and the quantum electrodynamics (QED) effect, an approximate formula has been obtained from the QCD sum rules (QSR);<sup>1)</sup>

$$\Delta_{np}(\rho) \simeq C_1 G(\rho) - C_2, \quad G(\rho) = \left( \frac{\langle \bar{q}q \rangle}{\langle \bar{q}q \rangle_0} \right)^{1/3}. \quad (3)$$

Here,  $\langle \bar{q}q \rangle$  and  $\langle \bar{q}q \rangle_0$  are, respectively, the isospin averaged in-medium and in-vacuum chiral condensate. The coefficient  $C_1$  is proportional to the  $u$ - $d$  quark mass difference  $\delta m$ , through the isospin-breaking constant  $\gamma \equiv \langle \bar{d}d \rangle_0 / \langle \bar{u}u \rangle_0 - 1$  as  $C_1 = -a\gamma$  with a positive constant  $a$  determined by the Borel QSR method.

The Skyrme-type CSB interaction is commonly adopted to study the ONS anomaly in the HF model;

$$\begin{aligned} V_{\text{CSB}}(\mathbf{r}) = & [s_0(1 + y_0 P_\sigma) \delta(\mathbf{r}) \\ & + \frac{s_1}{2}(1 + y_1 P_\sigma) (\mathbf{k}^\dagger \delta(\mathbf{r}) + \delta(\mathbf{r}) \mathbf{k}^2) \\ & + s_2(1 + y_2 P_\sigma) \mathbf{k}^\dagger \cdot \delta(\mathbf{r}) \mathbf{k}] \frac{\tau_{1z} + \tau_{2z}}{4}, \quad (4) \end{aligned}$$

where  $\tau_{iz}$  is the  $z$  component of isospin operator,  $\vec{k} = (\vec{\nabla}_1 - \vec{\nabla}_2)/2i$ ,  $\vec{r} = \vec{r}_1 - \vec{r}_2$ , and  $P_\sigma = (1 + \vec{\sigma}_1 \cdot \vec{\sigma}_2)/2$  is the spin-exchange operator. In Eq. (4),  $s_i$  and  $y_i$  are the strength parameters of the CSB and its spin exchange interactions. Equation (4) gives contributions to the energy difference of mirror nuclei  $\delta_{\text{ONS}}$  as

$$\delta_{\text{Skyrme}}(\rho) = -\frac{\tilde{s}_0}{4} \rho - \frac{1}{10} \left( \frac{3\pi^2}{2} \right)^{2/3} (\tilde{s}_1 + 3\tilde{s}_2) \rho^{5/3}, \quad (5)$$

where we have defined the effective coupling strengths,

$$\tilde{s}_0 \equiv s_0(1 - y_0), \quad \tilde{s}_1 \equiv s_1(1 - y_1), \quad \tilde{s}_2 \equiv s_2(1 + y_2). \quad (6)$$

Our approach is to constrain  $\tilde{s}_{0,1,2}$  from the low-energy constants in QCD,  $\gamma$  and  $\sigma_{\pi N}$ , by matching  $\delta_{\text{Skyrme}}(\rho)$  in Eq. (5) and  $\delta_{\text{chiral}}(\rho)$  expanded up to  $\mathcal{O}(\rho^{5/3})$  at low densities. Then, we obtain

$$\tilde{s}_0 = -\frac{4C_1\sigma_{\pi N}}{3f_\pi^2 m_\pi^2}, \quad \tilde{s}_1 + 3\tilde{s}_2 = \frac{1}{m_N^2} \frac{C_1\sigma_{\pi N}}{f_\pi^2 m_\pi^2}. \quad (7)$$

The  $\tilde{s}_0$  and  $\tilde{s}_1 + 3\tilde{s}_2$  are thus determined as  $\tilde{s}_0 = -15.5_{-12.5}^{+8.8}$  MeVfm<sup>3</sup> and  $\tilde{s}_1 + 3\tilde{s}_2 = 0.52_{-0.19}^{+0.42}$  MeVfm<sup>5</sup> with the uncertainty from  $\sigma_{\pi N}$  and  $C_1$  values.

We apply these QCD-based CSB interactions to the ONS anomaly problem, and the results are shown in Fig. 1. They show good agreement with the experimental data and cure largely the ONS anomaly within the theoretical uncertainty.

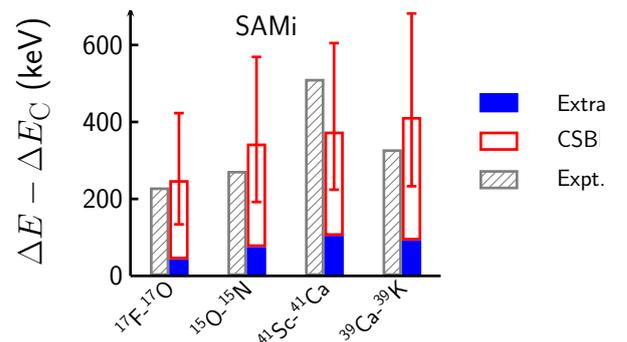


Fig. 1. Comparisons of the experimental ONS anomaly  $\Delta E_{\text{Expt.}} - \Delta E_C$  (grey hatched bars) and the corresponding theoretical estimates in SAMi EDF. The contribution from the QCD-based CSB interaction (CSB) and the extra contributions are indicated by the red bars with error bars and the blue bars, respectively.

## Reference

- 1) T. Hatsuda *et al.*, Phys. Rev. Lett. **66**, 2851 (1991).

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