RESPQ Manual

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1 What is RESPQ

The computer program RESPQ calculats the isovector spin-scalar, and spin-vector response functions of nuclei as a function of the transferred momentum q and transferred energy ω, including Δ isobar degrees of freedom. It treats nuclear correlations by a continuum random phase approximation (CRPA).

The code is written in FORTRAN. This program is originated from the program coded by Izumoto in 1980, and has been extended to various directions by Ken Kawahigashi, Thomas Sams, Kimiaki Nishida, Yasushi Nakaoka, Tomotugu Wakasa and Munetake Ichimura [1]-[7]. Ichimura takes the full responsibility throughout the development.

2 What can RESPQ calculate

2.1 Calculated quantities

The program calculates

2.1.1 Response functions

(1) Isovector spin-scalar response function $R_{\rm S}(q,\omega)$
(2) Isovector spin-vector response function $R_{\rm V}(q,\omega)$
(3) Isovector spin-longitudinal response function $R_{\rm L}(q,\omega)$
(4) Isovector spin-transeverse response function $R_{\rm T}(q,\omega)$

where q and ω are the intrinsic momentum and energy transfers.

*) Exactly speaking "spin-vector response function" should be called "trace of spin-vector-spin-vector response function". See subsec.7.3

2.1.2 Sum rule values

(1) Non-energy weighted sums (NEWS) $S_{\rm S}^{(0)}(q), S_{\rm V}^{(0)}(q), S_{\rm L}^{(0)}(q), S_{\rm T}^{(0)}(q)$
(2) Energy weighted sums (EWS) $S_{\rm S}^{(1)}(q), S_{\rm V}^{(1)}(q), S_{\rm L}^{(1)}(q), S_{\rm T}^{(1)}(q)$

