Twist analysis of the spin-orbit correlation in QCD†

Y. Hatta[*1,*2] and J. Schoenleber[*1]

Spin-orbit coupling $\vec{L}\cdot\vec{S}$ is a ubiquitous phenomenon across many areas of science including chemistry, condensed matter physics, and atomic and sub-atomic physics. It is a textbook material that the spin-orbit coupling of an electron in a hydrogen atom partly contributes to the hydrogen fine structure. Also, in atomic nuclei, the spin-orbit coupling of protons and neutrons orbiting around the nuclear mean field potential is crucial to explain the nuclear magic number. Similarly, quarks and gluons that constitute the proton are also subject to spin-orbit 'correlation' L_3S_3 between the longitudinal components of $\vec{L}$ and $\vec{S}$ which may be thought of as the relativistic generalization of $\vec{L}\cdot\vec{S}$. While this observation was made in the context of the nucleon structure more than a decade ago,[1)] its precise properties have been largely unexplored. In this work we present a rigorous QCD analysis.

The spin-orbit correlation of quarks (q) and gluons (g) having a momentum fraction x of the parent hadron is defined by

$$C_q(x) = \int d^2k_\perp d^2b_\perp \epsilon^{ij} b^i k^j \tilde{f}_q(x,k_\perp,b_\perp), \qquad (1)$$

$$xC_g(x) = \int d^2k_\perp d^2b_\perp \epsilon^{ij} b^i k^j \tilde{f}_g(x,k_\perp,b_\perp), \qquad (2)$$

where $\tilde{f}_{q,g}(x,k_\perp,b_\perp)$ are the polarized quark and gluon Wigner distributions representing the phase space distribution in transverse momentum $k_\perp$ and impact parameter $b_\perp$ spaces. Positive (negative) $C_{q,g}(x)$ indicates that the helicity and OAM of partons carrying a momentum fraction x are aligned (anti-aligned).

$C_{q,g}(x)$ are twist-three parton distribution functions. As such, they consist of the Wandzura–Wilczek part related to the twist-two PDFs and the genuine twist-three part represented by quark-gluon $q\bar{q}g$ and three-gluon ggg correlation functions. We have analyzed the twist structure of $C_{q,g}(x)$ using the QCD equations motion and the Lorentz invariant relations. The result for $C_q(x)$ reads, in massless QCD,

$$\begin{aligned} C_q(x) &= x\int_x^{\epsilon(x)} \frac{dx'}{x'}\Delta q(x') - x\int_x^{\epsilon(x)} \frac{dx'}{x'^2} q(x') \\ &+ x\int_x^{\epsilon(x)} dx_1 \int_{-1}^1 dx_2 \frac{\tilde{\Psi}_{Fq}(x_1,x_2)}{x_1-x_2} P\frac{3x_1-x_2}{x_1^2(x_1-x_2)} \\ &+ x\int_x^{\epsilon(x)} dx_1 \int_{-1}^1 dx_2 \Psi_{Fq}(x_1,x_2) P\frac{1}{x_1^2(x_1-x_2)}. \end{aligned} \qquad (3)$$

where $q(x)$ and $\Delta q(x)$ are the unpolarized and polarized quark PDFs, respectively, and Ψ_F, $\tilde{\Psi}_F$ are the genuine twist-three, quark-gluon correlators of the form $\langle \bar{q}\gamma^+ F^{+i} q\rangle$, $\langle \bar{q}\gamma^+\gamma_5 \tilde{F}^{+i} q\rangle$. A similar relation has been derived for the gluon $C_g(x)$ featuring the unpolarized $g(x)$ and polarized $\Delta g(x)$ gluon PDFs and the genuine three, three-gluon correlators $\langle F^{+\mu} F^{+i} F^{+}_{\mu}\rangle$.

Our derivation of $C_{q,g}(x)$ is in parallel with that of the quark and gluon orbital angular momentum distributions $L_{q,g}(x)$.[2)] The latter are building blocks of the Jaffe–Manohar sum rule for the nucleon spin

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g. \qquad (4)$$

Following this analogy, we have derived a novel momentum sum rule. By eliminating the unpolarized PDFs $q(x), g(x)$, we find for spinless hadrons

$$\begin{aligned} 1 \;=\; & -3\sum_q C_q^{(2)} - \frac{3}{2}C_g^{(2)} + \frac{3}{2}\int_{-1}^1 dx dx' \\ & \times \left[2x\sum_q \frac{\Psi_{Fq}(x,x') + \tilde{\Psi}_{Fq}(x,x')}{x-x'} + \frac{\tilde{\Psi}_G(x,x')}{x-x'} \right], \end{aligned} \qquad (5)$$

where $C_{q,g}^{(2)}$ are the second moment of $C_{q,g}(x)$. They can be interpreted as kinetic energies. The genuine twist-three correlators Ψ's all feature the operator representing the color Lorentz force in the transverse plane.

$$F_a^{+i} = \frac{1}{\sqrt{2}}(\vec{E} + \vec{v}\times\vec{B})_a^i. \qquad (6)$$

The remaining three terms can be interpreted as potential energies associated with the color Lorentz force.

By construction, (5) can be viewed as the momentum version of the Jaffe–Manohar sum rule[3)] for the nucleon spin. In the nucleon structure community, there have been two famous spin sum rules, the Jaffe–Manohar sum rule and the Ji sum rule. On the other hand, only one (trivial) momentum sum rule has been known.

$$1 = \sum_q A_{q+\bar{q}} + A_g. \qquad (7)$$

We now have two momentum sum rules in one-to-one correspondence with the two spin sum rules. Phenomenological consequences of this will be explored elsewhere.

References

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*1 RIKEN BNL Research Center
*2 Physics Department, Brookhaven National Laboratory

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