Analytical method for betatron tune in multi-frequency RFQ

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An electron scattering technique with a self-confining RI ion target (SCRIT)[1)] has been successfully verified for ^{137}Cs.[2)] Subsequently, the SCRIT facility is going to research short-lived unstable nuclei, particularly ^{132}Sn. To realize the electron scattering from online-produced ^{132}Sn, an isobar filter is necessary. The electron-beam-driven radioactive isotope separator for SCRIT (ERIS)[3)] can provide sufficient amounts of ^{132}Sn; however, the desired isotope is delivered together with its isobar ^{132}Sb, with which it has a small mass difference of only 0.002%. Therefore, we need a separator with a mass-resolving power greater than 40000.

As a mechanism of the high-resolution mass separator, we are planning to utilize a betatron resonance induced by skew components of the electromagnetic field in a multi-frequency RFQ (MRFQ). We identified a sharp unstable line stemming from the skew in numerical calculation. We study this instability via analytical computation and strive to find the optimum operation point for effective isobar separation.

We have conducted an analytical approximation of the betatron tune values. When applying two colors of RF, $n_1\omega_0$ and $n_2\omega_0$, the equation of motion in the MRFQ (whose bore radius is r_0) with a dimensionless timescale $\tau = \omega_0 t/2 = \pi t/T$ is written as

$$x'' + k(\tau)x = 0,\ y'' - k(\tau)y = 0,$$

$$k(\tau) = \frac{8\ eV(t)}{m\omega_0^2 r_0^2} = a + 2q_1\cos(2n_1\tau) + 2q_2\cos(2n_2\tau + \phi),$$

where $n_1 : n_2$ is the ratio of two frequencies, and the dimensionless factors a, q_1, and q_2 are proportional to the DC, first RF, and second RF voltages, respectively. The skew component is neglected. Owing to the linearity and the periodicity of the differential equation, the method of transfer matrix, M_u, is available, *i.e.*,

$$\begin{pmatrix} u(T) \\ u'(T) \end{pmatrix} = M_u \begin{pmatrix} u(0) \\ u'(0) \end{pmatrix} \quad (u = x, y).$$

If a periodic solution exists, the tune ν_u is defined as $\mathrm{Tr}\, M_u = 2\cos 2\pi\nu_u$. With a small skew, it is known that the resonance lines of instability occur along $\nu_x + \nu_y = k$ $(k = 1, 2, 3, \cdots)$ lines. An analytical formulation of tunes is expected to provide the positions and the nature of the instability in the stability diagram. Therefore, we focus on the tunes.

First, we calculate a low-voltage approximation by integration of the transfer matrix for one priod with induction and extending it using Chebyshev polynomials T_n:[4)]

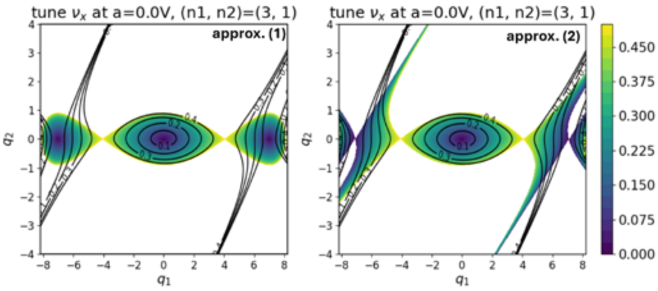

Fig. 1. Results of approximations by Eq. (1) and Eq. (2). The black line represents the contours of the tune obtained via numerical calculation, whereas the colored map shows those obtained using the approximations. The difference between the approximation results appears clearly for ratios of $n \geq 3$.

$$\mathrm{Tr}\, M_x = \sum_{i=1}^{2}\left\{2T_{n_i}\left(1 - \pi^2\frac{q_i^2}{n_i^4}\right)\right\} - 2 - 2\pi^2 a + \frac{\pi^4}{3}a^2 + \mathcal{O}(q^3). \quad (1)$$

Second, we develop a square wave approximation. The exact values of tunes can be obtained when square waves are applied instead of sinusoidal ones. If an amplitude of a rectangular wave is determined to have the same trace of the transfer matrix as one in a sinusoidal wave based on the approximation in Eq. (1), the tunes in the rectangular waves will serve as an approximation for those in the MRFQ. We calculate the tunes after simplifying the RFs as follows:

$$\cos 2x \to \begin{cases} +\sqrt{6}/\pi & (0 < \cos 2x) \\ -\sqrt{6}/\pi & (\cos 2x < 0), \end{cases} \quad (2)$$

where $x = n_i\tau$. These two calculations result in the tune distribution in the (q_1, q_2) plane as shown in Fig. 1. They successfully reproduce the values calculated from numerical calculations in the low-voltage region. Equation (2) derives the outline of the tune distribution more precisely than Eq. (1) does, although Eq. (2) still has deviations in the high-voltage region.

For our next step, we will not only improve the order of the analytic calculations shown herein but also consider the benefits of higher harmonics in square waves instead of those in sine waves in MRFQ. Furthermore, we are going to conduct a demonstration experiment of the mass separation via the use of skew components.

References

1) M. Wakasugi *et al.*, Nucl. Instrum. Methods Phys. Res. B **317**, 668 (2013).
2) K. Tsukada *et al.*, Phys. Rev. Lett. **131**, 092502 (2023).
3) T. Ohnishi *et al.*, Nucl. Instrum. Methods Phys. Res. B **317**, 357 (2013).
4) Y. Saito, M. Wakasugi, "Analytic Approximation of Beam Dynamics in Multi-RFQ for Isobar Separation", Poster Presented at LEES2024, Oct. 28 – Nov. 1, 2024, Sendai, Japan.

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DOI:10.34448/RIKEN.APR.58-054