Spin entanglement of two protons emitted from ^{6}Be†

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In nuclear physics, quantum entanglement can play a role in various context. Sakai *et al.*[1] demonstrated a strong spin correlation between two protons by measuring spin-singlet protons in the reaction of ^{2}H+$p \longrightarrow ^2$He+n. The spin-correlation function, represented by the Clauser-Horne-Shimony-Holt (CHSH) indicator, is determined as $S_{\rm expt} = 2.83 \pm 0.24_{\rm stat} \pm 0.07_{\rm sys}$, exceeding beyond the capability of local-hidden-variable (LHV) theory.[2] Recently, the quantum tomography method has been proposed as feasible.[3]

Two-proton ($2p$) radioactive emission is one possible example to observe the nuclear spin entanglement.[4] In so-called "prompt" $2p$ emission, such as ^{6}Be $\rightarrow ^4$He $+2p$,[5] the dominance of spin-singlet configuration is expected, owing to the effective $2p$ interaction in finite systems.

In this work,[4] I compute the spin correlation S of two protons and simulate the CHSH examination for ^{6}Be $\rightarrow ^4$He $+2p$. First, the options of measurement by so-called "Alice" and "Bob" conventionally are introduced:

$$\hat{A}_{1,\theta}(1) \otimes \hat{B}_{1,\theta}(2), \quad \hat{A}_{2,\theta}(1) \otimes \hat{B}_{1,\theta}(2),$$
$$\hat{A}_{1,\theta}(1) \otimes \hat{B}_{2,\theta}(2), \quad \hat{A}_{2,\theta}(1) \otimes \hat{B}_{2,\theta}(2). \quad (1)$$

Alice (Bob) observes a single fermion with one option selected from $\hat{A}_{1,\theta}$ and $\hat{A}_{2,\theta}$ ($\hat{B}_{1,\theta}$ and $\hat{B}_{2,\theta}$). These operators with the polarization angle θ are expressed as

$$\begin{aligned} \hat{A}_{1,\theta}(1) &= \hat{\sigma}_z(1), \\ \hat{A}_{2,\theta}(1) &= \hat{\sigma}_z(1)\cos 2\theta + \hat{\sigma}_x(1)\sin 2\theta, \end{aligned} \quad (2)$$

for Alice, whereas

$$\begin{aligned} \hat{B}_{1,\theta}(2) &= \hat{\sigma}_z(2)\cos\theta + \hat{\sigma}_x(2)\sin\theta, \\ \hat{B}_{2,\theta}(2) &= \hat{\sigma}_z(2)\cos\theta - \hat{\sigma}_x(2)\sin\theta, \end{aligned} \quad (3)$$

for Bob. In this work, $\theta = \pi/4$ (Tsirelson's bound).[6] For the $2p$ state $|\Psi(1,2)\rangle$, I calculate

$$\langle A_i B_j \rangle = \left\langle \Psi(1,2) \mid \hat{A}_{i,\theta}(1) \otimes \hat{B}_{j,\theta}(2) \mid \Psi(1,2) \right\rangle. \quad (4)$$

The CHSH indicator S is determined as[2]

$$S = \max\left\{ |S_{mppp}|, |S_{pmpp}|, |S_{ppmp}|, |S_{pppm}| \right\}, \quad (5)$$

where

$$S_{mppp} = -\langle A_1B_1\rangle + \langle A_2B_1\rangle + \langle A_1B_2\rangle + \langle A_2B_2\rangle,$$
$$S_{pmpp} = \langle A_1B_1\rangle - \langle A_2B_1\rangle + \langle A_1B_2\rangle + \langle A_2B_2\rangle,$$
$$S_{ppmp} = \langle A_1B_1\rangle + \langle A_2B_1\rangle - \langle A_1B_2\rangle + \langle A_2B_2\rangle,$$
$$S_{pppm} = \langle A_1B_1\rangle + \langle A_2B_1\rangle + \langle A_1B_2\rangle - \langle A_2B_2\rangle. \quad (6)$$

Note that, if $S > 2$, the LHV theory cannot explain it.

Figure 1 and Table 1 summarize my results obtained from the time-dependent three-body model.[4,5] Here, I use the tuning factor for the $2p$ interaction, $v_{pp} \rightarrow f_{pp} v_{pp}$, where the setting of $f_{pp} = 1$ reproduces the experimental energy, $E_{2p} = 1.37$ MeV.[7] Consequently, a strong spin correlation, $S \cong 2.6$, is suggested at the experimental energy. The CHSH measurement offers a novel probe into the effective interaction in finite nuclei. Two-proton emitters can provide a platform for identical-particle entanglement in quantum operations.[8]

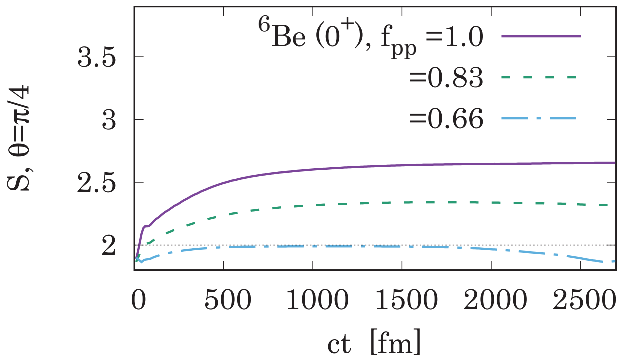

Fig. 1. Spin correlation function as the CHSH indicator for the $2p$-emitting decay of ^{6}Be $\rightarrow ^4$He $+2p$. The LHV-theory limit, $S = 2$, is indicated by the dotted line.

Table 1. The $2p$ energy, width, and CHSH indicator S of ^{6}Be for several f_{pp} values. The corresponding lifetime is given by $\tau = \hbar/\Gamma_{2p}$. Experimental data read $E_{2p} = 1.372(5)$ MeV and $\Gamma_{2p} = 0.092(6)$ MeV.[7]

f_{pp}	E_{2p} [MeV]	Γ_{2p} [MeV] (τ [s])	S
1.00	1.356	0.055 (1.2×10^{-20})	2.65
0.83	2.056	0.177 (3.7×10^{-21})	2.34
0.66	2.616	0.428 (1.5×10^{-21})	1.98

References

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