

DCP2 - Manual

**Antisymmetric Distorted Wave Impulse Approximation
Calculations for Composite Particle Scattering**

by

T. Udagawa, A. Schulte and F. Osterfeld

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Contents

1	What is DCP2?	4
2	What can DCP2 calculate?	4
2.1	Reactions	4
2.2	Reaction mechanism	4
2.3	Outline of calculation	5
3	Structure of the Program	6
3.1	Flow of the program	6
3.2	Compile and Run	7
4	How to make a Input Data File	8
4.1	Outline of input form	8
4.1.1	Structure	8
4.2	Detailed description of input data	8
4.2.1	Calculation options	8
4.2.2	Output options	9
4.2.3	Nuclei in each channel, and incident beam lab energy	9
4.2.4	Angular momenta in the reaction	10
4.2.5	Optical model parameters (OMP)	11
4.2.6	Partial wave expansions in each channel	11
4.2.7	Radial integration information	12
4.2.8	Angle information	12
4.2.9	Form factor calculations	13
4.2.10	Love-Franey NN interaction information	14
4.2.11	Single particle and hole state information	14
4.3	Definition of input parameters	16
4.3.1	Optical model potential	16
4.3.2	Love-Franey NN effective interaction	16
4.3.3	Single particle potential	16
4.4	Input Examples	17
4.4.1	$^{12}\text{C}(p, n)^{12}\text{N}$ (1^+ , 18.1 MeV) at $E_{lab}=200$ MeV	17
4.4.2	$^{90}\text{Zr}(^3\text{H}, t)^{90}\text{Nb}$, (0^- , 27.9 MeV) at $E_{lab}=600$ MeV	18
4.5	Output Examples	19
4.5.1	$^{12}\text{C}(p, n)^{12}\text{N}$ (1^+ , 18.1 MeV) at $E_{lab}=200$ MeV	19
4.5.2	$^{90}\text{Zr}(^3\text{H}, t)^{90}\text{Nb}$, (0^- , 27.9 MeV) at $E_{lab}=600$ MeV	23
5	Formulas	26
5.1	Differential cross section	26
5.2	Form factors	27
5.2.1	Direct form factor	27
5.2.2	Direct form factor for the nucleon-nucleus scattering	27
5.2.3	Exchange form factor	28
5.2.4	Exchange form factor for the nucleon-nucleus scattering	28
5.2.5	Exchange form factor in the no-recoil approximation	29
5.2.6	Exchange form factor in the plane wave approximation	29
5.3	Relativistic kinematics	30
5.4	Projectile Density distribution	30
5.4.1	Deuteron	30
5.4.2	Helium	31
5.5	Folding optical potential	31
A	Subroutines and their functions	32
A.1	Structure of the program	32
A.2	Functions of the subroutine	33
A.2.1	DCP2 -Main Routine	33
A.2.2	BSAXON and UNCPST - Single particle states	34
A.2.3	DCHECK1 - Checking field lengths of variables	34
A.2.4	OPT, FLGLCH, POTEN, and DISWAVE - Distorted waves	35
A.2.5	AFACAL - α -coefficients	35
A.2.6	EFFINT - Effective interaction components	36
A.2.7	PDENST - Projectile density function	36
A.2.8	FFCALD - Direct form factor	37
A.2.9	PACALD - Direct overlap integral and transition amplitudes	38

A.2.10	XLMCAL - Distortion factor	38
A.2.11	FFCALDM - Modified direct form factor	38
A.2.12	DCHECK2 - Checking field lengths of variables for the exchange part	39
A.2.13	TRECAL, DENST, TRECALM, DENSTM - Non-local projectile and target densities for exchange part	39
A.2.14	GFAC, GFACM - Vector coupling factors for exchange part	40
A.2.15	FFCALE - Exchange form factors	41
A.2.16	TENSOR - Tensor interaction for exchange part	42
A.2.17	PACALE - Exchange overlap integral and transition amplitudes	43
A.2.18	FFCALEM - Exchange overlap integral and transition amplitudes	43
A.2.19	CROSS - The differential cross sections	44
A.2.20	YLCAL, GAUSF, BESSEL, CLEB, CLEBZ, RAC7, NINEJ, DSPLS3 - Supporting subroutines	45
B	Variables and their definitions	47
B.1	PARAMETER variables	47
B.2	COMMON variables	48
B.2.1	CNST - Constants	48
B.2.2	CNTRL - Control of the calculation and output	48
B.2.3	SPSTAT - Particle-hole states in the target	48
B.2.4	BSX, UNCPA, CBST - Bound state wave functions	49
B.2.5	DWCC, POTCC, COUCC, RELKIN, OPTO, DISW - Distorted waves	51
B.2.6	ANGCC - Angle information	54
B.2.7	SPECFC - α -coefficients	54
B.2.8	FFCC, CDENS CCKF - Preparation of form factor calculations	54
B.2.9	CGFACA, CGFACB, TREDEN - Exchange form factor calculations	56
B.2.10	CGGR, AMPD, OVDE - Form factors and Transition amplitudes	57
B.2.11	CPWFAC, CGFACC, CXLM - Plane wave approximation for exchange form factor	58
B.3	List of Key variables	59
B.4	Variables on the TAPE	60

1 What is DCP2?

DCP is the abbreviation for **D**istorted wave impulse approximation (DWIA) calculations for **C**omposite **P**article scattering. DCP2 is a computer program, which performs a DWIA calculation for the light-ion induced inelastic scattering and charge exchange reactions to the discrete final states at intermediate energies. It permits the exact calculation of the so-called knock-on exchange transition amplitudes. It can calculate the exchange part in the no-recoil approximation, and can restore the recoil effect but in the plane wave approximation.

This program is originated from the program DCP1, coded by T. Udagawa, A. Schulte and F. Osterfeld in the middle of nineteen eighties. The original version of DCP1 was not published, and was not kept anywhere unfortunately.

B. T. Kim collected some of original subroutines from the Udagawa's Texas group, rebuilt the program with helps of Profs. H. Sakai and M. Ichimura during his stay at RIBF, Nishina Center, RIKEN, Japan in the winters of 2012 and 2013, and renamed it DCP2. Thus, B. T. Kim has the full responsibility for the present form of DCP2.

The code is written in FORTRAN. The program is written with capital letters. In this manual, the name consisting of capital letters denotes generally that of a subroutine, a variable, or a common block.

2 What can DCP2 calculate?

2.1 Reactions

DCP2 calculates the differential cross sections for the inelastic scattering

$$A(a, a')B$$

where $A(B)$ is the target (residual) nucleus, while a is the projectile nucleus and a' in its excited state, and the charge exchange reactions

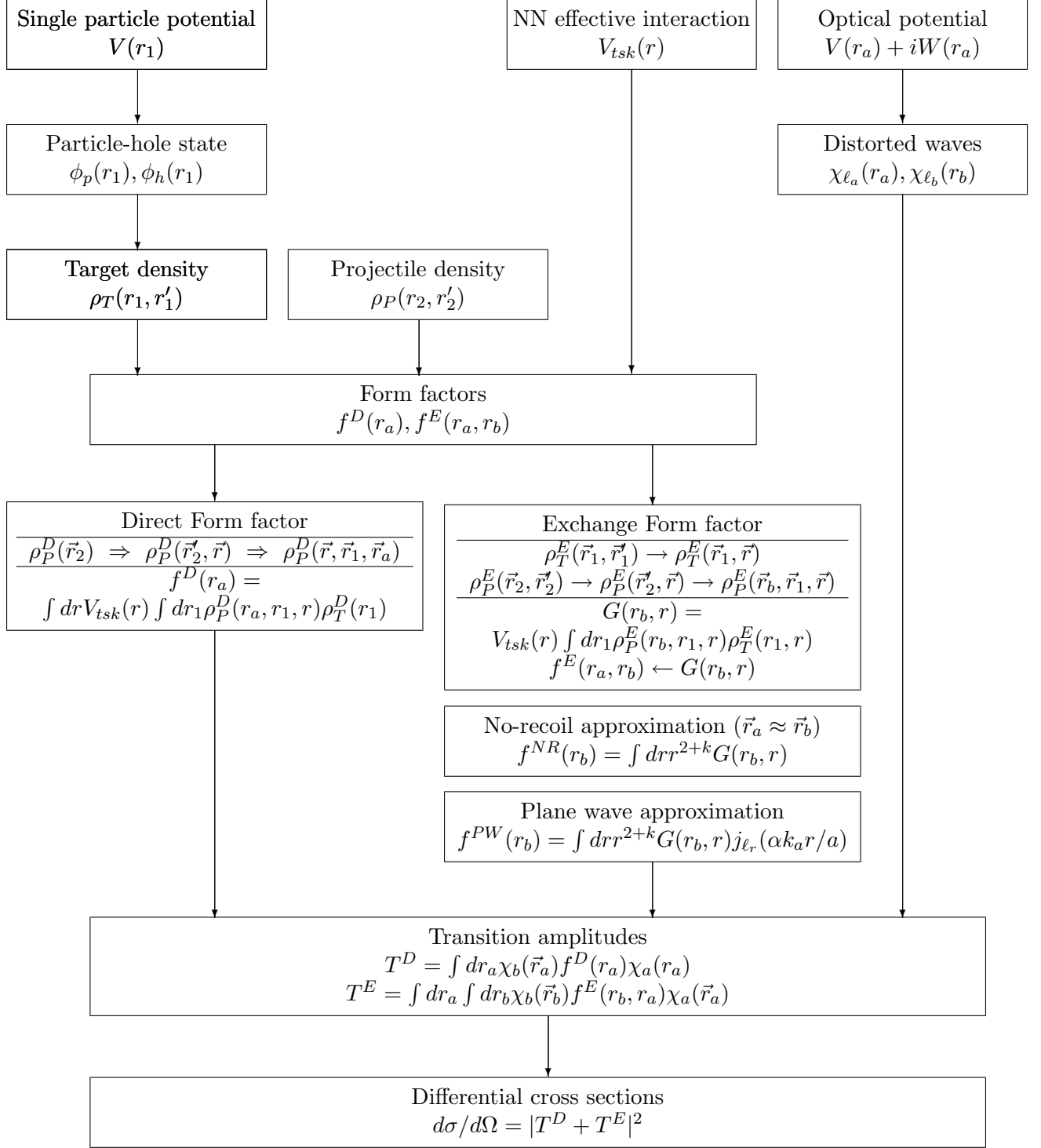
$$A(a, b)B$$

where b is the ejectile nucleus, by exchanging a charge with a , say, (p, n) , $({}^3\text{He}, t)$, etc.

2.2 Reaction mechanism

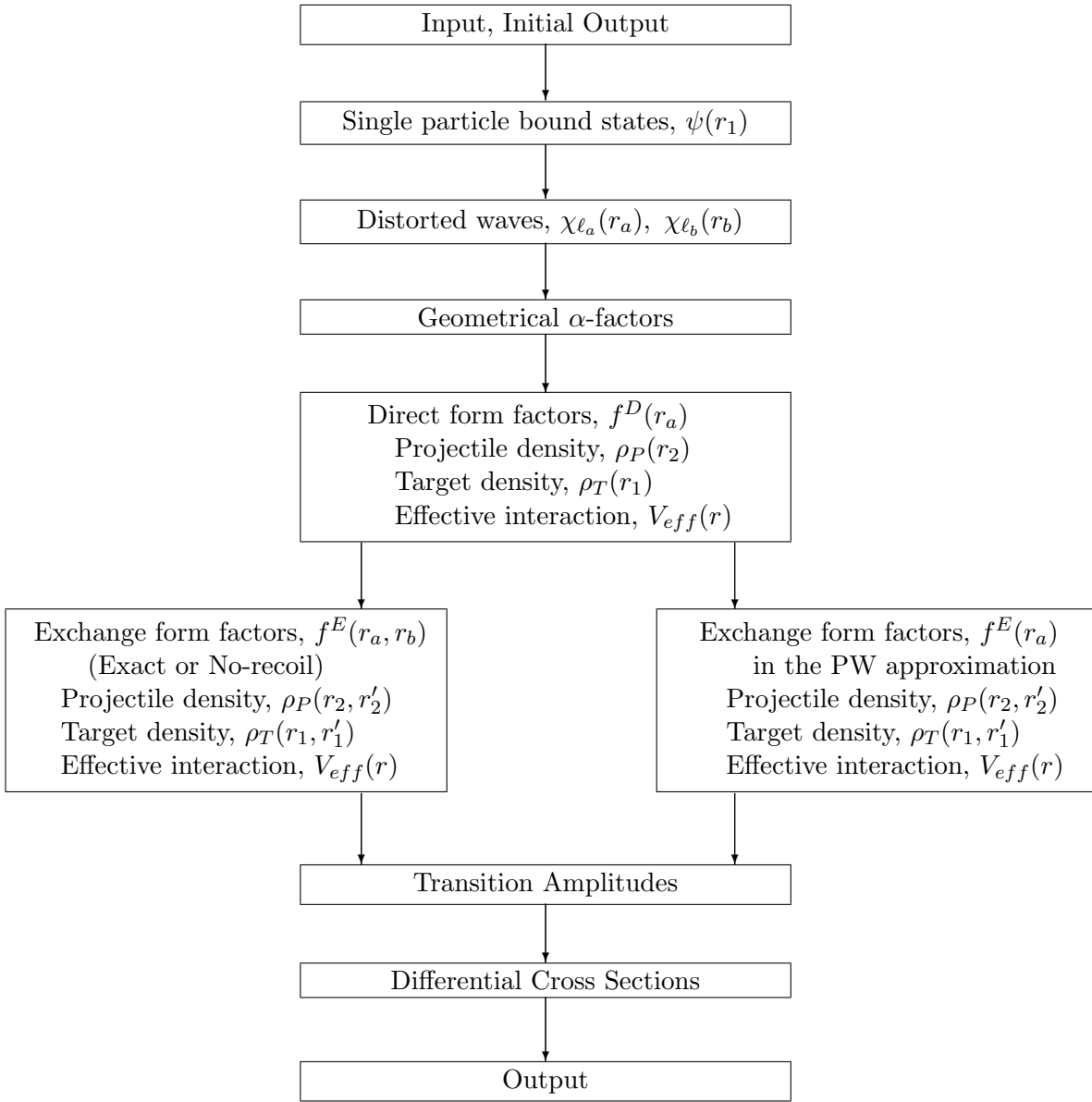
The antisymmetrized DWIA with the NN effective interactions with direct and exchange parts is the basic reaction mechanism. The effective interaction are assumed to be local and to contain central and tensor terms with spin-spin and isospin-isospin interactions. The excited and charge exchanged states are assumed to be in the single particle model state in a Woods-Saxon potential.

2.3 Outline of calculation



3 Structure of the Program

3.1 Flow of the program



3.2 Compile and Run

DCP2 consists of 8 fortran source files which contain subroutines as follows;

Source file	Subroutines
dcp2.f	MAIN, DCHECK1, DCHECK2
cmm.f	BSAXON, UNCPST, OPT, POTEN, FLGLCH, DISWAVE, RAC7, CLEB, CLEBZ, NINEJ, GAUSF, BESSEL, YLCAL, DSPLS3
crs.f	CROSS
ffc.f	AFACAL, EFFINT, PDENST
ffd.f	FFCALD, PACALD
ffdm.f	FFCALDM, XLMCAL
ffe.f	FFCALE, TRECAL, DENST, GFAC, TENSOR, PACALE
ffem.f	FFCALEM, TRECALM, DENSTM, GFACM

DCP2 can be compiled by calling a fortran compiler f77 or g77 on the shell of LINUX.

As an example,

```
"g77 dcp2.f cmm.f crs.f ffc.f ffd.f ffdm.f ffe.f ffem.f -o dcp2"
```

compiles DCP2 and creates a executable file "dcp2".

Before the run, one should prepare input data file (unit 7), "dcp.dat" on the shell. To execute, simply type

```
"./dcp2"
```

4 How to make a Input Data File

4.1 Outline of input form

4.1.1 Structure

Line #	Function
1	1. Calculation options
2	2. Output options
3-4	3. Nuclei in each channel, and incident beam lab energy
5-9	4. Angular momenta in the reaction
10-11	5. Optical model parameters of each channel
12	6. Partial wave information in each channel
13-14	7. Radial integration information
15	8. Angle information
16-19	9. Form factor calculations
20-32	10. NN Love-Franey interaction information
33-	11. Single particle and hole state information

4.2 Detailed description of input data

4.2.1 Calculation options

Line 1 : READ(7,10) (KTRLD(N),N=1,9)
FORMAT(24I3)
Stored in COMMON /CNTRL/

Variable	N	Value	Explanation
KTRLD(N)	1	0	Normal composite particle case with exact finite range form factor.
		1	Recoil effects in the plane wave approximation.
		2	No-recoil calculation is made.
	2	0	DWIA calculation is made.
		1	PWIA calculation is made.
	3	0	Normal optical model potential.
		1	Single folding optical model potential.
	4	0	Tensor force is included.
		1	Tensor force is neglected. (Tensor force is set equal to zero in EFFINT)
	5	0	Exchange effect is calculated.
		1	Exchange effect is neglected.
	9	0	Nonrelativistic kinematics is used.
		1	Relativistic kinematics is used.

4.2.2 Output options

Line 2 : READ(7,10) (KTLOUT(N),N=1,24)
 FORMAT(24I3)
 Stored in COMMON /CNTRL/

Variabe	N	Value	Explanation
KTLOUT(N)	1	1	Output of bound state wave functions.
	2	1	Output of distorted waves.
	3	1	Output of form factors and transition amplitudes in PACALD and PACALE.
	4	1	Output in GFAC.
	5	1	Output of elastic scattering information in OPT.
	7	1	Output of Coulomb wave functions in FLGLCH.
	8	1	Output of ph form factors.
	13	10	Output of detailed form factors, at every (KTLOUT(13) \times mesh size) point.

4.2.3 Nuclei in each channel, and incident beam lab energy

Line 3 : READ(7,12) TMI,Z1I,PMI,Z2I,ELI
 FORMAT(10F7.3)
 Stored in COMMON /DWCC/ by changing the names as
 TMASA,TZA,PMASA,PZA, respectively.

Variable	Explanation
TMI	Mass number of initial target, m_A .
Z1I	Charge number of target, Z_A .
PMI	Mass number of projectile, m_a . (Presently up to 3. Otherwise, increase parameter "NXA")
Z2I	Charge number of projectile, Z_a .
ELI	Incident beam LAB energy in MeV.

Line 4 : READ(7,12) TMI,Z1I,PMI,Z2I
 FORMAT(10F7.3)
 Stored in COMMON /DWCC/ by changing the names as
 TMASB,TZB,PMASB,PZB, respectively.

Variable	Explanation
TMF	Mass number of residual nucleus, m_B .
Z1F	Charge number of residual nucleus, Z_B .
PMF	Mass number of ejectile, m_b . (Presently up to 3. Otherwise, increase parameter "NXA")
Z2F	Charge number of ejectile, Z_b .

4.2.4 Angular momenta in the reaction

Line 5 : READ(7,10) JATW,ISATW
 FORMAT(24I3)
 Stored in COMMON /FFCC/

Variable	Explanation
JATW	Twice of the total angular momentum of the target A , I_A .
ISATW	Twice of the spin of the projectile a , s_a .

Line 6 : READ(7,17) JT,KPARIT,IST,EET
 FORMAT(3I3,3F7.3)
 Stored in COMMON /SPSTAT/

Variable	Explanation
JT	Transferred total angular momentum, j_t .
KPARIT	= 0; No parity change in the reaction, = 1; Parity change.
IST	Transferred spin, $s_t (= s_1 = s_2)$.
EET	Transferred energy in MeV.

Line 7 : READ(7,10) NOLTR, (LTR(N),N=1,NOLTR)
 FORMAT(24I3)
 Stored in COMMON /SPSTAT/

Variable	Explanation
NOLTR	Maximum number of transferred orbital angular momenta.
LTR(NOLTR)	Transferred orbital angular momenta, ℓ_t .

Line 8 : READ(7,10) NLSMAX, (ITR(I),ISR(I),L1R(I),I=1,NLSMAX)
 FORMAT(24I3)
 Stored in COMMON /SPSTAT/

Variable	Explanation
NLSMAX	Maximum number of $\{\ell_1 s_1 t_1\}$ sets in the target system.
ITR(NLSMAX)	Isospin, $t_1 = t_2$.
ISR(NLSMAX)	Spin, $s_1 (= s_2 = s_t)$.
L1R(NLSMAX)	Orbital angular momentum, ℓ_1 .

Line 9 : READ(7,10) MXMAX,MJMAX
 FORMAT(24I3)
 Stored in COMMON /FFCC/ for MXMAX
 Stored in COMMON /CTFAC/ for MJMAX

Variable	Explanation
MXMAX	Maximum value of m_{ℓ_t} .
MJMAX	Maximum value of m_{j_t} in [Eq.(12)].

4.2.5 Optical model parameters (OMP)

Line 10 : READ(7,12) VA,WA,WAS,ARA,AIA,AISA,RZRA,RZIA,RZISA,RZCA
 FORMAT(10F7.3)
 Stored in COMMON /POTCC/

Variable	Explanation (Woods-Saxon OMP in the incident channel)
VA	Depth parameter of real potential in MeV.
WA	Depth parameter of imaginary potential.
WAS	Depth parameter of imaginary surface potential.
ARA	Diffuseness parameter of real potential in fm.
AIA	Diffuseness parameter of imaginary potential.
AISA	Diffuseness parameter of imaginary surface potential.
RZRA	Reduced radius parameter of real potential in fm.
RZIA	Reduced radius parameter of imaginary potential.
RZIAS	Reduced radius parameter of imaginary surface potential.
RZCA	Reduced radius parameter of Coulomb potential.

Line 11 : READ(7,12) VB,WB,WBS,ARB,AIB,AISB,RZRB,RZIB,RZISB,RZCB
 FORMAT(10F7.3)
 Stored in COMMON /POTCC/

Variable	Explanation (Woods-Saxon OMP in the exit channel)
VB	Depth parameter of real potential in MeV.
WB	Depth parameter of imaginary potential.
WBS	Depth parameter of imaginary surface potential.
ARB	Diffuseness parameter of real potential in fm.
AIB	Diffuseness parameter of imaginary potential.
AISB	Diffuseness parameter of imaginary surface potential.
RZRB	Reduced radius parameter of real potential in fm.
RZIB	Reduced radius parameter of imaginary potential.
RZIBS	Reduced radius parameter of imaginary surface potential.
RZCB	Reduced radius parameter of Coulomb potential.

4.2.6 Partial wave expansions in each channel

Line 12 : READ(7,10) (LDWMIR(I),LDWMXR(I),LDWSTR(I),I=1,2)
 FORMAT(24I3)
 Stored in COMMON /DWCC/

Variable	Explanation (Number of partial waves, ℓ_a, ℓ_b)
LDWMIR(1)	Starting partial wave in the incident channel, ℓ_a .
LDWMXR(1)	Ending partial wave.
LDWSTR(1)	Step of partial wave.
LDWMIR(2)	Starting partial wave in the exit channel, ℓ_b .
LDWMXR(2)	Ending partial wave.
LDWSTR(2)	Step of partial wave.

4.2.7 Radial integration information

Line 13 : READ(7,11) (NXMIR(I),NXMXR(I),I=1,4),NHDMX,NHEMX
 FORMAT(14I5)
 Stored in COMMON /DWCC/ for NXMIR(I),NXMXR(I)
 Stored in COMMON /FFCC/ for NHDMX,NHEMX

Variable	Explanation
NXMIR(1)	Starting mesh point in the incident channel, r_a .
NXMXR(1)	Ending mesh point.
NXMIR(2)	Starting mesh point in the exit channel, r_b .
NXMXR(2)	Ending mesh point.
NXMIR(3)	Starting mesh point in the projectile system, r_2 .
NXMXR(3)	Ending mesh point.
NXMIR(4)	Starting mesh point in the target system, r_1 .
NXMXR(4)	Ending mesh point.
NHDMX	Interaction range in the direct form factor calculation, r^D .
NHEMX	Interaction range in the exchange form factor calculation, r^E .

Line 14 : READ(7,12) (XMESR(I),I=1,4),XMESH D,XMESHE
 FORMAT(10F7.3)
 Stored in COMMON /DWCC/ for XMESR
 Stored in COMMON /FFCC/ for XMESH D,XMESHE

Variable	Explanation
XMESR(1)	Mesh size in the incident channel, Δr_a .
XMESR(2)	Mesh size in the exit channel, Δr_b .
XMESR(3)	Mesh size in the projectile system, Δr_2 .
XMESR(4)	Mesh size in the target system, Δr_1 .
XMESH D	Mesh size in the direct form factor calculation, Δr^D .
XMESHE	Mesh size in the exchange form factor calculation, Δr^E .

