Proposal of isober-separation system using betatron resonance in a multi-frequency RFQ (MRFQ)

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We succeeded in the first-ever electron scattering off online-produced radioactive isotope ^{137}Cs at the SCRIT facility.[1)] The purity of the target ion in this experiment was greater than 95%. However, the case of electron scattering off Cs is quite unusual and typically requires an isober-separation system to purify the target ion. Thus, we propose herein a novel isober-separation system based on the betatron resonance in a multi-frequency RFQ (MRFQ).

Multi-color radio frequencies $n_1 f_0$, $n_2 f_0$ ..., with amplitudes V_1, V_2, ..., and DC voltage V_{dc} are applied to appropriately formed quadrupole electrodes in the MRFQ, where n_1, n_2, ... are integers. MRFQ provides many more degrees of freedom to the periodic structures controlling the ion motion than are available for the single-frequency case. The ion motion within a unit period ($\tau = 1/f_0$) is characterized by beta functions (β_x, β_y) and tune values (ν_x, ν_y). Betatron resonance does not occur in a perfect quadrupole field, but a strong sum resonance ($\nu_x + \nu_y = integer$) is excited by actively applying a skew field. The resonances are sometimes sufficiently sharp to distinguish isotopes with mass differences of approximately $\Delta M/M \sim 10^{-5}$. Such sharp resonances can be found within a large parameter space defined by a combination of RF, relative phase, and DC voltage.

The ion motion equation in MRFQ is as follows:

$$\frac{\partial^2 u}{\partial t^2} + \left(a + \sum_i q_i \cos(2\pi n_i f_i t + \phi_i) \right) u = 0, \quad (1)$$

where $u = x$ or y, $a = 2qV_{dc}/mr_0^2$, $q_i = 2qV_i/mr_0^2$, m is the mass, and r_0 is the bore radius of the quadrupole. This is the Hill's equation and is expressed in terms of the unit-period transfer matrix A as follows:

$$\begin{pmatrix} u(\tau) \\ u'(\tau) \end{pmatrix} = A \begin{pmatrix} u(0) \\ u'(0) \end{pmatrix}, \quad (2)$$

where $u' = dx/dt$ or dy/dt. The stability condition is given by $|Tr(A)| < 2$, and the beta functions and tune values are estimated from the transfer matrix analysis.

For example, given two frequencies $f_1 = 1.2$ MHz and $f_2 = 0.8$ MHz ($n_1 = 3$ and $n_2 = 2$), DC voltage $V_{dc} = 180$ V, and phase difference $\Delta\phi_{12} = 0$, the calculated stability diagram in the (V_1, V_2) space is shown in Fig. 1(a). The yellow and blue regions indicate the horizontally and vertically stable regions, respectively. The extended view of the overlap region indicated by a circle is as shown in Fig. 1(b), and the expected resonance lines are denoted by the red lines. The diagonal crossing line is the second-order sum resonance, $\nu_x + \nu_y = 2$.

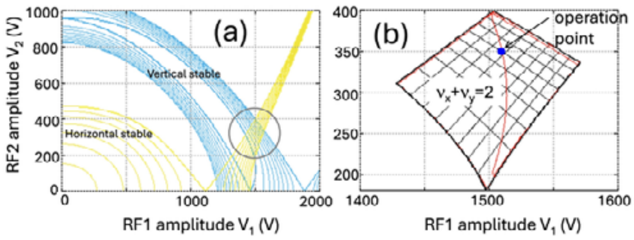

Fig. 1. Stability diagram (a) and the extended view (b) in (V_1, V_2) space.

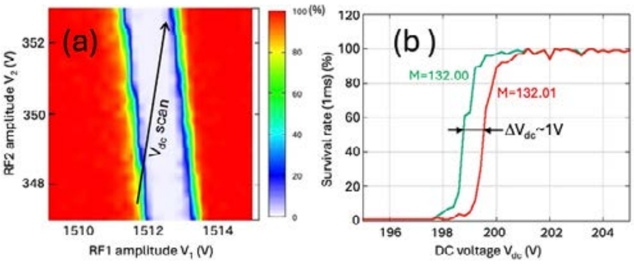

Fig. 2. Survival rate for 1-ms trapping with skew field (a) and those for the isotopes with mass numbers of 132.00 and 132.01 as a function of V_{dc} (b).

Based on an operation point, such as that indicated in Fig. 1(b) and applying a skew field generated by tilting the quadrupole RF field by only 5 mrad, the survival-rate mapping after trapping for 1 ms is obtained via particle simulation (Fig. 2(a)). The sum resonance appears clearly, with extremely sharp edges on both sides. Under this condition, the survival rate for two isotopes with a mass difference of $\Delta M/M \sim 6 \times 10^{-5}$ were calculated as a function of V_{dc} (Fig. 2(b)). The V_{dc} scan corresponds to moving the diagram along an allow, as shown in Fig. 2(a). The difference in DC voltage between the two edges is sufficiently large that, with at a DC voltage of ∼199.3 V, these isotopes can be separated in as short as 1 ms.

The high flexibility in creating the periodic structure of the MRFQ also allows us to search for other more suitable operating points. We have started research using the prototype to confirm the availability of the MRFQ as an isober separator.[2)]

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References
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2) H. Kobayashi *et al.*, in this report.

DOI:10.34448/RIKEN.APR.58-046