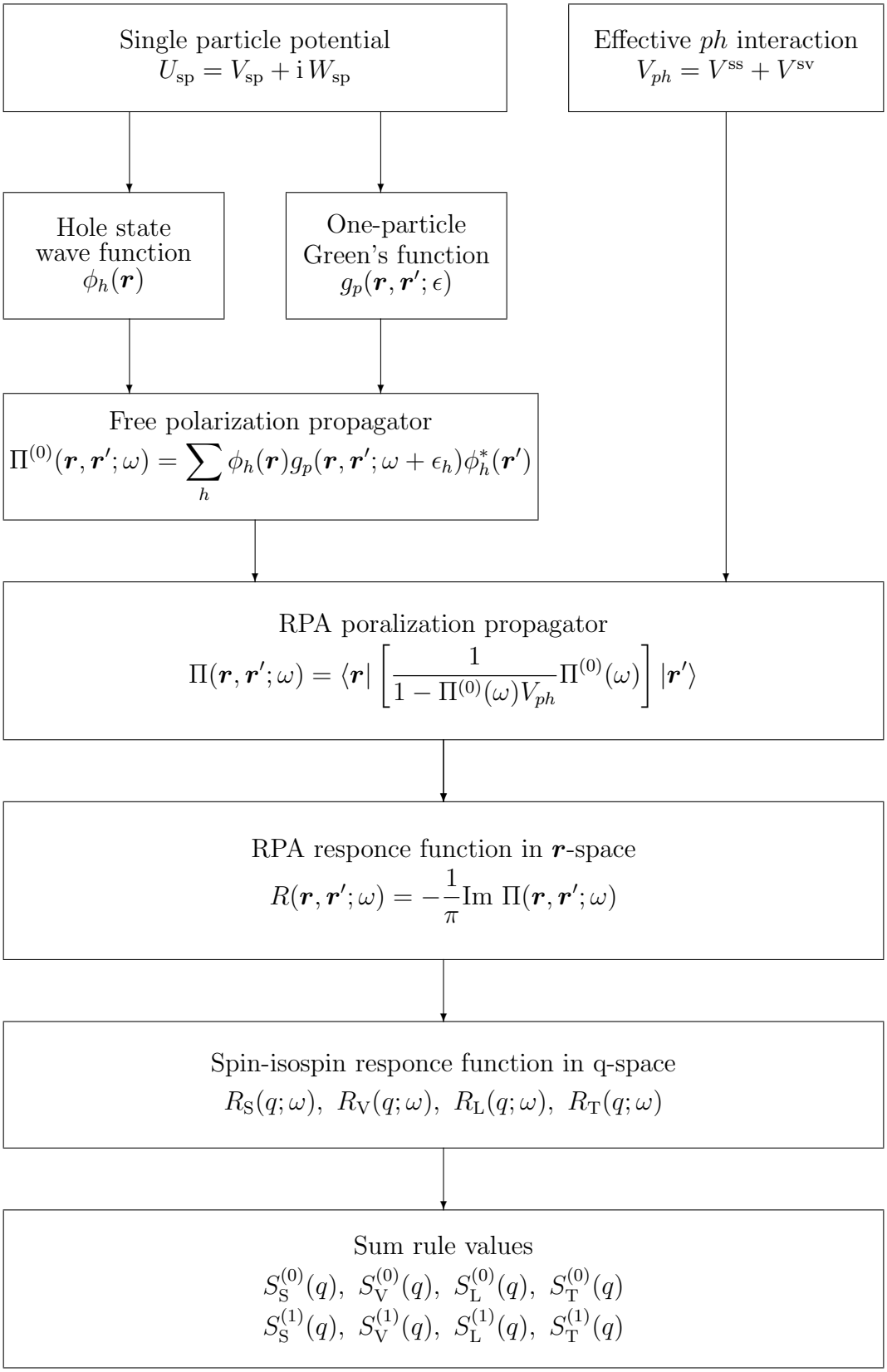
2.2 Theories

RESPQ utilizes following theoretical frameworks

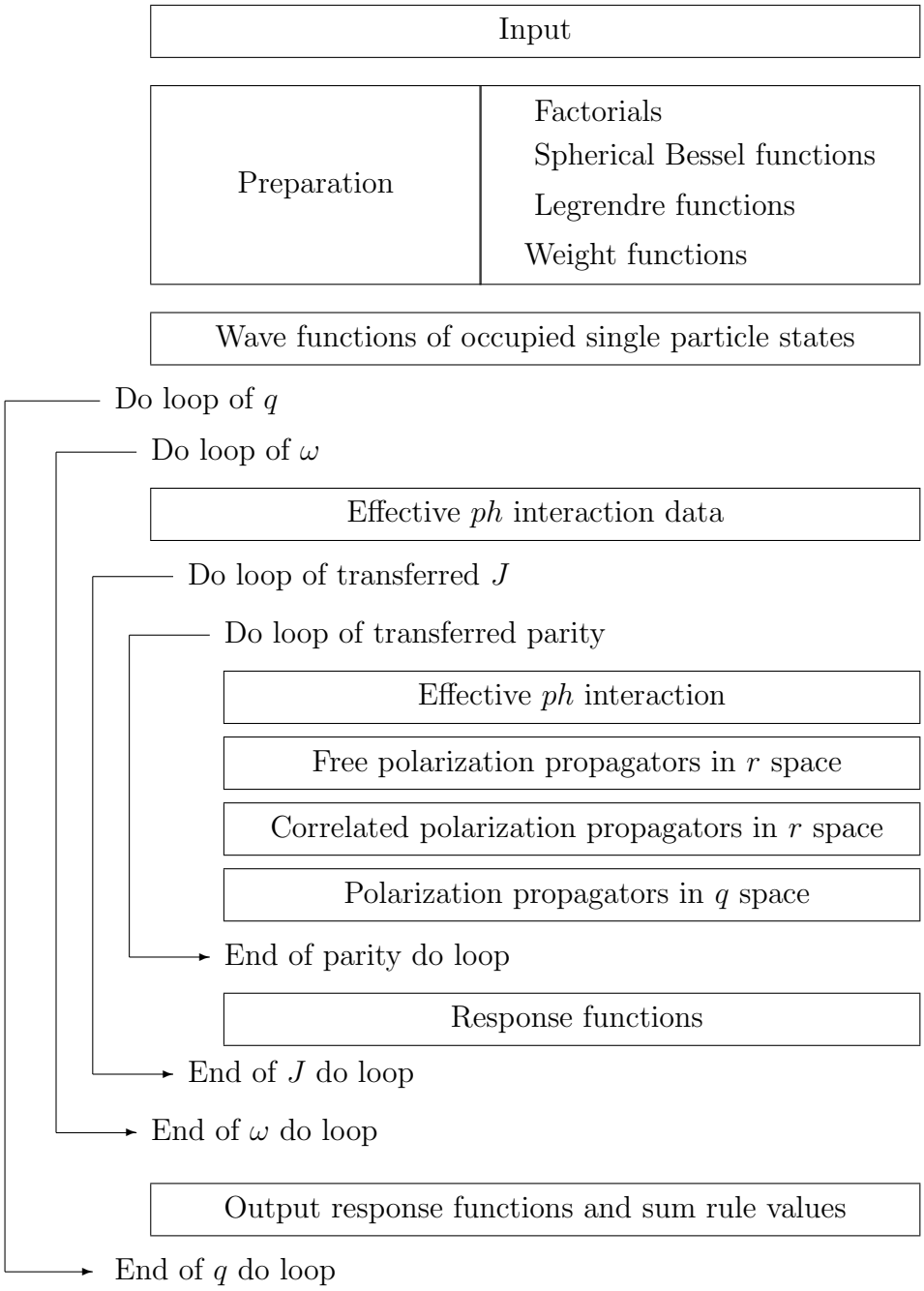
(1) A single particle model in a (complex) Woods-Saxon potential with effective masses and spreading widths
(2) A continuum RPA and TDA with the orthogonality condition for nuclear correlations in the isovector channels
(3) The f' Landau-Migdal interaction for the isovector spin-scalar channel
(4) The $\pi + \rho + g' + h'$ interaction for the isovector spin-vector channel
(5) The Δ degrees of freedom can be included.