4.2.8 Angle information

Line 15 : READ(7,12) THEB,THEBMX,DTHEB,THMIN,THMAX,THIND
 FORMAT(10F7.3)
 Stored in COMMON /ANGCC/

Variable	Explanation
THEB	Starting angle in the differential cross section calculations.
THEBMX	Ending angle.
DTHEB	Step in angle.
THMIN	Starting angle in elastic cross section calculations.
THMAX	Ending angle.
THIND	step in angle.

4.2.9 Form factor calculations

Line 16 : READ(7,10) (KCETN(N),N=1,2)
 FORMAT(24I3)
 Stored in COMMON /FFCC/

Variable	Value	Explanation
KCETN(1)	0	Central exchange form factor is calculated.
	1	Not considered.
KCETN(2)	0	Tensor exchange form factor is calculated.
	1	Not considered.

Line 17 : READ(7,11) NBCMI,NBCMX,NBSTPD,NBSTPE
 NONAR,NASTEP,N1STEP,NGAUSR,
 FORMAT(14I5)
 Stored in COMMON /FFCC/

Variable	Explanation
NBCMI	Minimum mesh point in the form factor.
NBCMX	Maximum mesh point.
NBSTPD	Mesh step in the direct form factor.
NBSTPE	Mesh step in the exchange form factor.
NONAR	Number of mesh point in r_a integration for exchange ff. (Integration is made only around $r_a = r_b$ with a range NONAR.)
NASTEP	Mesh step in the incident channel radius, r_a .
N1STEP	Mesh step in the target system radius, r_1 . (For (p,n) or (p,p'), better put all steps unity.)
NGAUSR	Number of gaussian integration points.

Line 18 : READ(7,11) LAMMXD(1),LAMMXD(2),LAMMXD(3)
 FORMAT(14I5)
 Stored in COMMON /CDENS/ for LAMMXD(I)

Variable	Explanation
LAMMXD(1)	Maximum value of λ_1 in exchange form factor.
LAMMXD(2)	Maximum value of λ_2 in exchange form factor.
LAMMXD(3)	Maximum value of λ in exchange form factor.

Line 19 : READ(7,14) LRP1MX,FACNR
 FORMAT(I7,6F7.2)
 Stored in COMMON /CPWFAC/

Variable	Explanation
LRP1MX	Maximum angular momentum of $\ell_r + 1$ in the PW approximation. (Inputting LRP1MX=1, and FACNR=0.0, gives NR approximation.)
FACNR	Recoil factor α . 1.0 is suggested. (See Formulation 3.3.)

4.2.10 Love-Franey NN interaction information

Line 20 : READ(7,14) KTRL1,(VRANG(N),N=1,6)
 FORMAT(I7,6F7.2)
 Stored in COMMON /FFCC/ for VRANG(N)

Variable	Explanation
KTRL1	=1; Yukawa type interaction, (Set equal to 1. Actually dummy.) =2; Gaussian type interaction. (Not used in the present program.)
VRANG(N)	6 different range parameters in the effective NN interaction.

Line 21-32 : READ(7,13) ((VSTR(K,N),N=1,6), K=1,12)
 FORMAT(7F10.4)
 Stored in COMMON /FFCC/

Variable	K	(<i>tsk</i>)	Strength parameters of Love-Franey interaction [W. G. Love and M. A. Franey, Phys. Rev. C24 (1981) 1073; C27 (1983) 438(E); C31 (1985) 488.]
VSTR(K,N)	K		N (=1 to 6) corresponds to 6 different ranges in VRANG(N).
	1	(000)	Real SO <i>t</i> -matrix interaction strength.
	2	(010)	Real TE <i>t</i> -matrix interaction strength.
	3	(012)	Real TNE <i>t</i> -matrix interaction strength.
	4	(100)	Real SE <i>t</i> -matrix interaction strength.
	5	(110)	Real TO <i>t</i> -matrix interaction strength.
	6	(112)	Real TNO <i>t</i> -matrix interaction strength.
	7	(000)	Imaginary SO <i>t</i> -matrix interaction strength.
	8	(010)	Imaginary TE <i>t</i> -matrix interaction strength.
	9	(012)	Imaginary TNE <i>t</i> -matrix interaction strength.
	10	(100)	Imaginary SE <i>t</i> -matrix interaction strength.
	11	(110)	Imaginary TO <i>t</i> -matrix interaction strength.
	12	(112)	Imaginary TNO <i>t</i> -matrix interaction strength.

4.2.11 Single particle and hole state information

Line 33 : READ(7,12) TMAS,PMAS,ZZT,ZZP
 FORMAT(10F7.3)
 Stored in COMMON /UNCPSA/

Variable	Explanation
TMAS	Mass of target system. (Target mass -1)
PMAS	Mass of valence nucleon.
ZZT	Charge of target system.
ZZP	Charge of valence nucleon.

Line 34 : READ(7,12) VSX,VSOR,DFNR,DFNSO,RZR,RZSO,RZC
 FORMAT(10F7.3)
 Stored in COMMON /BSX/ except VSX
 Stored in COMMON /UNCPSA / for VSX which will be searched in BSAXON.

Variable	Bound state potential parameters
VSX	Depth parameter.
VSOR	Spin-orbit depth parameter.
DFNR	Diffuseness parameter.
DFNSO	Spin-orbit diffuseness parameter.
RZR	Reduced radius parameter.
RXSO	Spin-orbit reduced radius parameter.
RZC	Coulomb reduced radius parameter.

Line 35 : READ(7,10) NOSP,NOSH
 FORMAT(2I3)
 Stored in COMMON /SPSTAT/

Variable	Explanation
NOSP	Number of particle states.
NOSH	Number of hole states.

Line 36 : READ(7,15) (ESP(N),NSP(N),LSP(N),JTWP(N),ITP(N),ITZP(N),
 + α N=1,NOSP)
 α =NOSP FORMAT(F10.5,5I5)
 Stored in COMMON /SPSTAT/

Variable	Single particle state information
ESP(N)	energy.
NSP(N)	Number of nodes.
LSP(N)	Orbital angular momentum.
JTWP(N)	Twice of total angular momentum.
ITP(N)	Isospin T . (Not used.)
ITZP(N)	= 1; neutron, = -1; proton.

Line 37+ α : READ(7,15) (ESH(N),NSH(N),LSH(N),JTW(H),ITH(N),ITZH(N),
 + β N=1,NOSH)
 β =NOSH FORMAT(F10.5,5I5)
 Stored in COMMON /SPSTAT/

Variable	Single hole state information
ESH(N)	energy.
NSH(N)	Number of nodes.
LSH(N)	Orbital angular momentum.
JTWH(N)	Twice of total angular momentum.
ITH(N)	Isospin T . (Not used.)
ITZH(N)	= 1; neutron, = -1; proton.

4.3 Definition of input parameters

4.3.1 Optical model potential

The optical model potential is of Woods-Saxon type, expressed as

$$\begin{aligned}
 U(r) &= -V_0 f_0(r) - i[W_I f_I(r) + 4W_S f_S(r)] + V_{Coul} \\
 f_0(r) &= \frac{1}{1 + \exp[(r - R_0)/a_0]} \\
 f_I(r) &= \frac{1}{1 + \exp[(r - R_I)/a_I]} \\
 f_S(r) &= \frac{1 + \exp[(r - R_S)/a_S]}{\{1 + \exp[(r - R_S)/a_S]\}^2}
 \end{aligned}$$

Note that **no spin-orbit interaction is applied**.

4.3.2 Love-Franey NN effective interaction

The Love-Franey nucleon-nucleon effective interaction is a superposition of Yukawa types as

$$\begin{aligned}
 V_{12}(r) &= V^C(r) + V^T(r)S_{12} \\
 V^C(r) &= \sum_{i=1}^6 V_i^C Y(r/R_i), \quad Y(x) = e^{-x}/x \\
 V^T(r) &= \sum_{i=1}^6 V_i^T r^2 Y(r/R_i) \\
 S_{12} &= \frac{3}{r^2} (\vec{\sigma}_1 \cdot \vec{r})(\vec{\sigma}_2 \cdot \vec{r}) - (\vec{\sigma}_1 \cdot \vec{\sigma}_2) \\
 &\equiv \left(\frac{24\pi}{5}\right)^{1/2} \sum_m (\vec{\sigma}_1 \vec{\sigma}_2)_{2m} Y_{2m}^*(\hat{r})
 \end{aligned}$$

4.3.3 Single particle potential

The single particle potential of Woods-Saxon type, is expressed as

$$\begin{aligned}
 V(r) &= -V_0 f_0(r) - \bar{\lambda}_\pi^2 (\vec{\sigma} \cdot \vec{\ell}) V_{SO} f_{SO}(r) + V_{Coul} \\
 f_0(r) &= \frac{1}{1 + \exp[(r - R_0)/a_0]} \\
 f_{SO}(r) &= \frac{1}{a_{SO} r} \frac{1 + \exp[(r - R_{SO})/a_{SO}]}{\{1 + \exp[(r - R_{SO})/a_{SO}]\}^2}
 \end{aligned}$$

where $\bar{\lambda}_\pi = \hbar/m_\pi c = \sqrt{2}$.

4.4 Input Examples

4.4.1 $^{12}\text{C}(p, n)^{12}\text{N} (1^+, 18.1 \text{ MeV})$ at $E_{lab}=200 \text{ MeV}$

```

0 0 0 0 0 0 0 0 1
0 0 0 0 0 0 0 0 0
12.0 6.0 1.0 1.0 200.0 0 0 0 0
12.0 7.0 1.0 0.0
0 1
1 0 1 18.1
2 0 2
2 1 1 0 1 1 2
4 1
11.00 14.00 0.000 0.69 0.59 0.867 1.20 1.17 1.260 1.20
11.00 14.00 0.000 0.69 0.59 0.867 1.20 1.17 1.260 1.20
0 22 1 0 22 1
0 1 120 1 120 1 80 1 120 300 300
0.10 0.10 0.10 0.10 0.03 0.150
1.0 50.0 1.0 10.0 70.0 1.0
0 0
1 120 1 1 59 1 1 32
2 0 2
4 1.000
1 1.40 0.70 0.55 0.40 0.25 0.15
-31.50 0.000 0.0 0.0 -193.752 4714.12 0.0
-10.50 0.000 0.0 0.0 -1216.75 3567.35 0.0
0.00 -69.7061 0.0 0.0 654.904 -24728.8 325631.0
-10.50 0.00 0.0 0.0 -2615.66 8396.19 0.0
3.50 0.00 0.0 0.0 -1439.49 7313.42 0.0
0.00 14.8573 0.0 0.0 334.910 -4099.14 50904.3
0.00 0.00 0.0 0.0 -563.008 -919.824 0.0
0.00 0.00 0.0 0.0 -2431.21 7419.73 0.0
0.00 -4.70474 0.0 0.0 478.422 -15805.0 212453.0
0.00 0.00 0.0 0.0 -469.589 1458.53 0.0
0.00 0.00 0.0 0.0 -217.827 -1076.89 0.0
0.00 3.33699 0.0 0.0 -303.689 9392.15 -147371.
11.0 1.00 6.0 1.00
46.55 1.91 0.765 1.000 1.260 1.260 1.250
5 2
26.00 0 0 1 1 -1
16.00 0 1 3 1 -1
2.000 0 1 1 1 -1
2.000 0 2 3 1 -1
2.000 0 3 5 1 -1
31.20 0 0 1 1 1
18.72 0 1 3 1 1
*****the end of the input data

```

4.4.2 $^{90}\text{Zr}(^3\text{H},t)^{90}\text{Nb}$, (0^- , 27.9 MeV) at $E_{lab}=600$ MeV

```

0 0 0 0 0 0 0 0 1
0 0 0 0 0 0 0 0 0
90.0 40.0 3.0 2.0 600.0 0 0 0
90.0 41.0 3.0 1.0
0 1
0 1 1 27.9
1 1 1 1
1 1 1 1
4 1
15.31 13.26 0.000 0.724 0.662 0.662 1.276 1.260 1.260 1.260
15.31 13.26 0.000 0.724 0.662 0.662 1.276 1.260 1.260 1.260
0115 1 0115 1 120
0.10 0.10 0.10 0.10 80 1 120 300 300 0
0.01 8.00 0.2 10.0 70.0 1.0
0 10 120 10 10 59 1 1 32
1 1 1 1 1
4 1.0000 0.70 0.55 0.40 0.25 0.15
1 1.40 0.000 0.0 0.0 -193.752 4714.12 0.0
-10.50 0.000 0.0 -1216.752 3567.35 0.0
0.00 -69.7061 0.0 654.904 -24728.8 325631.0
-10.50 0.00 0.0 -2615.66 8396.19 0.0
3.50 0.00 0.0 -1439.49 7313.42 0.0
0.00 14.8573 0.0 334.910 -4099.14 50904.3
0.00 0.00 0.0 -563.008 -919.824 0.0
0.00 0.00 0.0 -2431.21 7419.73 0.0
0.00 -4.70474 0.0 478.422 -15805.0 212453.0
0.00 0.00 0.0 -469.589 1458.53 0.0
0.00 0.00 0.0 -217.827 -1076.89 0.0
0.00 3.33699 0.0 -303.689 9392.15 -147371.
89.0 1.00 40. 1.00
51.00 11.20 0.670 0.670 1.270 1.270 1.270
7 9
5.160 1 1 1 1 -1
2.000 0 4 4 9 1 -1
1.000 0 4 4 7 1 -1
1.000 1 2 2 5 1 -1
1.000 1 2 2 3 1 -1
1.000 2 0 0 1 1 -1
1.000 0 5 5 11 1 -1
14.30 0 3 3 7 1 -1
8.866 1 1 1 3 1 -1
7.63 0 3 5 1 1 -1
5.16 1 3 5 1 1 -1
19.22 0 3 3 7 1 1
14.96 1 1 3 3 1 1
13.42 1 0 3 5 1 1
12.56 1 1 1 1 1 1
11.97 0 4 4 9 1 1
***** The end of the input data *****

```

4.5 Output Examples

4.5.1 $^{12}\text{C}(p, n)^{12}\text{N}$ (1^+ , 18.1 MeV) at $E_{lab}=200$ MeV

```

DWIA DIRECT-EXCHANGE CROSS SECTION CALCULATION FOR THE REACTION
      T      TZ      P      PZ
A-CHANNEL 12.000  6.000  1.000  1.000
B-CHANNEL 12.000  7.000  1.000  0.000
E=200.0 MEV
INITIAL STATE JATW, ISATW= 0 1
FINAL STATE J,K,E= 1 0 18.1 MEV
SPIN TRANSFER IST= 1
TOTAL L TRANSFER = 0 2
KTRLD(1-9) = 0 0 0 0 0 0 0 0 1
KTLOUT(1-20)= 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
** RELATIVISTIC KINEMATICS IS USED
NN INTERACTION POTENTIAL
KTRL1, VRAN= 1 1.40 0.70 0.55 0.40 0.25 0.15
K= 1 VSTR= 31.50 0.00 0.00 -193.75 4714.12 0.00
K= 2 VSTR= -10.50 0.00 0.00 -1216.75 3567.35 0.00
K= 3 VSTR= 0.00 -69.71 0.00 654.90 -24728.80 325631.00
K= 4 VSTR= -10.50 0.00 0.00 -2615.66 8396.19 0.00
K= 5 VSTR= 3.50 0.00 0.00 -1439.49 7313.42 0.00
K= 6 VSTR= 0.00 14.86 0.00 334.91 -4099.14 50904.30
K= 7 VSTR= 0.00 0.00 0.00 -563.01 -919.82 0.00
K= 8 VSTR= 0.00 0.00 0.00 -2431.21 7419.73 0.00
K= 9 VSTR= 0.00 -4.70 0.00 478.42 -15805.00 212453.00
K= 10 VSTR= 0.00 0.00 0.00 -469.59 1458.53 0.00
K= 11 VSTR= 0.00 0.00 0.00 -217.83 -1076.89 0.00
K= 12 VSTR= 0.00 3.34 0.00 -303.69 9392.15 -147371.00
SINGLE P AND H STATES IN THE TARGET SYSTEM
NOSP, NOSH= 5 2
      N      L      2J      2S      T      BE
PARTICLE 1 0 1 1 -1 26.000
PARTICLE 2 0 1 1 -1 16.000
PARTICLE 3 0 1 1 -1 2.000
PARTICLE 4 0 2 3 1 -1 2.000
PARTICLE 5 0 3 5 1 -1 2.000
HOLE 1 0 0 1 1 31.200
HOLE 2 0 1 3 1 18.720
** CALCULATED PH WAVEFUNCTION: KTLOUT(1)
NPAIR= 5
NP, NH= 1 1 4 1 2 2 3 2 5 2
NPAIR= 5
NP, NH= 1 1 4 1 2 2 3 2 5 2
PAIR STATES (IT IS L1)=( 1 1 0)
(NP, NH, SF)= 1 1 1.0 4 1 1.0 2 2 1.0 3 2 1.0 5 2 1.0
PAIR STATES (IT IS L1)=( 1 1 2)
(NP, NH, SF)= 1 1 1.0 4 1 1.0 2 2 1.0 3 2 1.0 5 2 1.0
DISTORTED WAVES FOR INITIAL AND FINAL CHANNELS
OMP PARAMETERS
      V      A      R      W      AI      RI      RC
A-CHANNEL 11.000  0.690  1.200 14.000  0.590  1.170  1.200
B-CHANNEL 11.000  0.690  1.200 14.000  0.867  1.260  1.200
      W      AI      RI      RC
      0.000  0.867  1.260
LDMI LDMX LDST NXMI NXMX XMES
A-CHANNEL 0 22 1 1 120 0.100
B-CHANNEL 0 22 1 1 120 0.100
E, WN, ETA FOR INCIDENT A-CHANNEL= 183.23 2.96 0.08
** OUTPUT OF DW : KTLOUT(2)=NXSTEP, ELASTIC SCAT : KTLOUT(5)
** OUTPUT OF COULOMB WF : KTLOUT(7)
OMP POTENTIAL IN STEPS OF 1.000
-0.605E+01 -0.138E+02 -0.561E+01 -0.131E+02 -0.411E+01 -0.102E+02
-0.134E+01 -0.460E+01 0.750E+00 -0.115E+01 0.134E+01 -0.227E+00
0.133E+01 -0.423E-01 0.120E+01 -0.779E-02 0.106E+01 -0.143E-02
0.948E+00 -0.263E-03 0.855E+00 -0.482E-04 0.778E+00 -0.886E-05
0.714E+00 -0.163E-05
E, WN, ETA FOR EXIT B-CHANNEL= 165.39 2.80 0.00
OMP POTENTIAL IN STEPS OF 1.000
-0.108E+02 -0.138E+02 -0.101E+02 -0.131E+02 -0.791E+01 -0.102E+02
-0.412E+01 -0.460E+01 -0.136E+01 -0.115E+01 -0.352E+00 -0.227E+00
-0.847E-01 -0.423E-01 -0.200E-01 -0.779E-02 -0.470E-02 -0.143E-02
-0.110E-02 -0.263E-03 -0.259E-03 -0.482E-04 -0.608E-04 -0.886E-05

```

-0.143E-04 -0.163E-05

ALPHA COEF. PROJ. SYSTEM IN EQ.(13) (AFACAL)

ALPHA(NLSK,NLT)=

NLT= 1	ALPHA=	-0.33E+00	0.00E+00	0.00E+00	-0.15E+00
NLT= 2	ALPHA=	0.00E+00	-0.15E+00	-0.15E+00	-0.88E-01

FFCALD IS CALLED

INPUT INFORMATION FOR DIRECT FORM FACTOR

N1MIN,N1MAX,N1STEP,XEMSH=	1	120	1	0.100
NBMIN,NBMAX,NBSTEP,XEMSH=	1	120	1	0.100
INTRAN,XMESHD=		300		0.030
MXMAX=	4			

FFCALE IS CALLED

INPUT INFORMATION FOR EXCHANGE FORM FACTOR

NAMIN,NAMAX,NASTEP,XEMSH=	1	120	1	0.100
NBCMI,NBCMX,NBSTEP,XEMSH=	1	120	1	0.100
N2MIN,N2MAX,N2STEP,XEMSH=	1	80	1	0.100
N1MIN,N1MAX,N1STEP,XEMSH=	1	120	1	0.100
INTRAN,XMESHE=		300		0.150
INTEG RANGE FOR RA=RB+-		59		
LAM1MX,LAM2MX,LAMDMX,MXMAX=	2	0	2	4
KCETN(1),KCETN(2)=	0	0		

FINAL DIFFERENTIAL CROSS SECTIONS

LT= 0 CONTRIBUTION TO THE CROSS SECTIONS

CM	ANGLE	MOM.	TRANS	DIRECT	EXCHANGE	TOTAL
0.100E+01	0.166E+00	0.756E+03	0.749E+02	0.358E+03		
0.200E+01	0.187E+00	0.730E+03	0.731E+02	0.344E+03		
0.300E+01	0.219E+00	0.689E+03	0.703E+02	0.321E+03		
0.400E+01	0.256E+00	0.635E+03	0.665E+02	0.293E+03		
0.500E+01	0.297E+00	0.572E+03	0.619E+02	0.259E+03		
0.600E+01	0.340E+00	0.503E+03	0.569E+02	0.223E+03		
0.700E+01	0.386E+00	0.432E+03	0.515E+02	0.187E+03		
0.800E+01	0.432E+00	0.362E+03	0.459E+02	0.152E+03		
0.900E+01	0.479E+00	0.297E+03	0.404E+02	0.120E+03		
0.100E+02	0.526E+00	0.237E+03	0.351E+02	0.910E+02		
0.110E+02	0.574E+00	0.185E+03	0.301E+02	0.669E+02		
0.120E+02	0.622E+00	0.141E+03	0.255E+02	0.472E+02		
0.130E+02	0.671E+00	0.104E+03	0.213E+02	0.319E+02		
0.140E+02	0.720E+00	0.749E+02	0.176E+02	0.204E+02		
0.150E+02	0.768E+00	0.521E+02	0.143E+02	0.122E+02		
0.160E+02	0.817E+00	0.350E+02	0.114E+02	0.672E+01		
0.170E+02	0.866E+00	0.225E+02	0.900E+01	0.326E+01		
0.180E+02	0.915E+00	0.138E+02	0.696E+01	0.129E+01		
0.190E+02	0.964E+00	0.783E+01	0.527E+01	0.341E+00		
0.200E+02	0.101E+01	0.403E+01	0.391E+01	0.534E-01		
0.210E+02	0.106E+01	0.177E+01	0.282E+01	0.150E+00		
0.220E+02	0.111E+01	0.574E+00	0.199E+01	0.437E+00		
0.230E+02	0.116E+01	0.784E-01	0.135E+01	0.785E+00		
0.240E+02	0.121E+01	0.124E-01	0.893E+00	0.112E+01		
0.250E+02	0.126E+01	0.180E+00	0.567E+00	0.139E+01		
0.260E+02	0.131E+01	0.444E+00	0.346E+00	0.157E+01		
0.270E+02	0.135E+01	0.715E+00	0.202E+00	0.168E+01		
0.280E+02	0.140E+01	0.939E+00	0.113E+00	0.170E+01		
0.290E+02	0.145E+01	0.109E+01	0.605E-01	0.165E+01		
0.300E+02	0.150E+01	0.116E+01	0.314E-01	0.155E+01		
0.310E+02	0.155E+01	0.116E+01	0.164E-01	0.141E+01		
0.320E+02	0.160E+01	0.109E+01	0.933E-02	0.124E+01		
0.330E+02	0.164E+01	0.990E+00	0.641E-02	0.106E+01		
0.340E+02	0.169E+01	0.863E+00	0.556E-02	0.884E+00		
0.350E+02	0.174E+01	0.728E+00	0.568E-02	0.713E+00		
0.360E+02	0.179E+01	0.596E+00	0.626E-02	0.558E+00		
0.370E+02	0.183E+01	0.476E+00	0.709E-02	0.424E+00		
0.380E+02	0.188E+01	0.371E+00	0.803E-02	0.312E+00		
0.390E+02	0.193E+01	0.283E+00	0.897E-02	0.223E+00		
0.400E+02	0.198E+01	0.212E+00	0.974E-02	0.154E+00		
0.410E+02	0.202E+01	0.155E+00	0.102E-01	0.103E+00		
0.420E+02	0.207E+01	0.111E+00	0.101E-01	0.660E-01		
0.430E+02	0.212E+01	0.769E-01	0.940E-02	0.408E-01		
0.440E+02	0.216E+01	0.516E-01	0.812E-02	0.242E-01		
0.450E+02	0.221E+01	0.330E-01	0.643E-02	0.136E-01		
0.460E+02	0.226E+01	0.198E-01	0.457E-02	0.722E-02		
0.470E+02	0.230E+01	0.110E-01	0.285E-02	0.353E-02		
0.480E+02	0.235E+01	0.539E-02	0.150E-02	0.156E-02		
0.490E+02	0.239E+01	0.233E-02	0.625E-03	0.639E-03		
0.500E+02	0.244E+01	0.103E-02	0.209E-03	0.331E-03		

LT= 2 CONTRIBUTION TO THE CROSS SECTIONS

CM	ANGLE	MOM.	TRANS	DIRECT	EXCHANGE	TOTAL
0.100E+01	0.166E+00	0.126E+01	0.179E+00	0.619E+00		
0.200E+01	0.187E+00	0.136E+01	0.191E+00	0.824E+00		
0.300E+01	0.219E+00	0.164E+01	0.213E+00	0.130E+01		

0.400E+01	0.256E+00	0.220E+01	0.249E+00	0.222E+01
0.500E+01	0.297E+00	0.316E+01	0.303E+00	0.370E+01
0.600E+01	0.340E+00	0.456E+01	0.377E+00	0.584E+01
0.700E+01	0.386E+00	0.636E+01	0.475E+00	0.859E+01
0.800E+01	0.432E+00	0.843E+01	0.598E+00	0.118E+02
0.900E+01	0.479E+00	0.106E+02	0.743E+00	0.152E+02
0.100E+02	0.526E+00	0.125E+02	0.909E+00	0.185E+02
0.110E+02	0.574E+00	0.140E+02	0.109E+01	0.213E+02
0.120E+02	0.622E+00	0.149E+02	0.128E+01	0.234E+02
0.130E+02	0.671E+00	0.151E+02	0.147E+01	0.245E+02
0.140E+02	0.720E+00	0.146E+02	0.165E+01	0.247E+02
0.150E+02	0.768E+00	0.134E+02	0.182E+01	0.239E+02
0.160E+02	0.817E+00	0.119E+02	0.196E+01	0.223E+02
0.170E+02	0.866E+00	0.100E+02	0.208E+01	0.202E+02
0.180E+02	0.915E+00	0.809E+01	0.216E+01	0.176E+02
0.190E+02	0.964E+00	0.627E+01	0.220E+01	0.150E+02
0.200E+02	0.101E+01	0.468E+01	0.221E+01	0.124E+02
0.210E+02	0.106E+01	0.337E+01	0.219E+01	0.100E+02
0.220E+02	0.111E+01	0.237E+01	0.213E+01	0.792E+01
0.230E+02	0.116E+01	0.166E+01	0.205E+01	0.617E+01
0.240E+02	0.121E+01	0.119E+01	0.194E+01	0.474E+01
0.250E+02	0.126E+01	0.912E+00	0.182E+01	0.361E+01
0.260E+02	0.131E+01	0.762E+00	0.168E+01	0.273E+01
0.270E+02	0.135E+01	0.696E+00	0.153E+01	0.206E+01
0.280E+02	0.140E+01	0.676E+00	0.138E+01	0.155E+01
0.290E+02	0.145E+01	0.677E+00	0.123E+01	0.117E+01
0.300E+02	0.150E+01	0.681E+00	0.108E+01	0.887E+00
0.310E+02	0.155E+01	0.680E+00	0.945E+00	0.671E+00
0.320E+02	0.160E+01	0.669E+00	0.814E+00	0.510E+00
0.330E+02	0.164E+01	0.648E+00	0.694E+00	0.389E+00
0.340E+02	0.169E+01	0.615E+00	0.584E+00	0.300E+00
0.350E+02	0.174E+01	0.572E+00	0.486E+00	0.234E+00
0.360E+02	0.179E+01	0.522E+00	0.400E+00	0.185E+00
0.370E+02	0.183E+01	0.465E+00	0.326E+00	0.149E+00
0.380E+02	0.188E+01	0.404E+00	0.262E+00	0.121E+00
0.390E+02	0.193E+01	0.342E+00	0.208E+00	0.992E-01
0.400E+02	0.198E+01	0.281E+00	0.163E+00	0.805E-01
0.410E+02	0.202E+01	0.223E+00	0.126E+00	0.644E-01
0.420E+02	0.207E+01	0.172E+00	0.965E-01	0.503E-01
0.430E+02	0.212E+01	0.127E+00	0.728E-01	0.384E-01
0.440E+02	0.216E+01	0.912E-01	0.542E-01	0.285E-01
0.450E+02	0.221E+01	0.631E-01	0.399E-01	0.208E-01
0.460E+02	0.226E+01	0.422E-01	0.290E-01	0.150E-01
0.470E+02	0.230E+01	0.274E-01	0.209E-01	0.110E-01
0.480E+02	0.235E+01	0.175E-01	0.151E-01	0.838E-02
0.490E+02	0.239E+01	0.110E-01	0.109E-01	0.674E-02
0.500E+02	0.244E+01	0.685E-02	0.793E-02	0.576E-02

TOTAL CROSS SECTIONS

CM ANGLE	MOM. TRANS.	DIRECT	EXCHANGE	TOTAL
0.100E+01	0.166E+00	0.758E+03	0.751E+02	0.358E+03
0.200E+01	0.187E+00	0.732E+03	0.733E+02	0.344E+03
0.300E+01	0.219E+00	0.691E+03	0.705E+02	0.323E+03
0.400E+01	0.256E+00	0.637E+03	0.667E+02	0.295E+03
0.500E+01	0.297E+00	0.575E+03	0.622E+02	0.263E+03
0.600E+01	0.340E+00	0.507E+03	0.572E+02	0.229E+03
0.700E+01	0.386E+00	0.438E+03	0.519E+02	0.196E+03
0.800E+01	0.432E+00	0.371E+03	0.465E+02	0.164E+03
0.900E+01	0.479E+00	0.307E+03	0.412E+02	0.135E+03
0.100E+02	0.526E+00	0.250E+03	0.360E+02	0.109E+03
0.110E+02	0.574E+00	0.199E+03	0.312E+02	0.882E+02
0.120E+02	0.622E+00	0.156E+03	0.268E+02	0.706E+02
0.130E+02	0.671E+00	0.119E+03	0.228E+02	0.564E+02
0.140E+02	0.720E+00	0.894E+02	0.192E+02	0.451E+02
0.150E+02	0.768E+00	0.656E+02	0.161E+02	0.362E+02
0.160E+02	0.817E+00	0.469E+02	0.134E+02	0.291E+02
0.170E+02	0.866E+00	0.325E+02	0.111E+02	0.234E+02
0.180E+02	0.915E+00	0.219E+02	0.912E+01	0.189E+02
0.190E+02	0.964E+00	0.141E+02	0.748E+01	0.153E+02
0.200E+02	0.101E+01	0.871E+01	0.612E+01	0.124E+02
0.210E+02	0.106E+01	0.514E+01	0.501E+01	0.102E+02
0.220E+02	0.111E+01	0.295E+01	0.412E+01	0.836E+01
0.230E+02	0.116E+01	0.174E+01	0.340E+01	0.695E+01
0.240E+02	0.121E+01	0.121E+01	0.283E+01	0.586E+01
0.250E+02	0.126E+01	0.109E+01	0.238E+01	0.499E+01
0.260E+02	0.131E+01	0.121E+01	0.202E+01	0.431E+01
0.270E+02	0.135E+01	0.141E+01	0.173E+01	0.374E+01
0.280E+02	0.140E+01	0.161E+01	0.149E+01	0.326E+01
0.290E+02	0.145E+01	0.177E+01	0.129E+01	0.283E+01
0.300E+02	0.150E+01	0.184E+01	0.112E+01	0.244E+01
0.310E+02	0.155E+01	0.184E+01	0.961E+00	0.208E+01
0.320E+02	0.160E+01	0.176E+01	0.824E+00	0.175E+01
0.330E+02	0.164E+01	0.164E+01	0.700E+00	0.145E+01
0.340E+02	0.169E+01	0.148E+01	0.590E+00	0.118E+01
0.350E+02	0.174E+01	0.130E+01	0.492E+00	0.947E+00
0.360E+02	0.179E+01	0.112E+01	0.407E+00	0.743E+00
0.370E+02	0.183E+01	0.941E+00	0.333E+00	0.573E+00
0.380E+02	0.188E+01	0.775E+00	0.270E+00	0.433E+00

0.390E+02	0.193E+01	0.625E+00	0.217E+00	0.322E+00
0.400E+02	0.198E+01	0.493E+00	0.173E+00	0.234E+00
0.410E+02	0.202E+01	0.378E+00	0.136E+00	0.167E+00
0.420E+02	0.207E+01	0.282E+00	0.107E+00	0.116E+00
0.430E+02	0.212E+01	0.204E+00	0.822E-01	0.792E-01
0.440E+02	0.216E+01	0.143E+00	0.623E-01	0.527E-01
0.450E+02	0.221E+01	0.961E-01	0.463E-01	0.344E-01
0.460E+02	0.226E+01	0.620E-01	0.336E-01	0.223E-01
0.470E+02	0.230E+01	0.384E-01	0.238E-01	0.146E-01
0.480E+02	0.235E+01	0.229E-01	0.166E-01	0.994E-02
0.490E+02	0.239E+01	0.133E-01	0.115E-01	0.738E-02
0.500E+02	0.244E+01	0.788E-02	0.814E-02	0.609E-02

4.5.2 $^{90}\text{Zr}(^3\text{H},t)^{90}\text{Nb}$, (0^- , 27.9 MeV) at $E_{lab}=600$ MeV

DWIA DIRECT-EXCHANGE CROSS SECTION CALCULATION FOR THE REACTION

A-CHANNEL 90.000 40.000 3.000 2.000
 B-CHANNEL 90.000 41.000 3.000 1.000

E=600.0 MEV

INITIAL STATE JATW,ISATW= 0 1
 FINAL STATE J,K,E= 0 1 27.9 MEV
 SPIN TRANSFER IST= 1
 TOTAL L TRANSFER = 1

KTRLD(1-9) = 0 0 0 0 0 0 0 0 1
 KTLOUT(1-20)= 0

** RELATIVISTIC KINEMATICS IS USED

NN INTERACTION POTENTIAL

KTRL1,VRAN=	1	1.40	0.70	0.55	0.40	0.25	0.15
K= 1 VSTR=	31.50	0.00	0.00	0.00	-193.75	4714.12	0.00
K= 2 VSTR=	-10.50	0.00	0.00	0.00	-1216.75	3567.35	0.00
K= 3 VSTR=	0.00	-69.71	0.00	0.00	654.90	-24728.80	325631.00
K= 4 VSTR=	-10.50	0.00	0.00	0.00	-2615.66	8396.19	0.00
K= 5 VSTR=	3.50	0.00	0.00	0.00	-1439.49	7313.42	0.00
K= 6 VSTR=	0.00	14.86	0.00	0.00	334.91	-4099.14	50904.30
K= 7 VSTR=	0.00	0.00	0.00	0.00	-563.01	-919.82	0.00
K= 8 VSTR=	0.00	0.00	0.00	0.00	-2431.21	7419.73	0.00
K= 9 VSTR=	0.00	-4.70	0.00	0.00	478.42	-15805.00	212453.00
K= 10 VSTR=	0.00	0.00	0.00	0.00	-469.59	1458.53	0.00
K= 11 VSTR=	0.00	0.00	0.00	0.00	-217.83	-1076.89	0.00
K= 12 VSTR=	0.00	3.34	0.00	0.00	-303.69	9392.15	-147371.00

SINGLE P AND H STATES IN THE TARGET SYSTEM

NOSP,NOSH=	7	9	N	L	2J	2S	T	BE
PARTICLE	1	1	1	1	1	1	-1	5.160
PARTICLE	2	0	4	4	9	1	-1	2.000
PARTICLE	3	0	4	4	7	1	-1	1.000
PARTICLE	4	1	2	2	5	1	-1	1.000
PARTICLE	5	1	2	2	3	1	-1	1.000
PARTICLE	6	2	0	0	1	1	-1	1.000
PARTICLE	7	0	5	5	11	1	-1	1.000
HOLE	1	0	3	3	7	1	-1	14.300
HOLE	2	1	1	1	3	1	-1	8.860
HOLE	3	0	3	3	5	1	-1	7.630
HOLE	4	1	1	1	1	1	-1	5.160
HOLE	5	0	3	3	7	1	1	19.220
HOLE	6	1	1	1	3	1	1	14.960
HOLE	7	0	3	3	5	1	1	13.420
HOLE	8	1	1	1	1	1	1	12.560
HOLE	9	0	4	4	9	1	1	11.970

** CALCULATED PH WAVEFUNCTION: KTLOUT(1)

NPAIR= 4
 NP,NH= 3 5 5 6 4 7 6 8
 PAIR STATES (IT IS L1)=(1 1 1)
 (NP,NH,SF)= 3 5 1.0 5 6 1.0 4 7 1.0 6 8 1.0

PROJECTILE DENSITY FOR FOLDING POT. CALCULATION

KMAS, NXMI, NXM, XMES3 = 3 1 80 0.100
 NORM PROJECTILE,RRMS,G1,FWID= 0.100E+01 0.160E+01 0.675E+00 0.420E+00

NORMAL DENSITY
 N2= 10 0.56E+00
 N2= 20 0.97E-01
 N2= 30 0.52E-02
 N2= 40 0.88E-04
 N2= 50 0.45E-06
 N2= 60 0.73E-09
 N2= 70 0.37E-12
 N2= 80 0.57E-16

DISTORTED WAVES FOR INITIAL AND FINAL CHANNELS

OMP PARAMETERS

	V	A	R	W	AI	RI	RC
A-CHANNEL	15.310	0.724	1.276	13.260	0.662	1.260	1.260
B-CHANNEL	15.310	0.724	1.276	13.260	0.662	1.260	1.260

	LDMI	LDMX	LDST	NXMI	NXM	XMES
A-CHANNEL	0	115	1	1	120	0.100
B-CHANNEL	0	115	1	1	120	0.100

E,W,ETA FOR INCIDENT A-CHANNEL= 578.71 9.39 1.02
 ** OUTPUT OF DW : KTLOUT(2)=NXSTEP, ELASTIC SCAT : KTLOUT(5)

** OUTPUT OF COULOMB WF : KTLOUT(7)

OM	POTENTIAL	IN STEPS OF	0.333				
0.153E+02	-0.133E+02	0.153E+02	-0.133E+02	0.152E+02	-0.133E+02		
0.150E+02	-0.132E+02	0.147E+02	-0.132E+02	0.144E+02	-0.132E+02		
0.141E+02	-0.132E+02	0.136E+02	-0.132E+02	0.132E+02	-0.131E+02		
0.127E+02	-0.130E+02	0.122E+02	-0.128E+02	0.118E+02	-0.126E+02		
0.114E+02	-0.122E+02	0.112E+02	-0.116E+02	0.112E+02	-0.107E+02		
0.115E+02	-0.950E+01	0.119E+02	-0.801E+01	0.125E+02	-0.636E+01		
0.131E+02	-0.475E+01	0.137E+02	-0.334E+01	0.141E+02	-0.224E+01		
0.142E+02	-0.145E+01	0.142E+02	-0.918E+00	0.140E+02	-0.571E+00		
0.137E+02	-0.351E+00	0.134E+02	-0.214E+00	0.130E+02	-0.130E+00		
0.126E+02	-0.791E-01	0.122E+02	-0.479E-01	0.118E+02	-0.290E-01		
0.114E+02	-0.175E-01	0.111E+02	-0.106E-01	0.108E+02	-0.641E-02		
0.104E+02	-0.388E-02	0.101E+02	-0.234E-02	0.984E+01	-0.142E-02		
0.957E+01	-0.856E-03						

E,WN,ETA FOR EXIT B-CHANNEL= 550.99 9.14 0.53

OM	POTENTIAL	IN STEPS OF	1.000				
0.378E+00	-0.133E+02	0.201E+00	-0.132E+02	-0.247E+00	-0.132E+02		
-0.801E+00	-0.130E+02	-0.903E+00	-0.121E+02	0.680E+00	-0.922E+01		
0.400E+01	-0.444E+01	0.634E+01	-0.133E+01	0.674E+01	-0.318E+00		
0.635E+01	-0.716E-01	0.581E+01	-0.159E-01	0.531E+01	-0.351E-02		
0.488E+01	-0.774E-03						

ALPHA COEF. PROJ. SYSTEM IN EQ.(13) (AFACAL)

ALPHA(NLSK,NLT)=

NLT= 1 ALPHA= 0.19E+00 0.19E+00

FFCALD IS CALLED

INPUT INFORMATION FOR DIRECT FORM FACTOR
 N1MIN,N1MAX,N1STEP,XEMSH= 1 120 1 0.100
 NBMIN,NBMAX,NBSTEP,XEMSH= 10 120 10 0.100
 **INTERPOLATION IN R_B IS MADE
 INTRAN,XMESHD= 300 0.030
 MXMAX= 4

FFCALE IS CALLED

INPUT INFORMATION FOR EXCHANGE FORM FACTOR
 NAMIN,NAMAX,NASTEP,XEMSH= 1 120 1 0.100
 NBCMI,NBCMX,NBSTPE,XMESH= 10 120 10 0.100
 **INTERPOLATION IN R_b IS MADE
 N2MIN,N2MAX,N2STEP,XEMSH= 1 80 1 0.100
 N1MIN,N1MAX,N1STEP,XEMSH= 1 120 1 0.100
 INTRAN,XMESHE= 300 0.015
 INTEG RANGE FOR RA=RB+- 59
 LAM1MX,LAM2MX,LAMDMX,MXMAX= 1 0 1 4
 KCETN(1),KCETN(2)= 0 0

FINAL DIFFERENTIAL CROSS SECTIONS

LT= 1 CONTRIBUTION TO THE CROSS SECTIONS

CM	ANGLE	MOM	TRANS	DIRECT	EXCHANGE	TOTAL
0.100E-01	0.247E+00	0.159E+03	0.488E+02	0.333E+02		
0.210E+00	0.249E+00	0.168E+03	0.490E+02	0.390E+02		
0.410E+00	0.256E+00	0.192E+03	0.495E+02	0.540E+02		
0.610E+00	0.266E+00	0.225E+03	0.503E+02	0.754E+02		
0.810E+00	0.279E+00	0.263E+03	0.511E+02	0.993E+02		
0.101E+01	0.296E+00	0.299E+03	0.518E+02	0.121E+03		
0.121E+01	0.315E+00	0.326E+03	0.521E+02	0.138E+03		
0.141E+01	0.336E+00	0.340E+03	0.518E+02	0.146E+03		
0.161E+01	0.359E+00	0.338E+03	0.508E+02	0.144E+03		
0.181E+01	0.383E+00	0.321E+03	0.489E+02	0.134E+03		
0.201E+01	0.408E+00	0.290E+03	0.461E+02	0.116E+03		
0.221E+01	0.434E+00	0.250E+03	0.425E+02	0.938E+02		
0.241E+01	0.461E+00	0.204E+03	0.382E+02	0.706E+02		
0.261E+01	0.489E+00	0.157E+03	0.334E+02	0.487E+02		
0.281E+01	0.517E+00	0.114E+03	0.283E+02	0.304E+02		
0.301E+01	0.546E+00	0.775E+02	0.231E+02	0.166E+02		
0.321E+01	0.575E+00	0.484E+02	0.181E+02	0.750E+01		
0.341E+01	0.604E+00	0.274E+02	0.136E+02	0.248E+01		
0.361E+01	0.634E+00	0.136E+02	0.970E+01	0.510E+00		
0.381E+01	0.663E+00	0.570E+01	0.651E+01	0.418E+00		
0.401E+01	0.694E+00	0.211E+01	0.410E+01	0.116E+01		
0.421E+01	0.724E+00	0.126E+01	0.245E+01	0.198E+01		
0.441E+01	0.754E+00	0.181E+01	0.150E+01	0.245E+01		
0.461E+01	0.785E+00	0.284E+01	0.113E+01	0.247E+01		
0.481E+01	0.816E+00	0.373E+01	0.119E+01	0.212E+01		
0.501E+01	0.847E+00	0.423E+01	0.153E+01	0.160E+01		
0.521E+01	0.877E+00	0.426E+01	0.200E+01	0.111E+01		
0.541E+01	0.909E+00	0.392E+01	0.248E+01	0.786E+00		
0.561E+01	0.940E+00	0.332E+01	0.285E+01	0.687E+00		
0.581E+01	0.971E+00	0.263E+01	0.308E+01	0.788E+00		
0.601E+01	0.100E+01	0.194E+01	0.312E+01	0.101E+01		
0.621E+01	0.103E+01	0.134E+01	0.298E+01	0.128E+01		

0.641E+01	0.106E+01	0.861E+00	0.269E+01	0.149E+01
0.661E+01	0.110E+01	0.521E+00	0.231E+01	0.160E+01
0.681E+01	0.113E+01	0.305E+00	0.188E+01	0.159E+01
0.701E+01	0.116E+01	0.191E+00	0.145E+01	0.147E+01
0.721E+01	0.119E+01	0.149E+00	0.107E+01	0.127E+01
0.741E+01	0.122E+01	0.153E+00	0.766E+00	0.103E+01
0.761E+01	0.125E+01	0.178E+00	0.546E+00	0.780E+00
0.781E+01	0.129E+01	0.206E+00	0.414E+00	0.562E+00

TOTAL CROSS SECTIONS

CM	ANGLE	MOM	TRANS	DIRECT	EXCHANGE	TOTAL
0.100E-01	0.247E+00	0.159E+03	0.488E+02	0.333E+02		
0.210E+00	0.249E+00	0.168E+03	0.490E+02	0.390E+02		
0.410E+00	0.256E+00	0.192E+03	0.495E+02	0.540E+02		
0.610E+00	0.266E+00	0.225E+03	0.503E+02	0.754E+02		
0.810E+00	0.279E+00	0.263E+03	0.511E+02	0.993E+02		
0.101E+01	0.296E+00	0.299E+03	0.518E+02	0.121E+03		
0.121E+01	0.315E+00	0.326E+03	0.521E+02	0.138E+03		
0.141E+01	0.336E+00	0.340E+03	0.518E+02	0.146E+03		
0.161E+01	0.359E+00	0.338E+03	0.508E+02	0.144E+03		
0.181E+01	0.383E+00	0.321E+03	0.489E+02	0.134E+03		
0.201E+01	0.408E+00	0.290E+03	0.461E+02	0.116E+03		
0.221E+01	0.434E+00	0.250E+03	0.425E+02	0.938E+02		
0.241E+01	0.461E+00	0.204E+03	0.382E+02	0.706E+02		
0.261E+01	0.489E+00	0.157E+03	0.334E+02	0.487E+02		
0.281E+01	0.517E+00	0.114E+03	0.283E+02	0.304E+02		
0.301E+01	0.546E+00	0.775E+02	0.231E+02	0.166E+02		
0.321E+01	0.575E+00	0.484E+02	0.181E+02	0.750E+01		
0.341E+01	0.604E+00	0.274E+02	0.136E+02	0.248E+01		
0.361E+01	0.634E+00	0.136E+02	0.970E+01	0.510E+00		
0.381E+01	0.663E+00	0.570E+01	0.651E+01	0.418E+00		
0.401E+01	0.694E+00	0.211E+01	0.410E+01	0.116E+01		
0.421E+01	0.724E+00	0.126E+01	0.245E+01	0.198E+01		
0.441E+01	0.754E+00	0.181E+01	0.150E+01	0.245E+01		
0.461E+01	0.785E+00	0.284E+01	0.113E+01	0.247E+01		
0.481E+01	0.816E+00	0.373E+01	0.119E+01	0.212E+01		
0.501E+01	0.847E+00	0.423E+01	0.153E+01	0.160E+01		
0.521E+01	0.877E+00	0.426E+01	0.200E+01	0.111E+01		
0.541E+01	0.909E+00	0.392E+01	0.248E+01	0.786E+00		
0.561E+01	0.940E+00	0.332E+01	0.285E+01	0.687E+00		
0.581E+01	0.971E+00	0.263E+01	0.308E+01	0.788E+00		
0.601E+01	0.100E+01	0.194E+01	0.312E+01	0.101E+01		
0.621E+01	0.103E+01	0.134E+01	0.298E+01	0.128E+01		
0.641E+01	0.106E+01	0.861E+00	0.269E+01	0.149E+01		
0.661E+01	0.110E+01	0.521E+00	0.231E+01	0.160E+01		
0.681E+01	0.113E+01	0.305E+00	0.188E+01	0.159E+01		
0.701E+01	0.116E+01	0.191E+00	0.145E+01	0.147E+01		
0.721E+01	0.119E+01	0.149E+00	0.107E+01	0.127E+01		
0.741E+01	0.122E+01	0.153E+00	0.766E+00	0.103E+01		
0.761E+01	0.125E+01	0.178E+00	0.546E+00	0.780E+00		
0.781E+01	0.129E+01	0.206E+00	0.414E+00	0.562E+00		

5 Formulas

The details of derivation of formulas can be found in a separate note "DCP2 - Formulation".