3 Structure of Program

3.1 Outline of calculation



3.2 Flow of the program



Detaile of the program strucure with subprogram names is given in Appendix A1

3.3 Compilation and run

An example to compile and run RESPQ by g77 on Linux

A. **Compile**

Execute the command "make" to work Makefile in the directory /src,

```
# Makefile for RESPQ
#
 FC          =  g77
 FFLAGS      =  -O2 -fno-automatic -finit-local-zero
 INCLUDE     =  ../include
 RM          =  rm
 OBJS        =  bound.o cleb.o clnqgj.o couplesv.o coupless.o cxbd.o cxfh.o \
                fxpotR.o gqq.o input0.o lgauss.o lgdr.o lgsbsl.o \
                main.o nonrpa.o ocmg.o phspec.o rac.o rsrtrl.o sbessl.o \
                slfzr.o spwd.o stfclg.o u9.o veffsv.o \
                wbound.o wconv.o weight.o write0.o wrsrtrl.o weffq.o \
.f.o:
        ${FC} ${FFLAGS} -I ${INCLUDE}  -c $ <
main: ${OBJS}
          ${FC} ${FFLAGS} -o $@ ${OBJS}
all: main
clean:
        ${RM} main *.o
```

B. **Run**

Link the unit 1 to the input data file, and run!

Shellscript example

```
# !  /bin/csh
ln -s  'input data file name'          fort.1
./main > 'output file name'
\rm fort.1
```

4 How to make input data

4.1 Outline of input form

4.1.1 Structure

line 1	Date, Run-number
line 2	Title
line 3	Calculation and output options
line 4 and below	Page style input : Data with page and subpage
last line	Blank: end of input data

4.1.2 Contents of data with page and subpage

page	subpage	content
1	-	Mass number, Charge, Transferred charge
2	-	Momentum transfer, Energy transfer
3	-	Transferred angular momentum and parity
4	-	Data for integration
5	1-5	Single particle potential parameters (real part)
6	1-5	Single particle potential parameters (imaginary part)
7	1-4	Effective mass and spreading width parameters
8	1,2	Number of single particle states, followed by table of state specification (nlj)
9	0-5	Effective ph interaction parameters

4.2 Detaild description of input data

4.2.1 Date, run number and title

Input format

read(1,'(2I2, I4, I2)') (numrun(i), i=1,4)
read(1,'(20a4)') ititle

numrun(1)	Month
numrun(2)	Day
numrun(3)	Year
numrun(4)	Run number
ititle	Title within 80 characters