5.1 Differential cross section

The differential cross section, Eq.(21) in the original paper (Hereafter, use as [Eq.(21)]), becomes

$$\frac{d\sigma}{d\Omega} = \frac{\mu_a \mu_b}{(2\pi\hbar^2)^2} \frac{k_b}{k_a} \frac{1}{(2I_A + 1)(2s_a + 1)} \left| \sum_i \sum_{k\ell_1 t_1} \alpha_{t_1 s_1 \ell_1 k \ell_t}^{j_t s_t \nu_1} T_{t_1 s_1 \ell_1 k \ell_t m_{\ell_t}}^i \right|^2 \quad [\text{Eq.}(21)]$$

where $\mu_a(\mu_b)$ is the reduced mass in the incident (exit) channel and i is $i = D$ for direct transitions and $i = E$ for exchange transitions.

The $\alpha_{t_1 s_1 \ell_2 k \ell_t}^{j_t s_t \nu_1}$ coefficients and the transition amplitudes are

$$\alpha_{t_1 s_1 \ell_2 k \ell_t}^{j_t s_t \nu_1} = W(s_t \ell_t s_t \ell_1; j_t k) \hat{s}_t^{-1} \hat{t}_1^{-1} \langle b || [c^\dagger c]_{s_1 t_1 \tilde{\nu}_1} || a \rangle, \quad \text{with } \hat{s} = \sqrt{2s + 1} \quad [\text{Eq.}(13)]$$

$$T_{t_1 s_1 \ell_1 k \ell_t m_{\ell_t}}^i = \frac{(4\pi)^{3/2}}{k_a k_b} \sum_{\ell_a \ell_b} i^{\ell_a - \ell_b + \pi} \hat{\ell}_a(\ell_a 0 \ell_t m_{\ell_t} | \ell_b m_{\ell_t}) O_{t_1 s_1 \ell_1 k \ell_t, \ell_a \ell_b}^i Y_{\ell_b m_{\ell_t}}(\hat{k}_b) \quad [\text{Eq.}(18)]$$

where the direct and exchange overlap integrals are

$$O_{t_1 s_1 \ell_1 k \ell_t, \ell_a \ell_b}^D = d_{\ell_a \ell_t \ell_b} \int dr_a \chi_{\ell_b}(r_a) f_{t_1 s_1 \ell_1 k \ell_t}^D(r_a) \chi_{\ell_a}(r_a) \quad [\text{Eq.}(19a)]$$

$$O_{t_1 s_1 \ell_1 k \ell_t, \ell_a \ell_b}^E = J \int dr_b \int dr_a r_b r_a \chi_{\ell_b}(r_b) f_{t_1 s_1 \ell_1 k \ell_t, \ell_a \ell_b}^E(r_b, r_a) \chi_{\ell_a}(r_a) \quad [\text{Eq.}(19b)]$$

$$d_{\ell_a \ell_t \ell_b} \equiv \frac{1}{\sqrt{4\pi}} \hat{\ell}_a \hat{\ell}_t \hat{\ell}_b^{-1} (\ell_a 0 \ell_t 0 | \ell_b 0) \quad [\text{Eq.}(19c)]$$

where $f_{t_1 s_1 \ell_1 k \ell_t}^D(r_a)$ and $f_{t_1 s_1 \ell_1 k \ell_t, \ell_a \ell_b}^E(r_b, r_a)$ are direct and exchange form factors, respectively. $\chi_{\ell_a}(r_a)(\chi_{\ell_b}(r_b))$ are the partial distorted waves in the incident (exit) channel. J is the Jacobian associated with the transformation of integral coordinate from \vec{r}_2 to \vec{r}_a . $\langle b || [c^\dagger c]_{s_1 t_1 \tilde{\nu}_1} || a \rangle$ is the usual spectroscopic amplitude for the spin-isospin wave function of projectile system.

The angular momentum couplings are as follows;

Form factor	Interaction	Couplings
Direct	Central	$k = 0, \{s_t \ell_t j_t\}, \delta(s_1, s_t), \delta(\ell_1, \ell_t), s_1 = s_t = 0$ or $1, \{\ell_a \ell_b \ell_t\}, \ell_a + \ell_b + \ell_t = \text{even}.$
	Tensor	$k = 2, \{s_t \ell_t j_t\}, \{s_1 \ell_1 j_t\}, \{\ell_1 \ell_t 2\}, \delta(s_1, s_t), s_1 = s_t = 1, \{\ell_a \ell_b \ell_t\}, \ell_a + \ell_b + \ell_t = \text{even}.$
Exchange	Central	$k = 0, \{s_t \ell_t j_t\}, \delta(s_1, s_t), \delta(\ell_1, \ell_t), s_1 = s_t = 0$ or $1, \{\ell_a \ell_b \ell_t\}.$
	Tensor	$k=2, \{s_t \ell_t j_t\}, \{s_1 \ell_1 j_t\}, \{\ell_1 \ell_t 2\}, \delta(s_1, s_t), s_1 = s_t = 1, \{\ell_a \ell_b \ell_t\}.$

5.2 Form factors

5.2.1 Direct form factor

We write the radial direct form factors as

$$\begin{aligned}
f_{t_1 s_1 \ell_1 k \ell_t}^D(r_a) &= i^{-\pi} (-)^{\ell_1} \hat{\ell}_t^{-1} \int r^2 dr V_{t_1 s_1 k}^D(r) \int r_1^2 dr_1 \rho_{P, k \ell_t \ell_1}^D(r_a, r_1, r) \rho_{T, \ell_1}^D(r_1) \quad [\text{Eq.}(27)] \\
\rho_{P, k \ell_t \ell_1}^D(r_a, r_1, r) &= \frac{2\pi}{\hat{k}^2} \sum_m \hat{\ell}_t(\ell_t 0 \ell_1 m | km) \int \rho_{P, k \ell_t \ell_1}^D(r_a, r_1, \mu, r) Y_{km}(\theta'_2, 0) Y_{\ell_1 m}^*(\theta, 0) d\mu \quad [\text{Eq.}(26)] \\
\rho_{T, \ell_1}^D(r_1) &= X(\ell_p \frac{1}{2} j_p, \ell_h \frac{1}{2} j_h; \ell_1 s_1 j_t) \langle I_B || [\hat{a}_{j_p \nu_p}^\dagger \hat{a}_{j_h \tilde{\nu}_h}]_{j_t} || I_A \rangle \sqrt{4\pi} d_{\ell_p \ell_h \ell_1} R_{\ell_p}(r_1) R_{\ell_h}(r_1)
\end{aligned}$$

where the force components $V_{t_1 s_1 k}^{D(E)}(r)$ are related to Love-Franey NN interaction $V_{tsk}^{D(E)}(r)$ by

$$V_{t_1 s_1 k}^i(r) = \sqrt{4\pi} f_k \hat{s}_1^2 \hat{t}_1^2 \sum_{st} \hat{s}^3 \hat{t}^3 \begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & s \\ \frac{1}{2} & \frac{1}{2} & s \\ s_1 & s_1 & k \end{Bmatrix} \begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & t \\ \frac{1}{2} & \frac{1}{2} & t \\ t_1 & t_1 & 0 \end{Bmatrix} P_i V_{tsk}^i(r) \quad [\text{Eq.}(16)]$$

Where $P_D = 1$ and $P_E = (-)^{s+t+1}$, while $f_0 = 1$ and $f_2 = \sqrt{8}$. The target density $\rho_{T, \ell_1}^D(r_1)$ is expressed in terms of the particle (hole) wave functions $R_{\ell_p}(r_1)(R_{\ell_h}(r_1))$, and $\langle I_B || [\hat{a}_{j_p \nu_p}^\dagger \hat{a}_{j_h \tilde{\nu}_h}]_{j_t} || I_A \rangle$ is the usual spectroscopic amplitude for the target system. The projectile density $\rho_P^D(r_2)$ is written as $\rho_{P, k \ell_t \ell_1}^D(r_a, r_1, r)$ in the multipole expansion method.

The angular momentum couplings from target density which appears both in the direct and exchange factors are

$$\{\ell_p \frac{1}{2} j_p\}, \{\ell_h \frac{1}{2} j_h\}, \{\ell_h \ell_p \ell_1\}, \quad \text{and} \quad \{j_h j_p j_t\}.$$

Note that the angular momenta, say, ℓ, ℓ_a, λ_2 , appeared in the multipole expansions of projectile density, are reduced to $\lambda_2 = k, \ell = \ell_t, \ell_q = \ell_1$ after angular integrations. (See Formulation 2.1.)

5.2.2 Direct form factor for the nucleon-nucleus scattering

For the nucleon-nucleus scattering, we adopt the following limits for the direct form factor

$$\begin{aligned}
\rho_P(\vec{r}_2) &= \delta(\vec{r}_2) = \sum_{\lambda_2} \rho_{P, \lambda_2}^D(r'_2, r) (-)^{\lambda_2} [Y_{\lambda_2} Y_{\lambda_2}]_{00} \\
\rho_{P, \lambda_2}(r'_2, r) &= \hat{\lambda}_2 (-)^{\lambda_2} \delta(r'_2 - r) / r^2
\end{aligned}$$

The radial direct form factor becomes

$$\begin{aligned}
f_{t_1 s_1 \ell_1 k \ell_t}^D(r_a) &= i^{-\pi} (-)^{\ell_1} \hat{\ell}_t^{-1} \int r^2 dr V_{t_1 s_1 k}^D(r) Q_{t_1 s_1 \ell_1 k \ell_t}^D(r_a, r) \\
Q_{t_1 s_1 \ell_1 k \ell_t}^D(r_a, r) &= \int r_1^2 dr_1 \rho_{T, \ell_1}^D(r_1) \frac{2\pi}{\hat{k}^2} \\
&\quad \times \sum_m \hat{\ell}_t(\ell_t 0 \ell_1 m | km) \hat{k}(-)^k \left(\frac{-1}{r_1 r_a r}\right) Y_{km}(\theta'_2, 0) Y_{\ell_1 m}^*(\theta, 0) \\
\mu \equiv \cos \theta &= \frac{r_a^2 + r_1^2 - r^2}{2r_1 r_a}, \quad \mu' \equiv \cos \theta'_2 = \frac{r_a^2 + r^2 - r_1^2}{2r r_a}
\end{aligned}$$

5.2.3 Exchange form factor

The exchange form factor can be written as,

$$f_{t_1 s_1 \ell_1 k \ell_t, \ell_b \ell_a}^E(r_b, r_a) = J 4\pi m_a^k \sum_{\lambda_a \lambda_b \ell_\alpha \ell_\beta} \left[\frac{(2k+1)!}{(2\lambda_a+1)!(2\lambda_b+1)!} \right]^{1/2} \delta_{\lambda_a+\lambda_b, k} (-r_a)^{\lambda_a} (r_b)^{\lambda_b} \\ \times X(\ell_\alpha \lambda_a \ell_a, \ell_\beta \lambda_b \ell_b; \ell_1 k \ell_t) d_{\ell_\alpha \lambda_a \ell_a} d_{\ell_\beta \lambda_b \ell_b} c_{t_1 s_1 \ell_1 k, \ell_\alpha \ell_\beta}(r_b, r_a) \quad [\text{Eq. (36)}]$$

$$c_{t_1 s_1 \ell_1 k, \ell_\alpha \ell_\beta}(r_b, r_a) = \frac{2\pi}{\hat{\ell}_1^2} \sum_{m_{\ell_1}} \hat{\ell}_\beta(\ell_\alpha m_{\ell_1} \ell_\beta 0 | \ell_1 m_{\ell_1}) \sum_{\ell \lambda} \hat{\ell}(\ell 0 \lambda m_{\ell_1} | \ell_1 m_{\ell_1}) \\ \times \int d\mu G_{t_1 s_1 \ell_1 \ell \lambda}^k(r_b, r) Y_{\lambda m_{\ell_1}}(\theta', \pi) Y_{\ell_\alpha m_{\ell_1}}^*(\theta, 0)$$

$$G_{t_1 s_1 \ell_1 \ell \lambda}^k(r_b, r) = \frac{1}{\sqrt{4\pi}} r^{-k} V_{t_1 s_1 k}^E(r) \sum_{\lambda_1 \lambda_2 \ell_c} (-)^\ell \hat{\lambda}_1 \hat{\lambda}_2 (\lambda_1 0 \lambda_2 0 | \lambda 0) W(\lambda_1 \lambda_2 \ell_1 \ell; \lambda \ell_c) \\ \times \int r_1^2 dr_1 \rho_{P, \lambda_2 \ell_c}^E(r_b, r_1, r) \rho_{T, \ell_1 \lambda_1 \ell_c}^E(r_1, r) \quad [\text{Eq. (33b)}]$$

$$\rho_{P, \lambda_2 \ell_c}^E(r_b, r_1, r) = \frac{2\pi}{\hat{\lambda}_2^2} \sum_m \hat{\ell}(\ell 0 \ell_q m | \lambda_2 m) \int \rho_{P, \lambda_2 \ell_q}^E(r_b, r_1, \mu, r) Y_{\lambda_2 m}(\theta'_2, 0) Y_{\ell_q m}^*(\theta, 0) d\mu \quad [\text{Eq. (31a)}]$$

$$\rho_{T, \ell_1 \lambda_1 \ell_c}^E(r_1, r) = \sum_{ph, \eta_1} i^{\ell_p + \ell_h - \pi} X(\ell_p \frac{1}{2} j_p, \ell_h \frac{1}{2} j_h; \ell_1 s_1 j_t) < I_B || [\hat{a}_{j_p \nu_p}^\dagger \hat{a}_{j_h \bar{\nu}_h}]_{j_t} || I_A > R_{\ell_p}(r_1) \\ \times (-)^{\eta_1} \hat{\ell}_h \hat{\ell}_c \hat{\eta}_1(\ell_c 0 \eta_1 0 | \ell_p 0) W(\ell_c \eta_1 \ell_1 \ell_h; \ell_p \lambda_1) \quad [\text{Eq. (31b)}] \\ \times \frac{2\pi}{\hat{\ell}_h^2} \sum_{m_1} \hat{\eta}_1(\eta_1 0 \lambda_1 m_1 | \ell_h m_1) \int R_{\ell_h}(r'_1) Y_{\ell_h m_1}(\theta, 0) Y_{\lambda_1 m_1}^*(\theta', 0) d\mu'$$

The angular momentum couplings appeared in the exchange form factor are as follows;

Interaction	Couplings
Central	$\delta(\ell_\alpha, \ell_a), \delta(\ell_\beta, \ell_b), \delta(\lambda_a, 0), \delta(\lambda_b, 0).$
Tensor	$\{\ell_\alpha \lambda_a \ell_a\}, \{\ell_\beta \lambda_b \ell_b\}, \{\ell_1 k \ell_t\}, \{\ell_\alpha \ell_\beta \ell_1\}, \{\lambda_a \lambda_b k\},$ $(\ell_\alpha + \lambda_a + \ell_a = \text{even}), (\ell_\beta + \lambda_b + \ell_b = \text{even})$

With this coupling scheme, the central exchange form factor just becomes the c -factor in the above equation,

$$f_{t_1 s_1 \ell_1 k=0, \ell_t, \ell_b \ell_a}^E(r_b, r_a) = c_{t_1 s_1 \ell_1 k=0, \ell_a \ell_b}(r_b, r_a).$$

Note that the angular momenta, say, $(\ell, \ell_q, \lambda_2)$, appeared in the multipole expansions of projectile density, and $(\eta_1, \ell_c, \lambda_1)$ in target density expansions, are disappeared after integrating over $d\hat{r}_1$ which gives $\ell_q = \ell_c$ and summing over the rest of angular momenta. (See Formulation 2.2.)

5.2.4 Exchange form factor for the nucleon-nucleus scattering

We adopt the following limits for the exchange form factor

$$\rho_P(\vec{r}_2, \vec{r}'_2) = \delta(\vec{r}_2) \\ \rho_{P, \lambda_2}(r'_2, r) = \hat{\lambda}_2 (-)^{\lambda_2} \delta(r'_2 - r) / r^2$$

The radial exchange form factor yields

$$f_{t_1 s_1 \ell_1 k \ell_t, \ell_b \ell_a}^E(r_b, r_a) = J 4\pi m_a^k \sum_{\lambda_a \lambda_b \ell_\alpha \ell_\beta} \left[\frac{(2k+1)!}{(2\lambda_a+1)!(2\lambda_b+1)!} \right]^{1/2} \delta_{\lambda_a+\lambda_b, k} (-r_a)^{\lambda_a} (r_b)^{\lambda_b} \\ \times X(\ell_\alpha \lambda_a \ell_a, \ell_\beta \lambda_b \ell_b; \ell_1 k \ell_t) d_{\ell_\alpha \lambda_a \ell_a} d_{\ell_\beta \lambda_b \ell_b} c_{t_1 s_1 \ell_1 k, \ell_\alpha \ell_\beta}(r_b, r_a)$$

The projectile density is just replaced by

$$\rho_{P,\lambda_2\ell_c}^E(r_b, r_1, r) = \frac{2\pi}{\hat{\lambda}_2^2} \sum_m \hat{\ell}(\ell_0\ell_c m|\lambda_2 m) \int \rho_{P,\lambda_2\ell_c}^E(r_b, r_1, \mu, r) Y_{\lambda_2 m}(\theta'_2, 0) Y_{\ell_c m}^*(\theta, 0) d\mu$$

$$\mu = \frac{r_b^2 + r_1^2 - r^2}{2r_1 r_b}, \quad \mu' = \frac{r_b - r_1 \mu}{r}$$

5.2.5 Exchange form factor in the no-recoil approximation

The no-recoil approximation was originally invented to simplify the cross section calculations for heavy-ion induced one and two nucleon transfer reactions. The essence of the approximation is to ignore the recoil momentum the target receives in the transfer process. In the same spirit we neglect here the recoil momenta which projectile and target pick up in the knock-on exchange process. Formally this approximation is obtained by replacing $\chi_a^{(+)}(\vec{k}_a, \vec{r}_a)$ in [Eq.(15)] through $\chi_a^{(+)}(\vec{k}_a, \vec{r}_b)$, i.e., ignoring the difference between the vectors \vec{r}_a and \vec{r}_b .

We write the radial exchange form factors in the no-recoil approximation as

$$f_{t_1 s_1 \ell_1 k \ell_t}^{NR}(r_b) = \sqrt{4\pi} (-)^k \hat{\ell}_1 \hat{\ell}_t^{-1} \int dr r^{k+2} G_{t_1 s_1 \ell_1 \ell_t \lambda}^k(r_b, r) \delta(k, \lambda) \quad [\text{Eq.(41)}]$$

Note that, in the no-recoil approximation, the angular momenta, say, (ℓ, λ) , appeared in the multipole expansions are reduced to $\ell = \ell_t, \lambda = k$. (See Formulation 3.2.)

5.2.6 Exchange form factor in the plane wave approximation

In the plane wave approximation, the recoil effect¹ is described by a recoil factor $\exp(-i\alpha \vec{k}_a \cdot \vec{r}/a)$. A simple reason is that in the plane wave approximation the incoming and outgoing waves are described by

$$\exp(i\vec{k}_a \cdot \vec{r}_a - i\vec{k}_b \cdot \vec{r}_b) = \exp[i(\vec{k}_a - \vec{k}_b) \cdot \vec{r}_b] \exp[-i\vec{k}_a \cdot \vec{r}/a]$$

where we use $\vec{r}_a = \vec{r}_b - \vec{r}/a$. Obviously $-\vec{k}_a/a$ is the change of linear momentum between the exchanged particles.

A possible improvement of the no-recoil approximation is then to replace f^{NR} by the following f^{PW} that takes into account the recoil factor within the plane wave approximation,

$$F_{t_1 s_1 \ell_1 k \ell_t m_{\ell_t}}^{PW}(\vec{r}_b) = J^{-1} f_{t_1 s_1 \ell_1 k \ell_t}^{PW}(r_b) Y_{\ell_t m_{\ell_t}}(\hat{r}_b) i^\pi = \int d\vec{r} f_{t_1 s_1 \ell_1 k \ell_t}^E(r_b) \exp(-i\alpha \vec{k}_a \cdot \vec{r}/a)$$

where a parameter α in the recoil factor is treated as an adjustable parameter. We fit it such that the resultant approximate cross section reproduces the exact cross section $\sigma(E)$ as closely as possible. It has turned out that a close fit is obtained with $\alpha = 1$.²

We obtain the radial exchange form factors with recoil effect in the plane wave approximation (See Formulation 3.3 for details.) as,

$$f_{t_1 s_1 \ell_1 k \ell_t}^{PW}(r_b) = \sqrt{4\pi} \sum_{\ell \lambda \ell_r} i^{\pi - \ell_r - \ell} \hat{k} \hat{\lambda} (k 0 \lambda 0 | \ell_r 0) \hat{\ell}_1 \hat{\ell}_r W(\ell \lambda \ell_t k : \ell_1 \ell_r)$$

$$\times (-)^{k + \ell_1 - \ell_t} (\ell m_{\ell_t} \ell_r 0 | \ell_t m_{\ell_t}) \int dr r^{k+2} G_{t_1 s_1 \ell_1 \ell \lambda}^k(r_b, r) j_{\ell_r}(\alpha k a r/a)$$

Inputting LRP1MX=1 ($\ell_r = 0$) and FACNR=0.0 ($\alpha = 0.0$), yields NR approximation.

¹T. Tamura, Phys. Rep. 14C, 59 (1974), Section 4.4.

²B. T. Kim, D. P. Knobles, S. A. Stotts, and T. Udagawa, Phys. Rev. C61, 044611 (2000).

5.3 Relativistic kinematics

What we want to do is the Lorentz transformation of Lab system with

$$\begin{aligned}
 E_{lab} & : \text{Kinetic energy of lab system} \\
 k_0 & : \text{Wave number} \\
 m_T & : \text{Target mass} \\
 m_P & : \text{Projectile}
 \end{aligned}$$

to c.m. system such that

$$(E_0 + m_T, k_0) \rightarrow (\omega, 0)$$

where $E_0 - m_P = E_{lab}$, and $E_0^2 = m_P^2 + k_0^2$. Thus we have

$$\begin{aligned}
 \begin{pmatrix} \omega \\ 0 \end{pmatrix} &= \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} E_0 + m_T \\ k_0 \end{pmatrix} \\
 \beta_{cm} &= \frac{\vec{k}_0}{E_0 + m_T} \quad (\text{Velocity of c.m. wrt lab frame}) \\
 \gamma_{cm} &= \frac{1}{\sqrt{1 - \beta^2}} = \frac{E_0 + m_T}{\sqrt{(E_0 + m_T)^2 - k_0^2}} = \frac{E_0 + m_T}{\sqrt{s}} \\
 s = \omega^2 &= (E_0 + m_T)^2 - k_0^2 \\
 &= (E_{lab} + m_P + m_T)^2 - (E_{lab} + m_P)^2 + m_P^2 \\
 &= (m_P + m_T)^2 + 2E_{lab}m_T
 \end{aligned}$$

We write the kinetic energy, wave number and masses in the center of mass system.

$$\begin{aligned}
 E_{cm} &= \omega - (m_T + m_P) = \sqrt{s} - (m_T + m_P) \\
 k_{cm}^2 &= \frac{m_T^2}{s} k_0^2 = \frac{m_T^2}{s} (E_0^2 - m_P^2) = \frac{m_T^2}{s} [(E_{lab} + m_P)^2 - m_P^2] = \frac{m_T^2}{s} (E_{lab}^2 + 2E_{lab}m_P) \\
 m_{cm,T} &= \gamma_{cm} m_T = \frac{E_0 + m_T}{\sqrt{s}} = \frac{m_T}{\sqrt{s}} (E_{lab} + m_P + m_T) \\
 m_{cm,P} &= \frac{1}{\sqrt{s}} [2E_{lab}^2 + 4E_{lab}m_P + m_P(m_P + m_T) + E_{lab}m_T] \\
 &\approx \frac{1}{\sqrt{s}} [m_P(m_P + m_T) + E_{lab}m_T]
 \end{aligned}$$

with $m_T \gg m_P$ and $m_T \gg E_{lab}$.

5.4 Projectile Density distribution

5.4.1 Deuteron

We use the Hulthen wavefunction of the deuteron

$$\begin{aligned}
 \rho_d(r) &= \phi_{Hulthen}^2 \\
 \phi_{Hulthen}(r) &= \frac{1}{\sqrt{4\pi}} \frac{1}{r} \sqrt{\frac{2\alpha\beta(\alpha + \beta)}{(\beta - \alpha)^2}} (e^{-\alpha r} - e^{-\beta r})
 \end{aligned}$$

where $\alpha^{-1} = 4.3$ fm, and $\beta = 7\alpha$.

5.4.2 Helium

We use the ${}^3\text{He}$ density from a paper by C.W. de Jager, H. de Vries and C. de Vries, Atomic data and nuclear data tables, **36** (1987) 495.

$$\rho_0(r) = \frac{z}{8\pi^{3/2}} \left[\frac{1}{a^3} \exp\left(\frac{-r^2}{4a^2}\right) - \frac{c^2(6b^2 - r^2)}{4b^7} \exp\left(\frac{-r^2}{4b^2}\right) \right]$$

with $a = 0.675$, $b = 0.836$, $c = 0.366$ fm.

5.5 Folding optical potential

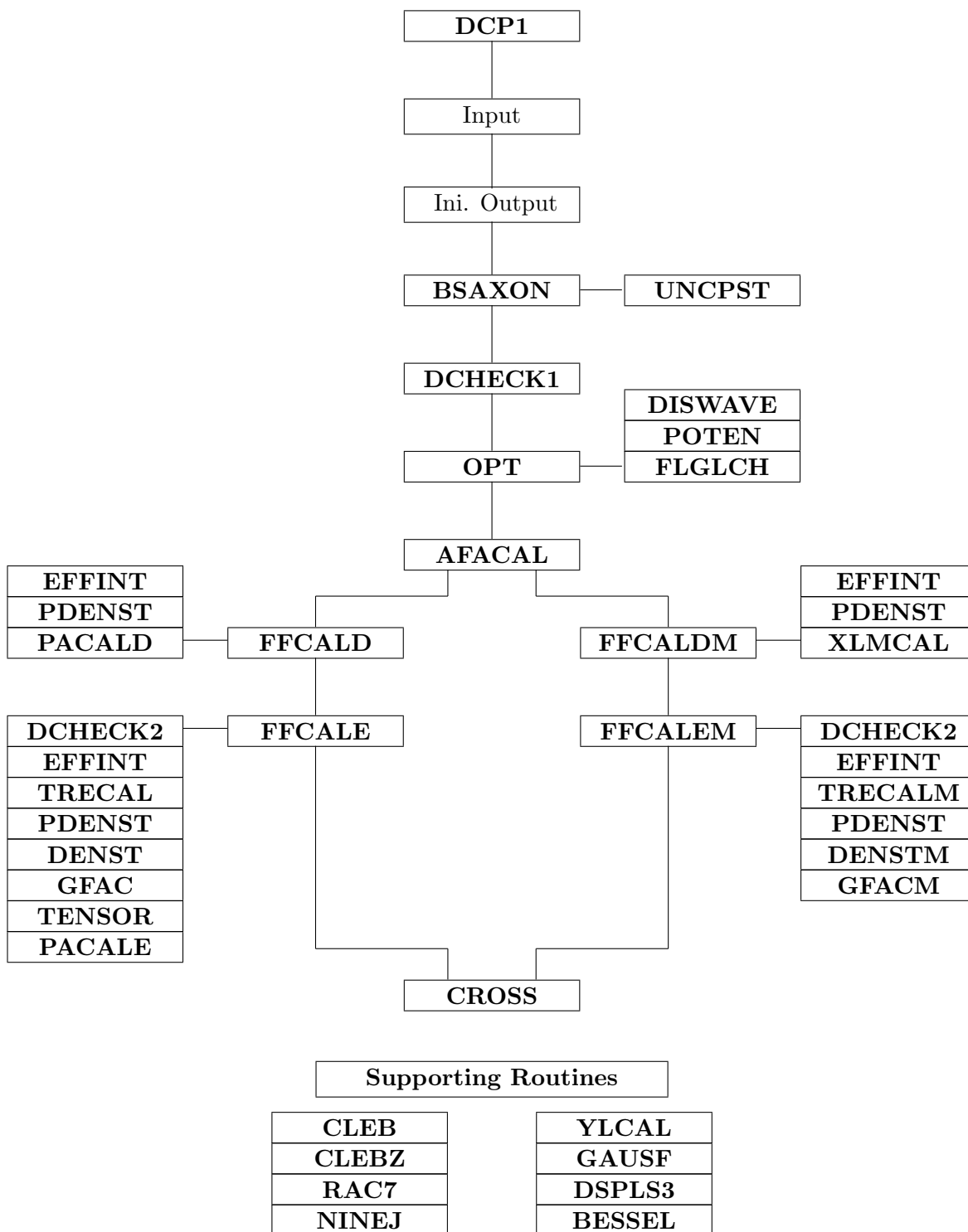
The single folding potential is defined as

$$U(r) = 4\pi \int dr_1 r_1^2 \rho(r_1) v_0(r, r_1),$$
$$v_0(r, r_1) = \frac{1}{2} \int_{-1}^1 v(|\vec{r} - \vec{r}_1|) d\mu$$

The last integration is done by the gaussian integration method. Here, the density function is normalized to the mass of the projectile; for example, $N_d = 2.0$ for the deuteron and $N_{He} = 3.0$ for the helium.

A Subroutines and their functions

A.1 Structure of the program



35 SUBROUTINES, 8300 LINES

A.2 Functions of the subroutine

A.2.1 DCP2 -Main Routine

The main routine DCP2 controls and performs whole calculation processes.

A.2.1.1 DCP2 defines first constants.

Variable	Definition
FACLOG(N)	Log value of factorial of integer N . Used for vector coupling coefficients.
PI	$\pi = 4 \arctan(1)$.
HBAR	$\hbar c = 197.327053$ in MeV · fm.
AMAS	atomic mass unit, $m_{amu} = 931.49432$ in MeV.
WNUNIT	wave number unit in 1/fm, $\sqrt{2m_{amu}}/\hbar c$.
FINE	fine structure constant, $e^2/\hbar c = 1/137.0359896$.

A.2.1.2 DCP2 reads input data.

A.2.1.3 DCP2 writes initial output.

A.2.1.4 DCP2 calls BSAXON to calculate the single particle and hole states.

A.2.1.5 DCP2 finds Particle-hole pairs, which satisfies

Parity	$\ell_p + \ell_h + \pi = \text{even}$
Triangle relations of	(j_p, j_h, j_t)
Triangle relations of	(ℓ_p, ℓ_h, ℓ_1) with $\vec{\ell}_1 = \vec{s}_1 + \vec{j}_t$

A.2.1.6 DCP2 calls DCHECK1 to check field lengths of variables.

A.2.1.7 DCP2 calculates the projectile wave functions and density. See Section 5.4 for nuclei less than 3, otherwise call BSAXON.

A.2.1.8 DCP2 calls OPT to generate distorted waves in the incident and exit channels.

This section first calculates the kinematical variables, such as E_{cm}, k, η and so on, in the incident and exit channels.

A.2.1.9 DCP2 calls AFACAL to calculate α coefficients of [Eq.(13)].

A.2.1.10 DCP2 calls XLMCAL to calculate the distortion factor.

A.2.1.11 DCP2 calculates direct form factors. It calls FFCALD or FFCALDM depending on KTRLD(1). If KTRLD(1)=1, it calls FFCALDM, otherwise FFCALD.

A.2.1.12 DCP2 calculates exchange form factors. It calls FFCALD or FFCALDM depending on KTRLD(1). If KTRLD(1)=1, it calls FFCALDM, otherwise FFCALD.

A.2.1.13 DCP2 calls CROSS to calculate differential cross sections.

A.2.2 BSAXON and UNCPST - Single particle states

A.2.2.1 BSAXON solves the Schrödinger equation to obtain the single particle (hole) radial wave functions

$$\left[-\frac{\hbar^2}{2\mu} \frac{d^2}{dr^2} + \frac{\hbar^2 \ell(\ell+1)}{2\mu r^2} + V(r)\right] u_{nj\ell s}(r) = -|E_0| u_{nj\ell s}(r)$$

where $u_{nj\ell s}(r)$ is defined as

$$\varphi(r, \theta, \phi) = R_{nj\ell s}(r) Y_{\ell m}(\theta, \phi) = \frac{u_{nj\ell s}(r)}{r} Y_{\ell m}(\theta, \phi)$$

For the details of $V(r)$, see Section 4.3. It searches the central potential depth parameter (Woods-Saxon type) for a given binding energy. (KTRL2=1 was set.)

A.2.2.2 UNCPST generates outward solutions from the origin by using the 4-point Stormer method and inward solutions from the far outside the well by a single step Stormer method and matches two solutions at a boundary point.

A.2.3 DCHECK1 - Checking field lengths of variables

A.2.3.1 DCHECK1 checks field lengths of variables with a given input data set.

The meaning of PARAMETER given in the program is following,

Parameter Name	Field length of Variable	Max. combinations
NHS	NOSH	
NPS	NOSP	
LXA	LDWMXR(1)	
LXB	LDWMXR(2)	
LPH	NPHMX	
LTL	L1R(NLSMAX)+LAMMXD(1)+1	
LM1	MP1MAX = MIN(LTR(NOLTR),MXMAX)+1	
LMM	MAX(LAMMXD(1,2,3))	
NXA	NXMXR(1)*PMASA	
NXB	NXMXR(2)	
NIN	MAX(NHDMX,NHEMX)	
N1X	NXMXR(4)/N1STEP+1	
KFM	Greater than KFMAX	
KCC	Greater than KCMAX	
LMJ	(MJMAX+1)*3	
KFMAX	NLSMAX*7	$\{t_1 s_1 \ell_1 k \ell_t\}$
KAMAX	(LAMMX+1)*(LAMMX+2)/2	$\{KF, \lambda \ell\}$
KBMAX	NLSMAX*LAM1MX*(LAM1MX+1)/2	$\{t_1 s_1 \ell_1 \lambda_1 \ell'_p\}$
KCMAX	KBMAX*LAM2MX*(LAM2MX+1)/2	$\{KB, \lambda_2 \ell\}$

A.2.4 OPT, FLGLCH, POTEN, and DISWAVE - Distorted waves

A.2.4.1 OPT(IDCHNL,NEXPT) calculates distorted waves for the incident(IDCHNL=1) and exit(IDCHNL=2) channels. NEXPT is set to be 1. It solves the Schrödinger equation for χ_ℓ with outgoing wave boundary conditions

$$\frac{\hbar^2}{2\mu} \frac{d^2 u_\ell(r)}{dr^2} + [E - U(r) - \frac{\hbar^2 \ell(\ell+1)}{2\mu r^2}] \chi_\ell(r) = 0$$

where χ_ℓ is related to the channel wave function as

$$\chi^{(+)}(\vec{k}, \vec{r}) = \frac{4\pi}{kr} \sum_{\ell m_\ell} i^\ell \chi_\ell(r) Y_{\ell m_\ell}(\hat{r}) Y_{\ell m_\ell}^*(\hat{k})$$

For the optical model potential $U(r)$, see Section 4.1.

These subroutines were written by Taro Tamura's group, and how to obtain distorted waves was written in a paper by T. Tamura and W. R. Coker, "Computers and the optical model for nuclear scattering", in the book edited by S. Fernbach and A. H. Taub, "Computers and their role in the physical sciences" (Gordon and Breach, 1967).

A.2.4.2 FLGLCH calculates the partial Coulomb waves. This subroutine is adopted by a paper by A. R. Barnett, D. H. Feng, J. W. Steed and L. J. B. Goldfarb, "Coulomb wave functions for all real η and ρ ", Comp. Phys. Comm. **8**, (1974) 377.

A.2.4.3 POTEN(IDCHNL,IDC) calculates the optical model Woods-Saxon potential $U(r)$ of Section 4.1. IDC is set to be 1 presently.

One may generate the optical potential by singly folding the potential for target-nucleon system (KTRLD(3)=1). The single folding potential (See Section 5.5.) is obtained by the gaussian integration method for which a supporting routine GAUSF(NGAUS,RTS,WGT) (See Section A.2.20.2.), where the abscissas (RTS) and weighting factors (WGT) for gaussian integration with the number of gaussian points, n =NGAUS are generated, is called.

A.2.4.4 DISWAVE calculates the distorted partial wave functions and the partial c-matrix by matching the internal partial wave functions generated from the origin and the Coulomb wave functions.

A.2.5 AFACAL - α -coefficients

A.2.5.1 AFACAL calculates the expansion coefficient $\alpha_{t_1 s_1 \ell_1 k \ell_t}^{j_t s_t \nu_1}$, [Eq.(13)], defined as

$$\begin{aligned} \alpha_{t_1 s_1 \ell_1 k \ell_t}^{j_t s_t \nu_1} &= W(s_t \ell_t s_t \ell_1; j_t k) \hat{s}_t^{-1} \hat{t}_1^{-1} \langle b | [c^\dagger c]_{s_1 t_1 \tilde{\nu}_1} | a \rangle \quad [\text{Eq.(13)}] \\ &= W(s_t \ell_t s_t \ell_1; j_t k) \times M \times L \\ L &= (1 - (-)^{s_a + t_a}) \hat{s}_a \hat{s}_b W(s_a \frac{1}{2} s_1 \frac{1}{2}; \frac{1}{2} s_b) \hat{t}_a \hat{t}_b W(t_a \frac{1}{2} t_1 \frac{1}{2}; \frac{1}{2} t_b) \\ M &= (-)^{t_1 + \nu_1 + t_a - \nu_a} \hat{t}_1^{-1} \langle t_a \nu_a t_b, -\nu_b | t_1 \nu_1 \rangle \end{aligned}$$

The overall phase factor $(-)^{\ell_1 + k + \ell_t}$ is also multiplied.

The L -values for different reactions are

Reaction	s_a	s_b	s_1	t_a	t_b	t_1	L
(p, p')	1/2	1/2	0	1/2	1/2	0	1
(p, n)	1/2	1/2	0	1/2	1/2	1	1
(d, d')	1	1	s_1	0	0	0	$\sqrt{(3+s_1)(2-s_1)}/2\delta(t_1, 0)$
$(d, 2p)$	1	s_b	s_1	0	1	1	$\sqrt{6}\hat{s}_b W(1\frac{1}{2}s_1\frac{1}{2}; \frac{1}{2}s_b)\delta(t_1, t_b)$
(h, t)	1/2	1/2	s_1	1/2	1/2	t_1	$4\delta(t_1, 0)\delta(s_1, 0) - (-)^{t_1+s_1}$
(α, α')	0	0	0	0	0	0	2

$$\sqrt{6}\hat{s}_b W(1\frac{1}{2}s_1\frac{1}{2}; \frac{1}{2}s_b) = \sqrt{(3+s_1)(2-s_1)}/2, \quad s_b = 1$$

AFACAL calls supporting subroutines CLEB (See A.2.20.4.) and RAC7 (See A.2.20.6.) whenever Clebsch-Gordan and Racah coefficients, respectively, are needed.

A.2.6 EFFINT - Effective interaction components

A.2.6.1 EFFINT(KEXCH) calculates the nucleon-nucleon effective interaction, $V_{t_1s_1k}^i(r)$, of [Eq.(16)], i.e.,

$$V_{t_1s_1k}^i(r) = \sqrt{4\pi}f_k\hat{s}_1^2\hat{t}_1^2 \sum_{st} \hat{s}^3\hat{t}^3 \begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & s \\ \frac{1}{2} & \frac{1}{2} & s \\ s_1 & s_1 & k \end{Bmatrix} \begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & t \\ \frac{1}{2} & \frac{1}{2} & t \\ t_1 & t_1 & 0 \end{Bmatrix} P_i V_{tsk}^i(r) \quad [\text{Eq.}(16)]$$

where $P_D = 1$ (KEXCH=0, Direct) and $P_E = (-)^{s+t+1}$ (KEXCH=1, Exchange), while $f_0 = 1$ and $f_2 = \sqrt{8}$. $V_{tsk}^i(r)$ is the Love-Franey effective interactions which are input.

EFFINT calls a supporting routine NINEJ (See A.2.20.7.) twice for 9-j symbols of the spin and isospin parts.

A.2.7 PDENST - Projectile density function

A.2.7.1 PDENST(KEXCH=0, direct form factor) calculates the expansion coefficients $\rho_{P,\lambda_2}^D(r'_2, r)$ of [Eq.(25)] for the multipole expansion of the local density function $\rho_P^D(r_2)$, i.e.,

$$\rho_P^D(r_2) = \sum_{\lambda_2} \rho_{P,\lambda_2}^D(r'_2, r)(-)^{\lambda_2}[Y_{\lambda_2}Y_{\lambda_2}]_{00} \quad [\text{Eq.}(23)]$$

$$\rho_{P,\lambda_2}^D(r'_2, r) = \sqrt{\pi} \int_{-1}^1 \rho_{P,\lambda_2}^D(r'_2, r, \mu) Y_{\lambda_2 0}(\theta, 0) d\mu, \quad [\text{Eq.}(25)]$$

$$r_2^2 = (r'_2)^2 + r^2 + 2r'_2 r \mu, \quad \mu \equiv \cos \theta = \hat{r} \cdot \hat{r}'_2$$

For the nucleon scattering (KTRLD(1)=3), it just returns.

It calls a supporting subroutine YLCAL (See Section A.2.20.1.) to obtain the spherical harmonics, and also calls GAUSF(NGAUS,RTS,WGT) (See Section A.2.20.2.) with NGAUS=16 for the gaussian integration. CLEB (See A.2.20.4.) is also called for the Clebsch-Gordan coefficients.

A.2.7.2 PDENST(KEXCH=1, exchange form factor) calculates the expansion coefficients $\rho_{P,\lambda_2}^E(r'_2, r)$ of [Eq.(31a)] for the multipole expansion of the nonlocal density function $\rho_P^E(\vec{r}_2, \vec{r}'_2)$, i.e.,

$$\begin{aligned}\rho_P^E(\vec{r}_2, \vec{r}'_2) &= \sum_{\lambda_2} \rho_{P,\lambda_2}^E(r'_2, r) (-)^{\lambda_2} [Y_{\lambda_2}(\hat{r}'_2) Y_{\lambda_2}(\hat{r})]_{00} \quad [\text{Eq.}(30a)] \\ \rho_{P,\lambda_2}^E(r'_2, r) &= \sum_{\ell_2 \eta_2} \hat{\ell}_2 \hat{\eta}_2 \omega_{\ell_2}(r'_2) (\eta_2 0 \lambda_2 0 | \ell_2 0) \\ &\quad \times \frac{2\pi}{2\ell_2 + 1} \sum_m \hat{\eta}_2 (\eta_2 0 \lambda_2 m | \ell_2 m) \int \omega_{\ell_2}(r_2) Y_{\ell_2 m}(\theta, 0) Y_{\lambda_2 m}^*(\theta', 0) d\mu \\ &\quad (\mu = \cos \theta = \vec{r}_2 \cdot \vec{r}'_2, \quad \text{and} \quad \cos \theta' = \vec{r}'_2 \cdot \vec{r}) \quad [\text{Eq.}(31a)]\end{aligned}$$

A.2.8 FFCALD - Direct form factor

A.2.8.1 FFCALD calls EFFINT(0) (See A.2.6.1.) to calculate the effective interaction for the direct form factors.

A.2.8.2 FFCALD calls PDENST(0) (See A.2.7.1.) to calculate the projectile density for the direct form factors.

A.2.8.3 FFCALD calculates the local target density $\rho_{T,\ell_1}^D(r_1)$, (See Formulation 1.5 for details.)

$$\rho_{T,\ell_1}^D(r_1) = X(\ell_p \frac{1}{2} j_p, \ell_h \frac{1}{2} j_h; \ell_1 s_1 j_t) \langle I_B || [\hat{a}_{j_p \nu_p}^\dagger \hat{a}_{j_h \nu_h}]_{j_t} || I_A \rangle \sqrt{4\pi} d_{\ell_p \ell_h \ell_1} R_{\ell_p}(r_1) R_{\ell_h}(r_1)$$

It calls a supporting routine NINEJ (See A.2.20.7.) to evaluate X -coefficients.

A.2.8.4 FFCALD further expands the resulting projectile density $\rho_{P,\lambda_2}^D(r'_2, r) Y_{\lambda_2 \mu_2}(\hat{r})$ of Eq.(23), obtained in PDENST, once more into

$$\begin{aligned}\rho_{P,k}^D(r'_2, r) Y_{km}(\hat{r}'_2) &= \sqrt{4\pi} \sum_{\ell_t \ell_1} \rho_{P,\lambda_2 \ell_t \ell_1}^D(r_a, r_1, r) [Y_{\ell_t}(\hat{r}_a) Y_{\ell_1}(\hat{r}_1)]_{km} \quad [\text{Eq.}(24)] \\ \rho_{P,k \ell_t \ell_1}^D(r_a, r_1, r) &= \frac{2\pi}{\hat{k}^2} \sum_m \hat{\ell}_t (\ell_t 0 \ell_1 m | km) \int \rho_{P,k \ell_t \ell_1}^D(r_a, r_1, \mu, r) Y_{km}(\theta'_2, 0) Y_{\ell_1 m}^*(\theta, 0) d\mu \\ (r'_2)^2 &= r_1^2 + r_a^2 - 2r_1 r_a \mu \quad \mu \equiv \cos \theta = \hat{r}_a \cdot \hat{r}_1, \\ &\quad \mu' \equiv \cos \theta' = \hat{r}_a \cdot \hat{r}'_2 = \frac{r_1 \mu - r_a}{r'_2} \quad [\text{Eq.}(26)]\end{aligned}$$

Here it calls GAUSF and YLCAL for the integration and CLEB for vector coupling coefficients. For the NN scattering,

$$\rho_{P,k \ell_t \ell_1}^D(r_a, r_1, r) = \frac{2\pi}{\hat{k}^2} \sum_m \hat{\ell}_t (\ell_t 0 \ell_1 m | km) \hat{k} (-)^k \left(\frac{-1}{r_1 r_a r} \right) Y_{km}(\theta'_2, 0) Y_{\ell_1 m}^*(\theta, 0)$$

Note that **the α -coefficients (See A.2.5) are multiplied in $\rho_{P,k \ell_t \ell_1}^D(r_a, r_1, r)$ here.**

A.2.8.5 FFCALD then calculates the direct form factors, i.e.,

$$f_{t_1 s_1 \ell_1 k \ell_t, m \ell_t}^D(r_a) = (-)^{\ell_1} \hat{\ell}_t^{-1} \int r^2 dr V_{t_1 s_1 k}^D(r) \int dr_1 \rho_{P,k \ell_t \ell_1}^D(r_a, r_1, r) \rho_{T,\ell_1}^D(r_1) \quad [\text{Eq.}(27)]$$

A.2.8.6 FFCALD calls PACALD(1) (See A.2.9.) to obtain the direct transition amplitudes.

A.2.9 PACALD - Direct overlap integral and transition amplitudes

A.2.9.1 PACALD(IDEXCH) calculates the overlap integrals of the direct form factor (IDEXCH=1) and the exchange form factor for no-recoil approximation (IDEXCH=2).

$$\begin{aligned} O_{t_1 s_1 \ell_1 k \ell_t, \ell_a \ell_b}^D &= d_{\ell_a \ell_t \ell_b} \int dr_a \chi_{\ell_b}(r_a) f_{t_1 s_1 \ell_1 k \ell_t}^D(r_a) \chi_{\ell_a}(r_a) \quad [\text{Eq.}(19a)] \\ O_{t_1 s_1 \ell_1 k \ell_t, \ell_a \ell_b}^{E, NR} &= d_{\ell_a \ell_t \ell_b} \int dr_a \chi_{\ell_b}(r_a) f_{t_1 s_1 \ell_1 k \ell_t}^{E, NR}(r_a) \chi_{\ell_a}(r_a) \\ d_{\ell_a \ell_t \ell_b} &\equiv \frac{1}{\sqrt{4\pi}} \hat{\ell}_a \hat{\ell}_t \hat{\ell}_b^{-1} (\ell_a 0 \ell_t 0 | \ell_b 0) \end{aligned}$$

It calls CLEBZ (See Section A.2.20.5.) for vector coupling coefficients.

A.2.9.2 PACALD then calculates direct transition amplitudes.

$$T_{t_1 s_1 \ell_1 k \ell_t m_{\ell_t}}^D = \frac{(4\pi)^{3/2}}{k_a k_b} \sum_{\ell_a \ell_b} i^{\ell_a - \ell_b + \pi} \hat{\ell}_a (\ell_a 0 \ell_t m_{\ell_t} | \ell_b m_{\ell_t}) O_{t_1 s_1 \ell_1 k \ell_t, \ell_a \ell_b}^D Y_{\ell_b m_{\ell_t}}(\hat{k}_b) \quad [\text{Eq.}(18)]$$

and sums over $(t_1 s_1 \ell_1, k = 0)$. Here it calls YLCAL for the spherical harmonics and CLEB for vector coupling coefficients.