4.2.2 Calculation and output options

Input format
READ(1,'(10I1)') KTOPTN

Variable Name	Input Variable	Value	Explanation
KTRPA	KTOPTN(1)	0	0th
		1	TDA
		2	RPA
		3	RPA for p-h, TDA for Δ-h
KTND	KTOPTN(2)	0	without Δ
		1	with Δ
KTRQCV	KTOPTN(3)	0	without convergence check
		1	with convergence check for $R_{\rm S}$ and $R_{\rm V}$
		2	Convergence check for $R_{\rm S}$
		3	Convergence check for $R_{\rm V}$
KTVEFF	KTOPTN(4)	0	Only LM term for effective ph interaction
		1	LM + OBEP for the effective ph interaction
KTHSPWD	KTOPTN(5)	0	without spreading widths of holes
		1	with spreading widths of holes
KTOUT_VEF	KTOPTN(6)	0	No output of the effective *ph* interaction
		1	Output the effective *ph* interaction
KTOUT_BDW	KTOPTN(7)	0	No output of occupied state wave functions
		N	Output occupied state wave func. in step of N

4.2.3 Page style input

Input format
READ(1,'(8f10.5)') (a(i), i=1,8)
page I=a(1), subpage ICH =(a(1)-I)*10

Explanations below are given in the form of
1st column : Internal variable name
2nd column : Input variable name
3rd column : Explanation
4th column : Notation in the formalism.

PAGE 1 Mass number and charge of the initial nuleus, and transferred charge

1.0	a(1)	Page	
Atg	a(2)	Mass number	A
Ztg	a(3)	Charge	Z
itrz	a(4)	Transferred charge	$Z_{\rm tr}$

PAGE 2 Transferred intrinsic momenta and energies

2.0	a(1)	Page	
nqmax	a(2)	Number of transferred momenta q	$N_q^{\max}$
q_int_min	a(3)	Minimum of q (fm^{-1})	$q_{\min}$
dq_int	a(4)	Increment of q (fm^{-1})	Δq
nwmax	a(5)	Number of transferred energy ω ($\leq$ nwdm)	$N_\omega^{\max}$
w_int_min	a(6)	minimum of ω (MeV)	$\omega_{\min}$
dw_int	a(7)	increment of ω (MeV)	$\Delta\omega$

*) nwdm is defined in include/dimm.fh/ nwdm=1001 in the present version

PAGE 3 Transferred angular momenta J, transferred parities $\Delta\pi$, etc.

3.0	a(1)	Page	
itrjmn	a(2)	Minimum transferred total angular momentum	$J_{\min}$
itrjmx	a(3)	Maximum transferred total angular momentum	$J_{\max}$
knmin	a(4)	Minimum parity index kn*	
knmax	a(5)	Maximum parity index kn	
KTLTR	a(6)	Orbital angular momentum transfer ($L_{\rm tr}$) option = 0: Standard = 1: Only lowest $L_{\rm tr}$ for the given J and $\Delta\pi$.	
EPSJ	a(7)	Criterion for the convergence check of the response functions with respect to J. If a(7)=0, EPSJ=0.005 (default)	

*) kn=1: natural parity excitation, kn=2: unnatural parity excitation

PAGE 4 Integration parameters

4.0	a(1)	Page	
rmax	a(2)	Maximum radius, r (fm)	$r_{\max}$
nsrmax	a(3)	Number of crude mesh* points	$N_{sr}^{\max}$
tkdmx	a(4)	Maximum momentum (k) for Fourier transformation (fm^{-1})	$k_{\max}$
nkmax	a(5)	Number of mesh points of k ($\leq$nkdm**)	$N_k^{\max}$

*) Fine mesh dr, crude mesh $dsr = r_{\max}/N_{sr}^{\max} = 2dr$
**) nsrdm and nkdm is defined in include/dimm.fh/.

PAGE 5 Single particle potential (real part)

5.ich	a(1)	Page.subpage ich=1: neutron hole, =2: proton hole, =3: neutron particle, =4: proton particle, =5: Δ	
vd(ich)	a(2)	Central potential depth (MeV). If vd=0, it is calculated from the given binding energy of the top state.	V_0^α
rrd(ich)	a(3)	Central potential radius parameter (fm)	$r_{\rm vr}^\alpha$
ard(ich)	a(4)	Central potential diffuseness parameter (fm)	$a_{\rm vr}^\alpha$
vsod(ich)	a(5)	Spin-orbit potential strength (MeV)	$V_{\rm LS}^\alpha$
rsod(ich)	a(6)	Spin-orbit potential radius paremeter (fm)	$r_{\rm so}^\alpha$
asod(ich)	a(7)	Spin-orbit potential diffuseness parameter (fm)	$a_{\rm so}^\alpha$
rcd(ich)	a(8)	Coulomb radius parameter (fm)	$r_{\rm c}^\alpha$