A.2.10 XLMCAL - Distortion factor

A.2.10.1 XLMCAL calculates the so-called distortion factor, defined as

$$I_{\ell_t m_{\ell_t}}(r_a) \equiv \frac{(4\pi)^{3/2}}{k_a k_b} \sum_{\ell_a \ell_b} i^{\ell_a - \ell_b + \pi} \hat{\ell}_a (\ell_a 0 \ell_t m_{\ell_t} | \ell_b m_{\ell_t}) d_{\ell_a \ell_t \ell_b} \chi_{\ell_b}(r_a) \chi_{\ell_a}(r_a)$$

This is a part of integrand of $T_{t_1 s_1 \ell_1 k \ell_t m_{\ell_t}}^D$ of Eq. (18) in the original paper, excepting the form factor in the overlap integral of Eq.(19a). It calls CLEB and CLEBZ for Clebsch-Gordan coefficients.

A.2.11 FFCALDM - Modified direct form factor

A.2.11.1 FFCALDM obtains exactly the same direct transition amplitudes as those of FFCALD. **The order of overlap integrations is interchanged between r_a and r_1 .** According to the paper by B. T. Kim, D. P. Knobles, S. A. Stotts, and T. Udagawa, Phys. Rev. C61, 044611 (2000), the modified direct form factor is defined as

$$f_{t_1 s_1 \ell_1 k \ell_t, m_{\ell_t}}^{D, M}(r_1) = (-)^{\ell_1} \hat{\ell}_t^{-1} \int r^2 dr V_{t_1 s_1 k}^D(r) \int dr_a \rho_{P, k \ell_t \ell_1}^D(r_a, r_1, r) I_{t_1 s_1 \ell_1 k \ell_t m_{\ell_t}}(r_a)$$

where $I_{t_1 s_1 \ell_1 k \ell_t m_{\ell_t}}(r_a)$ is the distortion factor calculated in XLMCAL.

A.2.11.2 FFCALDM then calculates overlap integrals.

$$O_{t_1 s_1 \ell_1 k \ell_t m_{\ell_t}}^{D, M} = \int r_1^2 dr_1 f_{t_1 s_1 \ell_1 k \ell_t, m_{\ell_t}}^{D, M}(r_1) \rho_{T, \ell_1}^D(r_1)$$

A.2.11.3 FFCALDM finally calculates direct transition amplitudes.

$$T_{t_1 s_1 \ell_1 k \ell_t m_{\ell_t}}^{D, M} = \frac{(4\pi)^{3/2}}{k_a k_b} \sum_{\ell_a \ell_b} i^{\ell_a - \ell_b + \pi} \hat{\ell}_a (\ell_a 0 \ell_t m_{\ell_t} | \ell_b m_{\ell_t}) O_{t_1 s_1 \ell_1 k \ell_t, \ell_a \ell_b}^{D, M}$$

and sums over $(t_1 s_1 \ell_1, k = 0)$.

A.2.12 DCHECK2 - Checking field lengths of variables for the exchange part

A.2.12.1 DCHECK2 checks field lengths of variables for the exchange form factor calculation with given input data.

The meaning of PARAMETER given in the program is following,

Parameter	Field length of Variable	Max. Combinations
NWTMAX	(LAM1MX+1)*(LAM1MX+2)/2	
KBMAX	NLSMAX*LAM1MX*(LAM1MX+2)/2	$\{t_1 s_1 \ell_1 \lambda_1 \ell'_p\}$
KB	(NLS,LAM1,LPP)	
KCMAX	KBMAX*NOLTR*LAM2MX*(LAM2MX+1)/2	$\{KB, \lambda_2 \ell\}$
KC	(LT,KB,LAM2,LL)	
KAMAX	LAMMX*(LAMMX+1)/2	$\{KF=(t_1 s_1 \ell_1 k \ell_t), \lambda \ell\}$
KA	(LL,LAM)	
KGMAX	KCMAX*LAMP1X	$\{KC, \lambda\}$
KG	(KC,LAM)	
NLS	(LT1,L1,S)	

A.2.13 TRECAL, DENST, TRECALM, DENSTM - Non-local projectile and target densities for exchange part

A.2.13.1 TRECAL first calls PDENST(1) (See A.2.7.) and calculates the non-local projectile density, $\rho_{P,\lambda_2}^E(r'_2, r)$ of [Eq.(31a)],

$$\begin{aligned} \rho_{P,\lambda_2}^E(r'_2, r) &= \sum_{\ell_2 \eta_2} \hat{\ell}_2 \hat{\eta}_2 \omega_{\ell_2}(r'_2) (\eta_2 0 \lambda_2 0 | \ell_2 0) \\ &\times \frac{2\pi}{2\ell_2 + 1} \sum_m \hat{\eta}_2 (\eta_2 0 \lambda_2 m | \ell_2 m) \int \omega_{\ell_2}(r_2) Y_{\ell_2 m}(\theta, 0) Y_{\lambda_2 m}^*(\theta', 0) d\mu \\ &(\mu = \cos \theta = \vec{r}_2 \cdot \vec{r}'_2, \quad \text{and} \quad \cos \theta' = \vec{r}_2 \cdot \vec{r}) \quad [\text{Eq.}(31a)] \end{aligned}$$

A.2.13.2 TRECAL then calculates the non-local target density, [Eq. (31b)],

$$\begin{aligned} \rho_{T,\ell_1 \lambda_1 \ell_c}^E(r_1, r) &= \sum_{ph, \eta_1} i^{\ell_p + \ell_h - \pi} X(\ell_p \frac{1}{2} j_p, \ell_h \frac{1}{2} j_h; \ell_1 s_1 j_t) < I_B || [\hat{a}_{j_p \nu_p}^\dagger \hat{a}_{j_h \nu_h}]_{j_t} || I_A > R_{\ell_p}(r_1) \\ &\times (-)^m \hat{\ell}_h \hat{\ell}_c \hat{\eta}_1 (\ell_c 0 \eta_1 0 | \ell_p 0) W(\ell_c \eta_1 \ell_1 \ell_h; \ell_p \lambda_1) \\ &\times \frac{2\pi}{2\ell_h + 1} \sum_{m_1} \hat{\eta}_1 (\eta_1 0 \lambda_1 m_1 | \ell_h m_1) \int R_{\ell_h}(r'_1) Y_{\ell_h m_1}(\theta, 0) Y_{\lambda_1 m_1}^*(\theta', 0) d\mu' \\ &(\mu = \cos \theta = \vec{r}_1 \cdot \vec{r}'_1, \quad \text{and} \quad \cos \theta' = \vec{r}_1 \cdot \vec{r}) \quad [\text{Eq.}(31b)] \end{aligned}$$

A.2.13.3 DENST(N,L) is called to calculate the last line of the above equation for a given $n_h = N, \ell_h = L$.

TRECAL and DENST call supporting subroutines NINEJ, RAC7, CLEBZ for the vector coupling coefficients.

A.2.13.4 TRECALM and DENSTM are basically the same as TRECAL and DENSTM except that r_1 is interpolated and that in the target densities the particle states are not added.

A.2.14 GFAC, GFACM - Vector coupling factors for exchange part

A.2.14.1 GFAC(K) defines various vector coupling factors for the exchange form factors for the given $K=1$ for the central interaction and $K=2$ for the tensor interaction. It first calculates a coupling factor in the projectile density of [Eq.(32)]

$$\begin{aligned}\rho_{P,\lambda_2\ell_b\ell}^E(r_b, r_1, r) &= \frac{2\pi}{2\lambda_2 + 1} \sum_m \hat{\ell}_b(\ell_b 0 \ell m | \lambda_2 m) \int \rho_{P,\lambda_2}^E(r_b, r_1, \mu, r) Y_{\lambda_2 m}(\theta'_2, 0) Y_{\ell m}^*(\theta, 0) d\mu \\ \text{CLEBQ(KC,M2)} &= \frac{2\pi}{2\lambda_2 + 1} \hat{\ell}_b(\ell_b 0 \ell m_2 | \lambda_2 m_2) \frac{2}{1 + \delta_{m_2 0}} \\ \text{KC} &= \{\text{KB} = (t_1 s_1 \ell_1 \lambda_1 \ell_b), \lambda_2 \ell\}\end{aligned}$$

A.2.14.2 GFAC(K) then calculates a coupling factor in the G^k of [Eq.(33b)]

$$\begin{aligned}G_{t_1 s_1 \ell_1 \ell_b \lambda}^k(r_b, r) &= \frac{1}{\sqrt{4\pi}} r^{-k} V_{t_1 s_1 k}^E(r) \sum_{\lambda_1 \lambda_2 \ell_c} (-)^{\ell_b} \hat{\lambda}_1 \hat{\lambda}_2 (\lambda_1 0 \lambda_2 0 | \lambda 0) W(\lambda_1 \lambda_2 \ell_1 \ell_b; \lambda \ell_c) \\ &\quad \times \int r_1^2 dr_1 \rho_{P,\lambda_2 \ell_b \ell_c}^E(r_b, r_1, r) \rho_{T,\ell_1 \lambda_1 \ell_c}^E(r_1, r) \\ \text{COEF(KG)} &= \frac{1}{\sqrt{4\pi}} (-)^{\ell_b} \hat{\lambda}_1 \hat{\lambda}_2 (\lambda_1 0 \lambda_2 0 | \lambda 0) W(\lambda_1 \lambda_2 \ell_1 \ell_b; \lambda \ell_c) \\ \text{KG} &= \{\text{KC}, \lambda\}\end{aligned}$$

A.2.14.3 GFAC(K) finally calculates two coupling factors in c of [(Eq.(35)]

$$\begin{aligned}c_{t_1 s_1 \ell_1 k, \ell_\alpha \ell_\beta}(r_b, r_a) &= \frac{2\pi}{\hat{\ell}_1^2} \sum_{m_{\ell_1}} \hat{\ell}_\beta(\ell_\alpha m_{\ell_1} \ell_\beta 0 | \ell_1 m_{\ell_1}) \sum_{\ell_c \lambda} \hat{\ell}_c(\ell_c 0 \lambda m_{\ell_1} | \ell_1 m_{\ell_1}) \\ &\quad \times \int d\mu G_{t_1 s_1 \ell_1 \ell_b \lambda}^k(r_b, r) Y_{\lambda m_{\ell_1}}(\theta', \pi) Y_{\ell_\alpha m_{\ell_1}}^*(\theta, 0) \\ &\quad (\mu = \cos \theta = \hat{r}_b \cdot \hat{r}'_a, \text{ and } \cos \theta' = \hat{r}_b \cdot \hat{r}) \\ \text{CLEBF(KA,M1,NLS)} &= \frac{2\pi}{2\ell_1 + 1} \hat{\ell}_c(\ell_c 0 \lambda m | \ell_1 m) \frac{2}{1 + \delta_{m 0}} (-)^m \\ \text{KA} &= \{\text{KF} = (t_1 s_1 \ell_1 k \ell_t), \lambda \ell\}, \ell_b \lambda\} \\ \text{NLS} &= \{t_1 s_1 \ell_1\} \\ \text{CCDD(JLS,M1)} &= \hat{\ell}_\beta(\ell_\alpha m \ell_\beta 0 | \ell_1 m) \\ \text{JLS} &= \{\ell_\alpha, \ell_\beta, \text{NLS}\}\end{aligned}$$

A.2.14.4 GFACM(K) calculates a coupling factor in the form factors for the recoil effects in the plane wave approximation, instead of the previous section A.2.13.3.

$$\begin{aligned}f_{t_1 s_1 \ell_1 k \ell_t}^{PW}(r_b) &= \sqrt{4\pi} \sum_{\ell_b \lambda \ell_r} i^{\pi - \ell_r - \ell_b} \hat{k} \hat{\lambda} (k 0 \lambda 0 | \ell_r 0) \hat{\ell}_1 \hat{\ell}_r W(\ell_b \lambda \ell_t k : \ell_1 \ell_r) \\ &\quad \times (-)^{k + \ell_1 - \ell_t} (\ell_b m_{\ell_t} \ell_r 0 | \ell_t m_{\ell_t}) \int dr r^{k+2} G_{t_1 s_1 \ell_1 \ell_b \lambda}^k(r_b, r) j_{\ell_r}(\alpha k_a r / a) \\ \text{CCDD(JLS)} &= i^{\pi - \ell_r - \ell_b} \hat{k} \hat{\lambda} (k 0 \lambda 0 | \ell_r 0) \hat{\ell}_1 \hat{\ell}_r W(\ell_b \lambda \ell_t k : \ell_1 \ell_r)\end{aligned}$$

A.2.15 FFCALC - Exchange form factors

A.2.15.1 FFCALC calls DCHECK2 (See A.2.12.) to check the field lengths of variables for the exchange form factors.

A.2.15.2 FFCALC calls EFFINT(1) (See A.2.6.) to calculate the effective interaction for the exchange form factors.

A.2.15.3 FFCALC calls TRECAL (See A.2.13.) to calculate the projectile and target density.

A.2.15.4 FFCALC starts to calculate the central and tensor exchange form factors.

This section has a big DO loop up to the statement number 1000, for the central exchange form factors (KCT=1) and the tensor exchange form factors (KCT=2). By inputting 1 for KCETN(KCT) (Input data Line #16), you may exclude the corresponding form factors.

It then calls GFAC(KCT) (See A.2.14.) to calculate the various coupling factors.

It starts next big DO loop up to the state number 900. In the DO loop, the following Section A.2.15.5 - A.2.15.8 for a given r_b are performed.

A.2.15.5 FFCALC performs integration in G-factors.

It first calculates the projectile density, $\rho_{P,\lambda_2\ell_a\ell}^E(r_b, r_1, r)$, in [Eq. (32)],

$$\rho_{P,\lambda_2\ell_a\ell}^E(r_b, r_1, r) = \frac{2\pi}{\hat{\lambda}_2^2} \sum_m \hat{\ell}_b(\ell_b 0 \ell m | \lambda_2 m) \int \rho_{P,\lambda_2}^E(r_b, r_1, \mu, r) Y_{\lambda_2 m}(\theta'_2, 0) Y_{\ell m}^*(\theta, 0) d\mu$$

The non-local target density from TRECAL (See A.2.13.) is multiplied to the projectile density and integration over r_1 is performed.

$$\int r_1^2 dr_1 \rho_{P,\lambda_2\ell_b\ell_c}^E(r_b, r_1, r) \rho_{T,\ell_1\lambda_1\ell_c}^E(r_1, r)$$

For a nucleon scattering, it becomes

$$\rho_{T,\ell_1\ell_b\lambda}^E(r_b, r) \times \hat{\lambda}_2(-)^{\lambda_2}$$

(See Formulation 3.3.2.)

A.2.15.6 FFCALC calculates the G-factor of [Eq. (33b)],

$$G_{t_1 s_1 \ell_1 \ell_b \lambda}^k(r_b, r) = \frac{1}{\sqrt{4\pi}} r^{-k} V_{t_1 s_1 k}^E(r) \sum_{\lambda_1 \lambda_2 \ell_c} (-)^{\ell_b} \hat{\lambda}_1 \hat{\lambda}_2 (\lambda_1 0 \lambda_2 0 | \lambda 0) W(\lambda_1 \lambda_2 \ell_1 \ell_b; \lambda \ell_c) \\ \times \int r_1^2 dr_1 \rho_{P,\lambda_2\ell_b\ell_c}^E(r_1, r) \rho_{T,\ell_1\lambda_1\ell_c}^E(r_1, r) \quad [\text{Eq. (33b)}]$$

Note that the α -coefficients (See A.2.5) are multiplied in $G_{t_1 s_1 \ell_1 \ell_b \lambda}^k(r_b, r)$ for the central interaction of the exchange form factor.

A.2.15.7 FFCALC calculates the c-factor of [Eq. (35)].

It first decomposes r into r_a and r_b in the G-factors,

$$G_{t_1 s_1 \ell_1 \ell_b \lambda}^k(r_b, r_a) = \frac{2\pi}{\hat{\ell}_t^2} \sum_{\ell_c \lambda} \hat{\ell}_c(\ell_c 0 \lambda m_{\ell_t} | \ell_t m_{\ell_t}) \int d\mu G_{t_1 s_1 \ell_1 \ell_b \lambda}^k(r_b, r) Y_{\lambda m_{\ell_t}}(\theta', \pi) Y_{\ell_c m_{\ell_t}}^*(\theta, 0)$$

It finally calculates the c-factor,

$$c_{t_1 s_1 \ell_1 k, \ell_\alpha \ell_\beta}(r_b, r_a) = \frac{2\pi}{\hat{\ell}_1^2} \sum_{m_{\ell_1}} \hat{\ell}_\beta(\ell_\alpha m_{\ell_1} \ell_\beta 0 | \ell_1 m_{\ell_1}) \sum_{\ell_c \lambda} \hat{\ell}_c(\ell_c 0 \lambda m_{\ell_1} | \ell_1 m_{\ell_1}) \\ \times \int d\mu G_{t_1 s_1 \ell_1 \ell_b \lambda}^k(r_b, r) Y_{\lambda m_{\ell_1}}(\theta', \pi) Y_{\ell_\alpha m_{\ell_1}}^*(\theta, 0) \quad [\text{Eq. (35)}]$$

Remember that this is **the exchange form factors for central interactions**. (See Formulation 2.2.)

It stores the results as "FFRIT(JLS), JLS=($\ell_\alpha, \ell_\beta, \text{NLS}$)", and writes on tape. (See B.4 for details.)

A.2.15.8 FFCALC also calculates the radial exchange form factor of [Eq. (41)] in the no-recoil approximation. (See Formulation 3.2.)

$$f_{t_1 s_1 \ell_1 k \ell_t}^{NR}(r_b) = \sqrt{4\pi} (-)^k \hat{\ell}_1 \hat{\ell}_t^{-1} \int dr r^{k+2} G_{t_1 s_1 \ell_1 \ell_t \lambda}^k(r_b, r) \quad [\text{Eq. (41)}]$$

A.2.15.9 FFCALC calls TENSOR (See A.2.16.) to calculate final form factors for the tensor interaction.

A.2.15.10 FFCALC calls PACALC (See A.2.17.) to obtain the exchange transition amplitudes.

A.2.15.11 FFCALC calls PACALD(2) (See A.2.9.) to obtain the exchange transition amplitudes in the no-recoil approximation.

A.2.16 TENSOR - Tensor interaction for exchange part

A.2.16.1 TENSOR calculates exchange form factor $f_{t_1 s_1 \ell_1 k \ell_t, \ell_b \ell_a}^E(r_b, r_a)$ in Eq.(36) in the original paper for $k=2$, i.e. for tensor interaction,

$$f_{t_1 s_1 \ell_1 k \ell_t, \ell_b \ell_a}^E(r_b, r_a) = J 4\pi m_a^k \sum_{\lambda_a \lambda_b \ell_\alpha \ell_\beta} \left[\frac{(2k+1)!}{(2\lambda_a+1)!(2\lambda_b+1)!} \right]^{1/2} \delta_{\lambda_a+\lambda_b, k} (-r_a)^{\lambda_a} (r_b)^{\lambda_b} \\ \times X(\ell_\alpha \lambda_a \ell_a, \ell_\beta \lambda_b \ell_b; \ell_1 \lambda \ell_t) d_{\ell_\alpha \lambda_a \ell_a} d_{\ell_\beta \lambda_b \ell_b} c_{t_1 s_1 \ell_1 k, \ell_\alpha \ell_\beta}(r_b, r_a) \quad [\text{Eq. (36)}]$$

Note that the multiplication of all geometrical factors becomes unity for central interaction ($k=0$) and the form factor becomes $c_{t_1 s_1 \ell_1 k, \ell_\alpha \ell_\beta}(r_b, r_a)$. (See Formulation 2.2.)

A.2.16.2 TENSOR first calculates the geometrical factor,

$$4\pi \left[\frac{(2k+1)!}{(2\lambda_a+1)!(2\lambda_b+1)!} \right]^{1/2} \delta_{\lambda_a+\lambda_b, k} X(\ell_\alpha \lambda_a \ell_a, \ell_\beta \lambda_b \ell_b; \ell_1 \lambda \ell_t) d_{\ell_\alpha \lambda_a \ell_a} d_{\ell_\beta \lambda_b \ell_b}$$

and saves as "C(KJLS)" in a linearized form. **Note that the α -coefficients (See A.2.5) are multiplied in this geometrical factor** for the tensor interaction.

A.2.16.3 TENSOR reads c-factor (FFRIT) from Tape 14, multiplies all factors and sums over $\lambda_a \lambda_b \ell_\alpha \ell_\beta$, and then saves as "GGRIT(JLS)", where JLS=($\ell_\alpha, \ell_\beta, \text{NLS}$). If the central interaction is considered (KCNTN(1).EQ.0), it reads c-factor for the central interaction from Tape 13, and adds to the tensor part. It then finally stores form factors as "GGRIT(JLS)" for every (r_b, r_a) , and writes on Tape 15.

A.2.17 PACALE - Exchange overlap integral and transition amplitudes

A.2.17.1 PACALE calculates the overlap integrals of [Eq.(19b)]

$$O_{t_1 s_1 \ell_1 k \ell_t, \ell_a \ell_b}^E = J \int dr_b \int dr_a r_b r_a \chi_{\ell_b}(r_b) f_{t_1 s_1 \ell_1 k \ell_t, \ell_a \ell_b}^E(r_b, r_a) \chi_{\ell_a}(r_a) \quad [\text{Eq. (19b)}]$$

During the integration processes, it reads exchange form factors $f_{t_1 s_1 \ell_1 k \ell_t, \ell_a \ell_b}^E(r_b, r_a)$ from Tape 15.

A.2.17.2 PACALE finally obtains transition amplitudes,

$$T_{t_1 s_1 \ell_1 k \ell_t, m_{\ell_t}}^E = \frac{(4\pi)^{3/2}}{k_a k_b} \sum_{\ell_a \ell_b} i^{\ell_a - \ell_b + \pi} \hat{\ell}_a(\ell_a 0 \ell_t m_{\ell_t} | \ell_b m_{\ell_t}) O_{t_1 s_1 \ell_1 k \ell_t, \ell_a \ell_b}^E Y_{\ell_b m_{\ell_t}}(\hat{k}_b) \quad [\text{Eq. (18)}]$$

and sums over $(t_1 s_1 \ell_1, k = 2)$. Here it calls YLCAL for the spherical harmonics and CLEB for vector coupling coefficients.

A.2.18 FFCALEM - Exchange overlap integral and transition amplitudes

A.2.18.1 FFCALEM obtains the exchange form factor with re-coil effects in the plane wave approximation. As in the case of direct form factor, the order of overlap integrations is interchanged between r_a and r_1 with the same spirit of the paper by B. T. Kim, *et. al.*. The first three steps are the same as FFCALE; namely, it calls DCHECK2 (See A.2.12.) to check the field lengths of variables for the exchange form factors, EFFINT(1) (See A.2.6.) to calculate the effective interaction for the exchange form factors, and TRECALM (See A.2.13.) to calculate the projectile and target density.

A.2.18.2 FFCALEM calls a supporting subroutine BESSEL (See A.2.20.3.) to obtain Bessel functions which is needed for the tensor exchange form factors in the plane wave approximation. (See Formulation 2.3.)

A.2.18.3 FFCALEM then calculates the central and tensor exchange form factors.

This section has a big DO loop up to the statement number 1000, for the central exchange form factors (KCT=1) and the tensor exchange form factors (KCT=2). By inputting 1 for KCETN(KCT) (Input data Line #16), you may exclude the corresponding form factors.

It then calls GFACM(KCT) (See A.2.14.) to calculate the various coupling factors.

It starts next big DO loop up to the state number 900. In the DO loop, the following Section A.2.18.4 and A.2.18.5 for a given r_1 are performed.

A.2.18.4 FFCALEM calculates the projectile density and integration over r_b .