*) α = ich = n^{-1}, p^{-1}, n, p, or Δ

PAGE 6 Single particle potential (imaginary part)

6.ich	a(1)	Page.subpage ich =3: neutron particle, =4: proton particle, =5: Δ	
wd(ich)	a(2)	Central potential depth (MeV). If wd=999.0, calculated by the spreading width formula. If wd is less than 0.1MeV, it is set to be 0.1MeV.	W_0^α
rid(ich)	a(3)	Central potential radius parameter (fm)	r_{vi}^α
aid(ich)	a(4)	Central potential diffuseness parameter (fm)	$a_{\rm vi}^\alpha$
WV0(ich)	a(5)	Relative strength of the volume type	$W_{\rm v}^\alpha$
WS0(ich)	a(6)	Relative strength of the surface type	$W_{\rm s}^\alpha$
RVD2(ICH)	a(7)	Surface potential radius parameter (fm)	$r_{\rm si}^\alpha$
AVD2(ICH)	a(8)	Surface potential diffuseness parameter (fm)	$a_{\rm si}^\alpha$

*) α = ich = n, p, or Δ

PAGE 7 Perey factor and spreading width parameters

7.ich	a(1)	Page.subpage ich=1: neutron hole, =2: proton hole, =3: neutron particle, =4: proton particle	
reffd(ich)	a(2)	Radius parameter of Perey factor (fm)	$r_{\rm eff}^\alpha$
aeffd(ich)	a(3)	Diffuseness parameter of Perey factor (fm)	$a_{\rm eff}^\alpha$
bdifd(ich)	a(4)	Strength parameter of Perey factor	b^α
alphsw(ich)	a(5)	Spreading width strength (MeV)	$\alpha_{\rm spwd}^\alpha$
eps0sw(ich)	a(6)	Spreading width parameter 1 (MeV)	ε_0^α
eps1sw(ich)	a(7)	Spreading width parameter 2 (MeV)	ε_1^α
BEFM(ich)	a(8)	Absolute value of Fermi energy (MeV)	$\lvert\epsilon_{\rm F}^\alpha\rvert$

*) α = ich = n^{-1}, p^{-1}, n or p

PAGE 8 Occupied single particle states

8.ich	a(1)	Page.subpage ich=1: neutron hole, =2: proton hole
nspmx(ich)	a(2)	Number of occupied states

Followed by reading the occupied state information with input format

```
      do n=1, nspmx(ich)
        read(1,'(2I5,2f10.5)') nspd(n,ich), lspd(n,ich),
     1             FJSPD(N,ICH), BERSPD(N,ICH)
      end do
```

nspd(n,ich)	Nodal quantum number	n^α
lspd(n,ich)	Orbital angular momentum	l^α
FJSPD(N,ICH)	Total angular momentum	j^α
BERSPD(N,ICH)	Binding energy (MeV) BERSPD = 0: binding energy is calculated $\neq$ 0: potential depth is calculated	

*) α = ich = n^{-1}, p^{-1}

PAGE 9 Effective *ph* interaction in isovector channels

9.0	a(1)	Spin-scalar contact interaction	
Vtautau	a(2)	Isovector spin-scalar interaction parameter (MeV fm^3)	V_τ

9.1	a(1)	π, ρ parameters	
facbs	a(2)	πNN coupling constant	$f^2_{\pi\mathrm{NN}}/(4\pi)$
fratio	a(3)	πNΔ, πNN coupling constant ratio	$f_{\pi\mathrm{N}\Delta}/f_{\pi\mathrm{NN}}$
crhoo	a(4)	ρ, π coupling ratio	C_ρ

9.2	a(1)	One-pion exchange interaction parameters	
epion	a(2)	Pion mass (MeV)	m_π
	a(3)	not used	
ctpi(1)	a(4)	Cut-off mass of NN interaction (MeV)	$\Lambda_{\pi\mathrm{NN}}$
nctpi(1)	a(5)	Power of form factor of NN interaction	$n_{\pi\mathrm{NN}}$
ctpi(2)	a(6)	Cut-off mass of NΔ interaction (MeV)	$\Lambda_{\pi\mathrm{N}\Delta}$
nctpi(2)	a(7)	Power of form factor of NΔ interaction	$n_{\pi\mathrm{N}\Delta}$