This section calculates the projectile density, $\rho_{P, \lambda_2 \ell_a \ell}^E(r_b, r_1, r)$,

$$\rho_{P, \lambda_2 \ell_a \ell}^E(r_b, r_1, r) = \frac{2\pi}{\hat{\lambda}_2^2} \sum_m \hat{\ell}_b(\ell_b 0 \ell m | \lambda_2 m) \int \rho_{P, \lambda_2}^E(r_b, r_1, \mu, r) Y_{\lambda_2 m}(\theta'_2, 0) Y_{\ell m}^*(\theta, 0) d\mu$$

For a nucleon scattering, it calculates the projectile density as, (See Formulation 3.1.)

$$\rho_{P, \lambda_2 \ell_a \ell}^E(r_b, r_1, r) = \frac{2\pi}{\hat{\lambda}_2^2} \sum_m \hat{\ell}_b(\ell_b 0 \ell m | \lambda_2 m) \hat{\lambda}_2(-)^{\lambda_2} \left(-\frac{1}{r_1 r_b r}\right) Y_{\lambda_2 m}(-\mu_p) Y_{\ell m}^*(\mu)$$

$$\mu_p = \frac{r_1^2 - r_b^2 - r^2}{2r r_b}, \quad \mu = \frac{r_b^2 + r_1^2 - r^2}{2r_1 r_b}$$

The distortion factor, $I_{t_1 s_1 \ell_1 k \ell_t m_{\ell_t}}(r_b)$ (See A.2.10.) is multiplied to the projectile density and integration is performed over r_b .

$$Z_{P, \lambda_2 \ell_b \ell}^E(r_1, r) = \int dr_b \rho_{P, \lambda_2 \ell_b \ell}^E(r_b, r_1, r) I_{t_1 s_1 \ell_1 k \ell_t m_{\ell_t}}(r_b)$$

A.2.18.4 FFCALEM calculates the G factor,

$$G_{t_1 s_1 \ell_1 \ell_b \lambda}^k(r_1, r) = \frac{1}{\sqrt{4\pi}} r^{-k} V_{t_1 s_1 k}^E(r) \sum_{\lambda_1 \lambda_2 \ell} (-)^{\ell_b} \hat{\lambda}_1 \hat{\lambda}_2 (\lambda_1 0 \lambda_2 0 | \lambda 0) W(\lambda_1 \lambda_2 \ell_1 \ell_b; \lambda \ell) \\ \times \int r_1^2 dr_1 Z_{P, \lambda_2 \ell_b \ell}^E(r_1, r) \rho_{T, \ell_1 \lambda_1 \ell}^E(r_1, r)$$

A.2.18.5 FFCALEM then defines the modified exchange form factor in the plane wave approximation as,

$$f_{t_1 s_1 \ell_1 k \ell_t}^{E, M}(r_1) = \sqrt{4\pi} \sum_{\ell_b \lambda \ell_r} i^{\pi - \ell_r - \ell_b} \hat{k} \hat{\lambda} (k 0 \lambda 0 | \ell_r 0) \hat{\ell}_1 \hat{\ell}_r W(\ell_b \lambda \ell_t k : \ell_1 \ell_r) \\ \times (-)^{k + \ell_1 - \ell_t} (\ell_b m_{\ell_t} \ell_r 0 | \ell_t m_{\ell_t}) \int dr r^{k+2} G_{t_1 s_1 \ell_1 \ell_b \lambda}^k(r_1, r) j_{\ell_r}(\alpha k_a r/a)$$

where ℓ_r and the recoil factor α values are input as "LRP1MX" and "FACNR", respectively. (See Section 4.2.9.)

A.2.18.6 FFCALEM then calculates overlap integrals.

$$O_{t_1 s_1 \ell_1 k \ell_t m_{\ell_t}}^{E, M} = \int r_1^2 dr_1 f_{t_1 s_1 \ell_1 k \ell_t, m_{\ell_t}}^{E, M}(r_1)$$

A.2.18.7 FFCALEM finally calculates direct transition amplitudes.

$$T_{t_1 s_1 \ell_1 k \ell_t m_{\ell_t}}^{E, M} = \frac{(4\pi)^{3/2}}{k_a k_b} \sum_{\ell_a \ell_b} i^{\ell_a - \ell_b + \pi} \hat{\ell}_a (\ell_a 0 \ell_t m_{\ell_t} | \ell_b m_{\ell_t}) O_{t_1 s_1 \ell_1 k \ell_t, \ell_a \ell_b}^{E, M}$$

and sums over $(t_1 s_1 \ell_1, k = 2)$.

A.2.19 CROSS - The differential cross sections

A.2.19.1 CROSS calculates the final differential cross sections of [Eq. (21)],

$$\frac{d\sigma}{d\Omega} = \frac{\mu_a \mu_b}{(2\pi \hbar^2)^2} \frac{k_b}{k_a} \frac{1}{(2I_A + 1)(2s_a + 1)} \left| \sum_i \sum_{k \ell_1 t_1} \alpha_{t_1 s_1 \ell_1 k \ell_t}^{j_t s_t \nu_1} T_{t_1 s_1 \ell_1 k \ell_t m_{\ell_t}}^i \right|^2 \quad [\text{Eq. (21)}]$$

A.2.20 YLCAL, GAUSF, BESSEL, CLEB, CLEBZ, RAC7, NINEJ, DSPLS3 - Supporting subroutines

A.2.20.1 YLCAL(A,L,M,R) calculates a part of spherical harmonics, $Y_{\ell m}(\mu)$,

$$\left[\frac{2\ell + 1}{4\pi} \frac{(\ell - m)!}{(\ell + m)!} \right]^{1/2} P_{\ell m}(\mu), \quad \mu = \cos \theta$$

by utilizing the recurrence relations of the associated Legendre functions, $P_{\ell m}(\mu)$,

$$\begin{aligned} (2\ell + 1)\mu P_{\ell m} &= (\ell - m + 1)P_{\ell+1,m} + (\ell + m)P_{\ell-1,m} \\ (2\ell + 1)(1 - \mu^2)^{1/2} P_{\ell m} &= P_{\ell-1,m+1} - P_{\ell+1,m+1}. \end{aligned}$$

The arguments are A= μ , L= ℓ , M= m , and R = answer.

A.2.20.2 GAUSF(N,R,W) generates abscissas and weighting factors for the Gaussian integration

$$\int_{-1}^1 f(x)dx = \sum_{i=1}^n w_i f(x_i) + \Delta$$

where

$$\begin{aligned} x_i &= i^{\text{th}} \text{ zero of Legendre function } P_n(x) \\ w_i &= \frac{2}{1 - x_i^2} [P'_n(x_i)]^2 \end{aligned}$$

The arguments are N= n , R= x_i , and W= w_i .

A.2.20.3 BESSEL(A,L,R1,R2) generates the spherical Bessel and Neumann functions, $j_n(x)$ and $n_n(x)$, by using the recurrence formula,

$$\begin{aligned} \frac{n+1}{z} j_n(z) + \frac{dj_n(z)}{dz} &= j_{n-1}(z) \\ \frac{n}{z} j_n(z) - \frac{dj_n(z)}{dz} &= j_{n+1}(z) \end{aligned}$$

The arguments are A= x , L= n , R1 = value of spherical Bessel function and R2 = that of Neumann.

A.2.20.4 CLEB(A,B,C,D,E,F,FACLOG,R) calculates the Clebsch-Gordan coefficients, i.e.,

$$\begin{aligned} (j_1 m_1 j_2 m_2 | j m_j) &= \delta_{m, m_1 + m_2} \sqrt{\frac{(2j+1)(j+j_1-j_2)!(j-j_1+j_2)!(j_1+j_2-j)!}{(j_1+j_2+j+1)!}} \\ &\times \sqrt{\frac{(j+m)!(j-m)!(j_1+m_1)!(j_1-m_1)!(j_2+m_2)!(j_2-m_2)!}{(j+m_1+m_2)!}} \\ &\times \sum_k \left[\frac{(-)^k}{k!(j_1+j_2-j-k)!(j_1-m_1-k)!(j_2+m_2-k)!} \right. \\ &\quad \left. \times \frac{1}{(j-j_2+m_1+k)!(j-j_1-m_2+k)!} \right] \end{aligned}$$

The summation is extended over all integral k for which the argument of every factorial is nonnegative.

The arguments are A= j_1 , B= j_2 , C= j , D= m_1 , E= m_2 , F= m , R = answer, and FACLOG is the log of integral value of n . $n = 500$ is set.

A.2.20.5 CLEBZ(A,B,C,FACLOG,R) calculates the Clebsch-Gordan coefficients with all m 's are zero, i.e.,

$$(j_1 0 j_2 0 | j 0) = (-)^{g+j} \hat{j} \sqrt{\frac{(j_1 + j_2 - j)!(j + j_1 - j_2)!(j + j_2 - j_1)!}{(2g + 1)!}} \frac{g!}{(g - j)!(g - j_1)!(g - j_2)!}$$

$$= 0 \quad \text{for } j + j_1 + j_2 = 2g = \text{ odd}$$

The arguments are A= j_1 , B= j_2 , C= j , R = answer, and FACLOG is the same as in CLEB.

A.2.20.6 RAC7(A,B,C,D,E,F,FACLOG,R) calculates the Racah coefficients, i.e.,

$$W(j_1 j_2 J j_3; j_{12} J_{23}) = \hat{J}_{12}^{-1} \hat{J}_{23}^{-1} \sum_{m_1 m_2 m_3} (j_1 m_1 j_2 m_2 | J_{12} M_{12})(J_{12} M_{12} j_3 m_3 | J M)$$

$$(j_2 m_2 j_3 m_3 | J_{23} M_{23})(j_1 m_1 J_{23} M_{23} | J M)$$

The arguments are A= j_1 , B= j_2 , C= J , D= j_3 , E= j_{12} , F= j_{23} , R = answer, and FACLOG is the same as in CLEB. In coding, it does not call CLEB, but use the analytic form of Clebsch-Gordan coefficients in A.2.20.4 directly.

A.2.20.7 NINEJ(A(9),FACLOG,R) calculates the 9- j symbols as

$$U \begin{pmatrix} a & b & e \\ c & d & e' \\ f & f' & g \end{pmatrix} = (-)^{\sigma} \sum_{\lambda} \hat{\lambda}^2 W(b c e f; \lambda a) W(b c f' e'; \lambda d) W(e f e' f'; \lambda g)$$

$$\sigma = a + b + c + d + e + e' + f + f' + g = \text{integer}$$

by calling the Racah coefficient routine RAC7.

The arguments are A(1)= a , A(2)= b , A(3)= c , A(4)= d , A(5)= e , A(6)= e' , A(7)= f , A(8)= f' , A(9)= g , R = answer, and FACLOG is the same as in CLEB.

A.2.20.8 DSPLS3(X,Y,N,XI,YI,M,Q,AU,IGO) interpolates a given set of $\{(x_i, y_i), i = 1, \dots, n\}$ to a set of $\{(x_j, y_j), j = 1, \dots, m\}$ by the cubic spline interpolation method.

The arguments are X= x_i , Y= y_i , N= n , XI= x_j , YI= y_j , and M= m . Q and AU are internally used, and IGO = 1 is set.

B Variables and their definitions

B.1 PARAMETER variables

Variable Name	Present #	The way counting number. (DCHEK1 and DCHECK2 check the present number whether that is good enough for the calculation.)
KAA	40	Max. possible combinations of $KA=\{KF=(t_1s_1l_1kl_t), \lambda\ell\}$
KBB	30	Max. possible combinations of $KB=\{t_1s_1l_1\lambda_1\ell'_p\}$
KCC	80	Max. possible combinations of $KC=\{KB, \lambda_2\ell\}$
KFM	10	Max. possible combinations of $KF=\{t_1s_1l_1kl_t\}$
KGK	125	Max. possible combinations of $\{KC, \lambda\}$
KJL	1200	Max. possible combinations of $\{t_1s_1l_1kl_t, l_a l_b\}$
KTE	10000	Max. possible combinations of $\{l_\alpha\lambda_a l_a, l_\beta\lambda_b l_b; l_1kl_t\}$ which appears in the X -factor of the exchange form factor.
LAB	2000	Max. possible combinations of $\{l_t l_a l_b\}$
LMJ	10	$(MJMAX+1)*3$
LMM	4	Max of (LAMMXD(1,2,3))
LM1	5	Max magnetic quantum in the transferred l_t , MP1MAX.
LM2	30	Max. possible combinations of $\{l_h\lambda_1\}$ in the nonlocal target density.
LPH	10	Max of ph pairs in the target, NPHMX.
LRP	11	Max # of partial waves in the PW approximation.
LTL	9	Max total angular momentum in the target, $L1R(NLSMAX)+LAMMXD(1)+1$.
LXA	150	Max of partial waves in the incident channel, LDWMXR(1).
LXB	150	Max of partial waves in the exit channel, LDWMXR(2).
L1X	10	NOLTR*MP1MAX
L2X	150	NOLTR*MP1MAX*NPAIRD
NHS	9	Max # of hole states, NOSH.
NIN	300	Max of (NHDMX,NHEMX)
NOU	300	NHEMX
NPS	9	Max # of particle states, NOSP.
NXA	420	NXMXR(1)*PMASA
NXB	140	NXMXR(2)
N1X	140	NXMXR(4)/N1STEP+1

B.2 COMMON variables

[Variables with * are input data and those in bold face are the key quantities.]

B.2.1 CNST - Constants

Variable name	Definition
FACLOG(N)	Log value of factorial of integer N . Used for vector coupling coefficients. Presently, $N=500$.
PI	$\pi = 4.0 \arctan(1.0)$.
HBAR	$\hbar c = 197.327053$ in MeV · fm.
AMAS	atomic mass unit, $m_{amu} = 931.49432$ in MeV.
WNUNIT	wave number unit in 1/fm, $\sqrt{2m_{amu}}/\hbar c$.
FINE	fine structure constant, $e^2/\hbar c = 1/137.0359896$.

Used subroutines : All subroutines except FLGLCH, DCHECK1,DCHECK2, and 8 supporting subroutines.

B.2.2 CNTRL - Control of the calculation and output

Variable name	Definition
KTRLD(9)*	Calculation options. See 4.2.1 for details.
KTLOUT(24)*	Output options. See 4.2.2 for details.
KRTYPE	= 21; for (d,2p) reaction, = 31: for (He,t) reaction

Used subroutines : All subroutines except DISWAVE, XLMCAL, and 8 supporting subroutines.

B.2.3 SPSTAT - Particle-hole states in the target

Variable name	Definition
EET*	Transferred energy
SPECTA(LPH)	Spectroscopic factor for ph pair. Presently set 1.0. (DCP2)
ESP(LPH)*	Binding energy of particle state.
ESH(LPH)*	Binding energy of hole state.
NSP(LPH)*	Number of nodes of particle state.
LSP(LPH)*	Orbital angular momentum of particle state.
JTWP(LPH)*	Twice of total angular momentum of particle state.
NSH(LPH)*	Number of nodes of particle state.
LSH(LPH)*	Orbital angular momentum of particle state.
JTWH(LPH)*	Twice of total angular momentum of particle state.
IPAIR	Dummy
NOSP*	Number of particle states.
NOSH*	Number of hole states.
NPST(LPH)	Particle state number of each ph pair. (DCP2)
NHST(LPH)	Hole state number of each ph pair. (DCP2)
NPAIRD	Number of ph pair. (DCP2)
L1R(8)*	Orbital angular momentum, ℓ_1 .
ISR(8)*	Spin, $s_1 (= s_2 = s_t)$.
ITR(8)*	Isospin, $t_1 = t_2$.
NLSMAX*	Number of $\{\ell_1 s_1 t_1\}$ sets in the target system.

Continued

Variable name	Definition
JT*	Transferred total angular momentum, j_t .
MJMAX*	Maximum magnetic quantum number of j_t , m_{j_t} .
KPARIT*	= 0; No parity change in the reaction, = 1; Parity change.
IST*	Transferred spin, $s_t (= s_1 = s_2)$.
NOLTR*	Maximum number of transferred orbital angular momenta.
N1	Dummy
LTR(8)*	Transferred orbital angular momenta, ℓ_t .
ITP(LPH)*	Isospin T of particle state. (Not used.)
ITH(LPH)*	Isospin T of hole state.
ITZP(LPH)*	= 1; neutron, = -1; proton of particle state.
ITZH(LPH)*	= 1; neutron, = -1; proton of hole state.

Used subroutines : DCP2, DCHECK1, DCHECK2, AFACAL, FFCALD, PACALD, FFCALDM, XLMCAL, FFCAL, TRECAL, DENST, GFAC, TENSOR, PACALE, FFCALM, TRECALM, DENSTM, GFACM, CROSS.

B.2.4 BSX, UNCPA, CBST - Bound state wave functions

B.2.4.1 COMMON block BSX

Variable name	Definition
URSAVE(500)	Calculated single particle wave function $R_p(r)$. Max. of mesh points=500. After calculation, URSAVE transfers to USAVP (particle state) and USAVH (hole), saves on another COMMON block CBST
ENEPT	Calculated binding energy (KTRL2=0).
VNEPT	Calculated potential depth (KTRL2=1). KTRL2=1 is set in the program. (Initial value is input as VSX (See 4.2.11.))
VSOR*	Spin-orbit depth parameter.
DFNR*	Diffuseness parameter.
DFNSO*	Spin-orbit diffuseness parameter.
RZR*	Reduced radius parameter.
RXSO*	Spin-orbit reduced radius parameter.
RZC*	Coulomb reduced radius parameter.
XMESH2	Mesh size. XMESH2=0.1 is set.
NODER	Number of nodal points except the origin.
LBTR	Orbital angular momentum.
JBTRTW	Twice of total angular momentum.
NXRAWF	Max. mesh point where wave function calculation is terminated. $R_{max} = R_C + 10.0 * a_C$ is set.
KTRL4	= 0; wave function itself stored in URSAVE. KTRL4=0 is set. = 1; wave function * $V(r)$. = 2; wave function * $r^{-\ell}$. = 3; wave function * $v(r) * r^{-\ell}$.
KOPOT	Output control for calculations. KOPOT=0 is set. (No output)

Used subroutines : DCP2, BSAXON, UNCPST.

B.2.4.2 COMMON block UNCPSA

UNCPSA is only used for communicating between BSAXON and UNSPST.

Variable name	Definition
KTRL3	= 1; $R=RZ*(A1**(1/3)+A2**(1/3))$
ITBEMX	Number of iteration. ITBEMX=15 is set. (DCP2)
ACURCY	Accuracy need for matching. ACURCY=0.001 is set. (DCP2)
AMUPMU	= 0; Atomic mass unit, =1, proton mass unit. AMUPMU=0.0 is set.
KTRL2	= 0; B.E. is searched, = 1; VSX is searched. KTRL2=1 is set. (DCP2)
KTRL8	Dummy
KEX2	With nonzero, NXCPL2=KEX2.
KEX4	With nonzero, NXRA=KEX4+NXCPL2.
KEX40	=0; Integration of the internal solution outwards to be matched at R_m . =1; Integration of the external solution inwards.
KEX41	=1; Search has been achieved.
KEX42	Calculated number of the nodal point minus that of previous calculation. Repeat searching until KEX42=0.
KEX43	Dummy
TMAS*	Mass of target system. (Target mass -1)
PMAS*	Mass of valence nucleon.
ZZT*	Charge of target system.
ZZP*	Charge of valence nucleon.
RMAS	Reduced mass =(TMAS*PMAS)/(TMAS+PMAS).
ZZ	=ZZT*ZZP.
XMES1	Mesh size for first 8 points in the Störmer method. =XMESH2/8 is set.
PERCNT	Ratio of searched step and searched value = 0.2 is set.
VSX*	Depth parameter of the central bound state potential.
ISTW	Twice of spin of valence nucleon. =1 is set.
NXRA	Total mesh points for the calculation. =NXRAWF+KEX2, or =NXRAWF+30 if KEX2=0.
NXRM	Matching mesh point. =(XBAR+.5*DFNR*FLOAT(NODE))/XMES2.
NXRMP1	=NXRM+1.
NXRMP2	=NXRM+2.
NXRMP3	=NXRM+3.
NXRMP4	=NXRM+4.
NXRMP5	=NXRM+5.
NODE	Number of present nodal point during the search.
KGES	Solve three times near a guessed value, $(a - \delta a, a, a + \delta a)$ Do loop number.
EGESRD	Dummy
EGES	Given binding energy =ESP(LPH) or ESH(LPH) which are input.
EGEST	Initial binding energy. Starts with EGES.
DELGES	δ_a in KGES explanation.
FKAPPA	Wave number outside the well. $k = \sqrt{2\mu E}/\hbar c$.
FKAPIN	Wave number inside the well. $k = \sqrt{2\mu(V_0 - E)}/\hbar c$.
URRMIN	Value of internal solution at matching point.
URRMEX	Value of external solution at matching point.
RGDLIN(3)	Slope of internal solution at matching point.
RGDLEX(3)	Slope of external solution at matching point.

Continued

Variable name	Definition
XMEM(514)	Radial distance. 14 more points are need for the Stömer method.
VCENTR(514)	Central potential.
VSPIN(514)	Spin-orbit potential.
VCOULM(514)	Coulomb potential.
PFORM1(2)	PFORM1(1)= $\exp(x - R_0)/a_0$, PFORM1(2)= $\exp(x - R_{SO})/a_{SO}$.
PFORM2(2)	PFORM2(N)= $1.0 / (1.0 + \text{PFORM1}(N))$.
PFORM3(2)	PFORM3(N)= $\text{PFORM1}(N) * \text{PFORM2}(N) * \text{PFORM2}(N)$.

Used subroutines : DCP2, BSAXON, UNCPST.

B.2.4.3 COMMON block CBST

Variable name	Definition
USAVP(NPS*NXA)	Final particle state functions $\phi_p(r_1)$, in the linearized form for NPS states.
USAVH(NHS*NXA)	Final hole state functions $\phi_h(r_1)$, in the linearized form for NHS states.

Used subroutines :DCP2, FFCALD, FFCALDM, TRECAL, DENST, FFCALEM, TRECALM, DENSTM.

B.2.5 DWCC, POTCC, COUCC, RELKIN, OPTO, DISW - Distorted waves

B.2.5.1 COMMON block DWCC

The variables with dimension 4, say A(4) has following meaning;

Variable	Meaning
A(1)	Variable in the incident channel.
A(2)	Variable in the exit channel.
A(3)	Variable in the target system. Not used.
A(4)	Variable in the projectile system. Not used.

Variable name	Definition
LDWMIR(4)*	Starting partial wave.
LDWMXR(4)*	Ending partial wave.
WN(4)	Wave number.
LDWSTR(4)*	Step of partial wave.
CE(4)	$\eta = \mu Z_1 Z_2 e^2 / \hbar^2 k$
CFUNIR(4)	Dummy
ECM(4)	Energy in the center of mass system.
XMESR(4)*	Mesh size
NXMXR(4)*	Maximum (ending) mesh point.
NXMIR(4)*	Minimum (starting) mesh point.
NOEXP	Dummy
MXSTEP	Dummy

Continued

Variable name	Definition
TMASA	Mass of the target, =TMI*.
TMASB	Mass of the residual nucleus, =TMF*.
TMASX	Dummy
TMASY	Dummy
PMASA	Mass of the projectile, =PMI*.
PMASB	Mass of the ejectile, =PMF*.
PMASX	Dummy
PMASY	Dummy
TZA	Charge of the target, =Z1I*.
TZB	Charge of the residual nucleus, =Z1F*.
TZX	Dummy
TZY	Dummy
PZA	Charge of the projectile, =Z2I*.
PZB	Charge of the ejectile, =Z2F*.
PZX	Dummy
PZY	Dummy
XBARD(4)	Radius parameter in the real optical potential.
XBARID(4)	Radius parameter in the imaginary optical potential.

Used subroutines : DCP2, DCHECK1, DCHECK2, OPT, POTEN, AFACAL, FFCALD, PACALD, PDENST, FFCALDM, XLMCAL, FFCALF, TRECAL, DENST, GFAC, TENSOR, PACALE, FFCALFEM, TRECALM, DENSTM, GFACM, CROSS.

B.2.5.2 COMMON block POTCC

The variables with dimension 4, say A(4) has the same meaning in DWCC.

Variable name	Definition
VD(4)*	Depth parameter of real potential in MeV.
WD(4)*	Depth parameter of imaginary potential.
WSD(4)*	Depth parameter of imaginary surface potential.
ARD(4)*	Diffuseness parameter of real potential in fm.
AID(4)*	Diffuseness parameter of imaginary potential.
AISD(4)*	Diffuseness parameter of imaginary surface potential.
RZRD(4)*	Reduced radius parameter of real potential in fm.
RZID(4)*	Reduced radius parameter of imaginary potential.
RZISD(4)*	Reduced radius parameter of imaginary surface potential.
RZCD(4)*	Reduced radius parameter of Coulomb potential.
IDCH(4)	=0; $R_0 = r_0 A_T^{1/3}$, =1; $R_0 = r_0 (A_T^{1/3} + A_P^{1/3})$, Presently zero is set.
VRIT(900)	Optical model potential in $r, U_0(r)$
NXMN	Minimum range of VRIT. =1 is set.
NXMX	Maximum range of VRIT. =NXMXR(ID)+2.

Used subroutines : DCP2, OPT, POTEN, DISWAVE.

B.2.5.3 COMMON block COUCC

Variable name	Definition
FC(130)	Regular Coulomb wave function, $F_\ell(r)$.
GC(130)	Irregular Coulomb wave function, $G_\ell(r)$.
FCP(130)	Derivative of regular Coulomb wave function, $F'_\ell(r)$.
GCP(130)	Derivative of irregular Coulomb wave function, $G'_\ell(r)$.
L5	Maximum ℓ .
L6	Dummy
ETA	$\eta(= Z_T Z_P e^2 / \hbar^2 k)$ -value.
SIGMAZ	S-wave Coulomb phase shift.
RD	kr .
Z	kR_m where R_m is the matching radius.
KTOUT7	Nonzero value prints Coulomb functions.
K9	Dummy
STP	# of steps in Runge-Kutta integration. =100 is set.
ACCR	Accuracy parameter of continued fractions. = 10^{-6} is set.

Used subroutines : OPT, FLGLCH, DISWAVE.

B.2.5.4 COMMON block RELKIN

Variable name	Definition
RMASA	Reduced mass in the incident channel in the relativistic kinematics.
RMASB	Reduced mass in the exit channel.
TREL1	Kinetic energy in the incident channel.
TREL2	Kinetic energy in the exit channel.
OMEGR(20)	energy

Used subroutines : DCP2, CROSS.

B.2.5.5 COMMON block OPTO

Variable name	Definition
FOLD(NXA)	Folding potential for the optical model potential. Calculated in DCP2 if KTRLD(3)=1.

Used subroutines : DCP2, POTEN.

B.2.5.6 COMMON block DISW

Variable name	Definition
DISWA(LXA*NXA)	Distorted waves in the incident channel, $\chi_{\ell_a}(r_a)$ in the linearized form.
DISWB(LXB*NXB)	Distorted waves in the exit channel, $\chi_{\ell_b}(r_b)$.

Used subroutines : DCP2, OPT, PACALD, XLMCAL, PACALE.

B.2.6 ANGCC - Angle information

Variable name	Definition
THEB*	Starting angle in the differential cross section calculations.
THEBMX*	Ending angle.
DTHEB*	Step in angle.
THMIN*	Starting angle in elastic cross section calculations.
THMAX*	Ending angle.
THIND*	step in angle.

Used subroutines : DCP2, OPT, XLMCAL, PACALD, CROSS.

B.2.7 SPECFC - α -coefficients

Variable name	Definition
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Used subroutines : AFACAL, FFCALD, FFCALDM, FFCAL, TENSOR, FFCALM.

B.2.8 FFCC, CDENS CCKF - Preparation of form factor calculations

B.2.8.1 COMMON block FFCC

Variable name	Definition
VRANG(6)*	6 different range parameters in the effective NN interaction.
VSTR(12,6)*	Strength parameters of Love-Franey interaction. See A.4.2.10 for details.
VV(NIN,12)	NN effective interaction of [Eq.(16)], $V_{t_1s_1k}^i(r)$. NIN= r , # of $(t_1s_1k) \times 2 = 12$.
XMESH	Mesh size of the ff calculation. =XMESH* or XMESHE*.
XMESH*	Mesh size in the direct ff calculation, Δr^D .
XMESHE*	Mesh size in the exchange ff calculation, Δr^E .
RHED(NIN)	Radial points where NN effective interaction is calculated. (DCP2)
NBCMI*	Minimum mesh point in the form factor.
NBCMX*	Maximum mesh point.
NONB	# of real mesh points of r_b in the ff calculation. =(NBCMX-NBCMI)/NBSTEP+1.
NBSTEP	Interpolation step in r_b .
LASTEP	Interpolation step in ℓ_a . =LDWSTR(1)*
LBSTEP	Interpolation step in ℓ_b . =LDWSTR(2)*
MXMAX	Max. value of m_{ℓ_t} .
MP1MAX	= Min(LTR(NOLTR),MXMAX)+1.
NOLA	# of real ℓ_a in the ff calculation. =(LAP1MX-LAP1MI)/LASTEP+1.
NOLB	# of real ℓ_b in the ff calculation. =(LBP1MX-LBP1MI)/LBSTEP+1.
INTRAN	Interaction range of $V_{t_1s_1k}^i(r)$. =NHDMX*, or NHEMX*.
KCETN(2)*	KCETN(1)= 0; Central exchange ff is calculated. =1; Not considered. KCETN(2)= 0; Tensor exchange ff is calculated. =1; Not considered.

Continued

Variable name	Definition
NONA	# of real mesh points of r_a in the ff calculation.
NONAH	Half of NONA. NONA and NONAH are defined in Sec. 4 of DCP2 (main) in the code.
NONAR*	# of mesh point in r_a integration for exchange ff. (Integration is made only around $r_a = r_b$ with a range NONAR.)
NASTEP*	Mesh step in the incident channel radius, r_a .
NHDMX*	Interaction range in the direct form factor calculation, r^D .
NHEMX*	Interaction range in the exchange form factor calculation, r^E .
NGAUSR*	# of gaussian integration points.
NBSTPD*	Mesh step in the direct form factor.
NBSTPE*	Mesh step in the exchange form factor.
N1STEP*	Mesh step in the target system radius, r_1 . (For (p,n) or (p,p'), better put all steps unity.)
JATW*	Twice of the total angular momentum of the target A , I_A .
ISATW*	Twice of the spin of the projectile a , s_a .

Used subroutines : DCP2, DCHEK1, DCHECK2, EFFINT, FFCALD, PACALD, FFCALDM, XLMCAL, FFFCALE, TRECAL, DENST, GFAC, TENSOR, PACALE, FFFCALEM, TRECALM, DENSTM, GFACM, CROSS.

B.2.8.2 COMMON block CDENS - Projectile density.

Variable name	Definition
RHOD(NXA,NIN,LMM)	Direct projectile multipole density, $\rho_{P,\lambda_2}^D(r'_2, r)$
or	NXA= r'_2 , NIN= r , LMM= λ_2 . Calculated in PDENST.
RHOE(NXA,NIN,LMM)	Exchange projectile multipole density, $\rho_{P,\lambda_2}^E(r'_2, r)$
USAVEX(4×NXA)	Projectile wave functions, $\psi_P(r_2)$.
SPEC(4)	Spectroscopic factor for the projectile system. =1.0 is set.
DENSTY(NXA)	Projectile density function, $\rho_P(r_2)$.
LAMMXD(3)*	LAMMXD(1)=Maximum value of λ_1 in ff. LAMMXD(2)=Maximum value of λ_2 in ff. LAMMXD(3)=Maximum value of λ in ff.
LBXD(4)	Orbital ℓ_2 for the composite projectile system . (In case of $m_P > 4$, BSAXON is called for $\psi_P(r_2)$.)
NOSTX	# of bounds states in case of $m_P > 4$.

Used subroutines : DCP2, DCHEK1, DCHECK2, PDENST, FFCALD, FFCALDM, XLMCAL, FFFCALE, TRECAL, DENST, GFAC, PACALE, FFFCALEM, TRECALM, DENSTM, GFACM.

B.2.8.3 COMMON block CCKF

Variable name	Definition
	Stores t_1, s_1, ℓ_1, k , and ℓ_t values in possible combinations of $\{t_1 s_1 \ell_1 k \ell_t\}$.
L1KFD(KFM)	$\ell_1, =\text{L1R(NLS)}^*$.
ISKFD(KFM)	$s_1, =\text{ISR(NLS)}^*$.
KKFD(KFM)	$k, =0$ or 2 .
LTKFD(KFM)	$\ell_t, =\text{LTR(NOLTR)}^*$.
KFMAX	# of all possible combinations of $\{t_1 s_1 \ell_1 k \ell_t\}$.
ITKFD(KFM)	$t_1, =\text{ITR(NLS)}^*$.
NLSKFD(KFM)	Corresponding # (NLS) of $\{\ell_1 s_1 t_1\}$ set.
NLTKFD(KFM)	Corresponding # (NTS) of ℓ_t .
LTMIM	Smallest ℓ_t value.
LTMAX	Largest ℓ_t value.

Used subroutines : DCP2, DCHEK1, DCHECK2, FFCALD, PACALD, FFCALDM, XLMCAL, FFCAL, TRECAL, GFAC, FFCALM, TRECALM, GFACM, CROSS.

B.2.9 CGFACA, CGFACB, TREDEN - Exchange form factor calculations

B.2.9.1 COMMON block CGFACA

Variable name	Definition
KAMAX	Max. possible combinations of $\text{KA}=\{\text{KF}=(t_1 s_1 \ell_1 k \ell_t), \lambda \ell\}$ in [Eq.(35)].
KCMAX	Max. possible combinations of $\text{KC}=\{\text{KB}=(t_1 s_1 \ell_1 \lambda_1 \ell'_p), \lambda_2 \ell\}$ in [Eq.(33b)].
KGMAX	Max. possible combinations of $\{\text{KC}, \lambda\}$ in [Eq.(33b)].
JLSMX	Max. possible number of $\{t_1 s_1 \ell_1 k \ell_t, \ell_a \ell_b\}$ in [Eq.(36)].
KBBS(10)	Stores KB value for a given $\{t_1 s_1 \ell_1\}$ set.
LAMD(KAA)	λ for a given KA set.
KBC(KCC)	Stores KB value for a given KC set.
LAM2PC(KCC)	λ_2 for a given KC set.
LPP1C(KCC)	ℓ'_p for a given KC set.
LLP1D(KCC)	$\ell + 1$ for a given KC set.
NINTC(KCC)	$3t_1 + s_1 + 1$ for a given KC set. Used for choosing correct array of the effective potential.
KAD(KGG,5)	Stores KA value for a given (KG,NLS) set. NLSMAX=5 is set.
KCD(KGG)	Stores KC value for a given KG set.
NLSD(KGG)	Stores NLS value for a given KG set.
CLEBF(KAA,5,5)	Calculated following geometrical factor in [Eq.(35)]. $\text{CLEBF(KA,M1,NLS)} = \frac{2\pi}{2\ell_1+1} \hat{\ell}(\ell_0 \lambda m \ell_1 m) \frac{2}{1+\delta_{m_0}} (-)^m$.
	Calculated in subroutine GFAC.
CLEBQ(KCC,5)	Calculated geometrical factor of partial projectile density in [Eq.(33b)]. $\text{CLEBQ(KC,M2)} = \frac{2\pi}{2\lambda_2+1} \hat{\ell}(\ell_0 \ell'_p m_2 \lambda_2 m_2) \frac{2}{1+\delta_{m_2 0}}$.
	Calculated in subroutine GFAC.
COEF(KGG)	Calculated following geometrical factor in [Eq.(33b)]. $\text{COEF(KG)} = \frac{1}{\sqrt{4\pi}} (-)^\ell \hat{\lambda}_1 \hat{\lambda}_2 (\lambda_1 0 \lambda_2 0 \lambda 0) W(\lambda_1 \lambda_2 \ell_1 \ell; \lambda \ell'_p)$ Calculated in subroutine GFAC.

Used subroutines : DCHECK2, FFCAL, TRECAL, GFAC, TENSOR, PACAL, FFCALM, TRECALM, GFACM.

B.2.9.2 COMMON block CGFACB

Variable name	Definition
JLSPMX	JLSMX for tensor part. Max. possible number of $\{t_1 s_1 \ell_1, k = 2, \ell_t, \ell_a \ell_b\}$.
JLSCMX	JLSMX for central part. Max. possible number of $\{t_1 s_1 \ell_1 = \ell_t, \ell_a \ell_b\}$.
NLTD(NLS)	Store NLT value for a given NLS. NLSMAX=5 is set.
KKD(KJL)	Store $KK = \lambda^* \text{NLSMAX} + \text{NLS}$ for a given $\{t_1 s_1 \ell_1 k \ell_t, \ell_a \ell_b\}$ set.
LAD(KJL)	ℓ_a value for a given $\{t_1 s_1 \ell_1 k \ell_t, \ell_a \ell_b\}$ set.
LBD(KJL)	ℓ_a value for a given $\{t_1 s_1 \ell_1 k \ell_t, \ell_a \ell_b\}$ set.
LYD(KJL)	=0 if $\ell_1 = \ell_t$, otherwise 1, for a given $\{t_1 s_1 \ell_1 k \ell_t, \ell_a \ell_b\}$ set.
LTD(KJL)	ℓ_t value for a given $\{t_1 s_1 \ell_1 k \ell_t, \ell_a \ell_b\}$ set.
NND(KJL)	Store $\text{LB} * \text{NOLTR} + \text{NLT}$ for a given $\{t_1 s_1 \ell_1 k \ell_t, \ell_a \ell_b\}$ set.
CLEBD(KJL,10)	Calculated the following factor in [Eq.(18)] for a given $\{t_1 s_1 \ell_1 k \ell_t, \ell_a \ell_b\}$ set. $\text{CLEBD}(\text{KJL}, \text{M1}) = i^{\ell_a - \ell_b + \pi} \hat{\ell}_a(\ell_a 0 \ell_t m_{\ell_t} \ell_b m_{\ell_t})$.
	Calculated in subroutine PACALE.
CCDD(KJL,10)	Calculated following geometrical factor in [Eq.(33b)]. $\text{CCDD}(\text{JLS}, \text{M1}) = \hat{\ell}_\beta(\ell_a m \ell_\beta 0 \ell_1 m)$ Calculated in subroutine GFAC.

Used subroutines : FFCAL, GFAC, TENSOR, PACALE.

B.2.9.3 COMMON block TREDEN

Variable name	Definition
GWT(N1X,NIN,LM2)	The following part of nonlocal target density of [Eq.(31b)]. $\frac{2\pi}{2\ell_h + 1} \sum_{m_1} \hat{\ell}'_h(\ell'_h 0 \lambda_1 m_1 \ell_h m_1) \int R_{\ell_h}(r'_1) Y_{\ell_h m_1}(\theta, 0) Y_{\lambda_1 m_1}^*(\theta', 0) d\mu'$ Argument (N1X,NIN,LM2) = $(r_1, r, \{\ell'_h, \lambda_1\})$. Calculated in subroutine DENST.
TRHO(N1X,NIN,KBB)	The nonlocal target density, $\rho_{T, \ell_1 \lambda_1 \ell_c}^E(r_1, r)$, of [Eq.(31b)]. Argument (N1X,NIN,KBB) = $(r_1, r, \text{KB} = \{t_1 s_1 \ell_1, \lambda_1 \ell_c\})$. Note that $\ell_c = \ell'_p$ in the original paper. Calculated in subroutine TRECAL.

Used subroutines : FFCAL, TRECAL, DENST.

B.2.10 CGGR, AMPD, OVDE - Form factors and Transition amplitudes

B.2.10.1 COMMON block CGGR

Variable name	Definition
GGRI(KFM,N1X)	Direct form factor, $f_{t_1 s_1 \ell_1 k \ell_t}^D(r_a)$, of [Eq.(27)]. Argument (KFM,N1X) = $(\{t_1 s_1 \ell_1 k \ell_t\}, r_a)$.
or	
GGRIE(KFM,N1X)	No-recoil exchange form factor, $f_{t_1 s_1 \ell_1 k \ell_t}^{NR}(r_b)$. Argument (KFM,N1X) = $(\{t_1 s_1 \ell_1 k \ell_t\}, r_b)$.

Used subroutines : FFCALD, PACALD, FFCAL, PACALE.

B.2.10.2 COMMON block AMPL

Variable name	Definition
AMPD(700,8)	The direct overlap integral, $O_{t_1 s_1 \ell_1 k \ell_t, \ell_a \ell_b}^D$ of [Eq.(19a)] multiplied by a geometrical factor CLEBD(KJL,M1) calculated in PACALD. Sums over $\{t_1 s_1 \ell_1 k, \ell_a\}$ are done. Argument (LBLT,M1) = $(\{\ell_b \ell_t\}, m_{\ell_t})$. LBLT=700, and M1=8 are set.
AMPE(700,8)	The same as AMPD, but for the no-recoil exchange form factors.

Used subroutines : PACALD, PACALE, TENSOR.

B.2.10.3 COMMON block OVDE

Variable name	Definition
OVDD(50,10,10)	The direct transition amplitude, $T_{t_1 s_1 \ell_1 k \ell_t m_{\ell_t}}^D$, of [Eq.(18)]. Argument (NT,LT,M1) = $(\theta_b, \ell_t, m_{\ell_t})$. LT=10, and M1=10 are set.
OVED(50,10,10)	The same as OVDD, but for the no-recoil exchange form factors.
TH(50)	Angle θ_b .
DELWN(50)	Momentum transfer corresponding the angle θ_b .

Used subroutines : PACALD, PACALE, CROSS.

B.2.11 CPWFAC, CGFACC, CXLM - Plane wave approximation for exchange form factor

B.2.11.1 COMMON block CPWFAC

Variable name	Definition
TFAC(3,10,10)	CG coefficient in the transition amplitude in [Eq.(12)]. TFAC(MT,MJT,LT) = $(s_t m_t j_t m_{j_t} \ell_t m_{\ell_t}) / (\sqrt{\frac{2}{1+\delta_{m_t,0}}})$ for a given $(s_t = 1, j_t)$ Argument (MT,MJT,LT) = (m_t, m_{j_t}, ℓ_t) .
LRP1MX*	Maximum angular momentum of $\ell_r + 1$ in the PW approximation. (By inputting LRP1MX=1, NR approximation can be obtained.)
FACNR*	Recoil factor α . 1.0 is suggested. (See Formulation 3.3.)

Used subroutines : DCP2, DCHECK2, FFCALDM, FFCALEM, GFACM.

B.2.11.2 COMMON block CGFACC

Variable name	Definition
KADD(KGG)	Stores KA value for a given KG set for the modified version.
CCDDD(KJL)	Calculated following geometrical factor for the modified version. CCDDD(JLS) = $i^{\pi-\ell_r-\ell_b} \hat{k} \hat{\lambda}(k0\lambda0 \ell_r 0) \hat{\ell}_1 \hat{\ell}_r W(\ell_b \lambda \ell_t k : \ell_1 \ell_r)$ Calculated in subroutine GFACM.

Used subroutines : FFCALEM, GFACM.

B.2.11.3 COMMON block CXLM

Variable name	Definition
XLMRI(LTL,LM1,NXA)	The distortion factor, $I_{\ell_t m_{\ell_t}}(r_a)$, defined as $\frac{(4\pi)^{3/2}}{k_a k_b} \sum_{\ell_a \ell_b} i^{\ell_a - \ell_b + \pi} \hat{\ell}_a(\ell_a) 0\ell_t m_{\ell_t} \ell_b m_{\ell_t} \rangle d_{\ell_a \ell_t \ell_b} \chi_{\ell_b}(r_a) \chi_{\ell_a}(r_a).$ Argument (LTL,LM1,NXA) = $(\ell_t, m_{\ell_t}, r_a)$.
LLP1MX	LLP1MX=L1R(NLSMAX)+LAMMXD(1)+1.

Used subroutines : DCHECK2, FFCALDM, XLMCAL, FFCALEM, GFACM.

B.3 List of Key variables

Variable	Function	Argument notation	Current dimension	Calculated subroutine	Stored common
USAVP(NP*N1)	$R_{\ell_p}(r_1)$	$(r_1 \times \# \text{ of } \ell_p)$	NPS*N1X	BSAXON	CBST
USAVH(NH*N1)	$R_{\ell_h}(r_1)$	$(r_1 \times \# \text{ of } \ell_h)$	NHS*N1X	BSAXON	CBST
DISWA(LA*NA)	$\chi_{\ell_a}(r_a)$	$(r_a \times \# \text{ of } \ell_a)$	LXA*NXA	OPT	DISW
DISWB(LB*NB)	$\chi_{\ell_b}(r_b)$	$(r_b \times \# \text{ of } \ell_b)$	LXB*NXB	OPT	DISW
ALPHA(NLSK,NT)	$\alpha_{t_1 s_1 \ell_1 k \ell_t}^{j_t s_t \nu_1}$	(κ, ℓ_t) $\kappa = \{t_1 s_1 \ell_1 k\}$	(30,6)	AFACAL	SPECFC
VV(NI,12)	$V_{t_1 s_1 k}^i(r)$	$(r, \{t_1 s_1 k\})$	(NIN,12)	EFFINT(I) I=0, Dir.	FFCC I=1, Ex.
DENTY(NA)	$\rho_P(r_2)$	r_2	NXA	DCP2	CDENS
RHOD(NX,NI,LM)	$\rho_{P,\lambda_2}^D(r'_2, r)$	(r'_2, r, λ_2)	(NXA,NIN,LMM)	PDENST(0) 0; Dir.	CDENS
RHOT(NH,LM)	$\rho_{P,\lambda_2 \ell_t \ell_1}^D(r_a, r_1, r)$ (fixed $r_a r_1; \ell_t \ell_1$)	(r, λ_2)	(NIN,LMM)	FFCALD	Dim.
RHOE(NX,NI,LM)	$\rho_{P,\lambda_2}^E(r'_2, r)$	(r'_2, r, λ_2)	(NXA,NIN,LMM)	PDENST(1) 1; Ex.	CDENS
RHOT(NH,LM)	$\rho_{P,\lambda_2 \ell_t \ell_1}^E(r_b, r_1, r)$ (fixed $r_b r_1; \ell_t \ell_1$)	(r, λ_2)	(NIN,LMM)	FFCALE	Dim.
TRHO(N1,NL)	$\rho_{T,\ell_1}^D(r_1)$	(r_1, ℓ_1)	(N1X,5)	FFCALD	Dim.
TRHO(N1,NI,KB)	$\rho_{T,\ell_1 \lambda_1 \ell_c}^E(r_1, r)$	(r_1, r, β) $\beta = \{\ell_1 \lambda_1 \ell_c\}$	(N1X,NIN,KBB)	TRECAL	TREDEN
GGRI(KF,NA)	$f_{\gamma}^D(r_a)$ $\gamma = \{t_1 s_1 \ell_1 k \ell_t\}$	(γ, r_a)	(KFM,NXA)	FFCALD	CGGR
GGRIE(KF,NA)	$f_{\gamma}^{NR}(r_a)$	(γ, r_a)	(KFM,NXA)	FFCALE	CGGR
GGRI(KA,NH,NL)	$G_{t_1 s_1 \ell_1 \ell_b \lambda}^k(r_b, r)$ (fixed r_b)	(α, r, η) $\alpha = \{\ell_b \lambda\}$ $\eta = \{t_1 s_1 \ell_1\}$	(KAA,NOU,L1X)	FFCALE	Dim.
FFRIT(JLS)	$f_{\gamma, \ell_b \ell_a}^E(r_b, r_a)$	$\{\gamma, \ell_b \ell_a\}$ (fixed r_b, r_a)	(KJL)	FFCALE	TAPE
AMPD(KF,M1)	$O_{\gamma, \ell_a \ell_b}^D$	(γ, m_{ℓ_t})	(700,8)	PACALD(1)	AMPL
AMPE(KF,M1)	$O_{\gamma, \ell_a \ell_b}^E$	(γ, m_{ℓ_t})	(700,8)	PACALE	AMPL
AMPE(KF,M1)	$O_{\gamma, \ell_a \ell_b}^{NR}$	(γ, m_{ℓ_t})	(700,8)	PACALD(2)	AMPL
OVDD(NT,LT,M1)	$T_{\ell_t m_{\ell_t}}^D(\hat{k}_b)$	$(\theta_b, \ell_t, m_{\ell_t})$	(50,10,10)	PACALD	OVED
OVED(NT,LT,M1)	$T_{\ell_t m_{\ell_t}}^E(\hat{k}_b)$	$(\theta_b, \ell_t, m_{\ell_t})$	(50,10,10)	PACALE	OVED

B.4 Variables on the TAPE

Variable	Arg.	Function	WR unit	Tape #	Write subroutine	Read subroutine	remarks
FFRIT	(JLS)	$f_{\gamma_0, \ell_b \ell_a}^E(r_b, r_a)$ (fixed r_b, r_a)	$\{\gamma_0, \ell_b \ell_a\}$ JLSCMC	13	FFCALE	TENSOR	Central Ex. ff
FFRIT	(JLS2)	$f_{\gamma_2, \ell_b \ell_a}^E(r_b, r_a)$ (fixed r_b, r_a)	$\gamma_1 = \{t_1 s_1 \ell_1, k = 0, \ell_t\}$ $\{\gamma_2, \ell_b \ell_a\}$ JLSPMX	14	FFCALE	TENSOR	Tensor Ex. ff
GGRIT	(JLS)	$f_{k=0}^E + f_{k=2}^E$ (fixed r_b, r_a)	$\gamma_2 = \{t_1 s_1 \ell_1, k = 2, \ell_t\}$ $\{\gamma_0, \ell_b \ell_a\}$	15	TENSOR	PACALE	Total Ex. ff