9.3	a(1)	One-ρ exchange interaction parameters	
erhoo	a(2)	ρ-meson mass (MeV)	m_ρ
	a(3)	not used	
ctrh(1)	a(4)	Cut-off mass of NN interaction (MeV)	$\Lambda_{\rho\mathrm{NN}}$
nctrh(1)	a(5)	Power of form factor of NN interaction	$n_{\rho\mathrm{NN}}$
ctrh(2)	a(6)	Cut-off mass of NΔ interaction (MeV)	$\Lambda_{\rho\mathrm{N}\Delta}$
nctrh(2)	a(7)	Power of form factor of NΔ interaction	$n_{\rho\mathrm{N}\Delta}$

9.4	a(1)	g' interaction parameters	
gpr(1,1)	a(2)	g'_{NN}	g'_{NN}
gpr(1,2)	a(3)	$g'_{\mathrm{N}\Delta}$	$g'_{\mathrm{N}\Delta}$
gpr(2,2)	a(4)	$g'_{\Delta\Delta}$	$g'_{\Delta\Delta}$
ctgpr	a(5)	Cutoff mass of g' form factor (MeV)	Λ_g
nctgpr	a(6)	Power of g' form factor	n_g

9.5	a(1)	h' interaction parameters	
hpr(1,1)	a(2)	h'_{NN}	h'_{NN}
hpr(1,2)	a(3)	$h'_{\mathrm{N}\Delta}$	$h'_{\mathrm{N}\Delta}$
hpr(2,2)	a(4)	$h'_{\Delta\Delta}$	$h'_{\Delta\Delta}$
cthpr	a(5)	Cutoff mass of h' form factor (MeV)	Λ_h
ncthpr	a(6)	Power of h' form factor	n_h

5 Definition of input parameters

5.1 Single particle potential

The single particle local potentials are written as

$$U^{\alpha}(\boldsymbol{r}) = -V_0^{\alpha} F_{\rm cr}^{\alpha}(r) - iW_0^{\alpha} F_{\rm ci}^{\alpha}(r) - V_{\rm so}^{\alpha} F_{\rm so}^{\alpha}(r)\, \boldsymbol{l}\cdot\boldsymbol{\sigma}^{\alpha} + V_{\rm coul}^{\alpha}(r) \tag{1}$$

where

$$\alpha = \text{ich} = \begin{cases} 1 & n \in \text{a occupied states} \\ 2 & p \in \text{a occupied states} \\ 3 & n \in \text{a unoccupied states} \\ 4 & p \in \text{a occupied states} \\ 5 & \Delta \end{cases} \tag{2}$$

The radial form factors $F_x^{\alpha}(r)$ and the Coulomb potential $V_{\rm coul}^{\alpha}(r)$ are defined as

$$F_{\rm cr}^{\alpha}(r) = \frac{1}{1+\exp[(r - r_{\rm vr}^{\alpha} A_{\rm c}^{1/3})/a_{\rm vr}^{\alpha}]} \tag{3}$$

$$\begin{aligned} F_{\rm ci}^{\alpha}(r) &= \frac{W_{\rm v}^{\alpha}}{W_{\rm v}^{\alpha}+W_{\rm s}^{\alpha}} \frac{1}{1+\exp[(r - r_{\rm vi}^{\alpha} A_{\rm c}^{1/3})/a_{\rm vi}^{\alpha}]} \\ &+ \frac{W_{\rm s}^{\alpha}}{W_{\rm v}^{\alpha}+W_{\rm s}^{\alpha}} \frac{4\exp[(r - r_{\rm si}^{\alpha} A_{\rm c}^{1/3})/a_{\rm si}^{\alpha}]}{\left(1+\exp[(r - r_{\rm si}^{\alpha} A_{\rm c}^{1/3})/a_{\rm si}^{\alpha}]\right)^2} \end{aligned} \tag{4}$$

$$F_{\rm so}^{\alpha}(r) = \left(\frac{\hbar}{m_{\pi}c}\right)^2 \frac{1}{a_{so}^{\alpha}} \frac{1}{r} \frac{\exp[(r - r_{\rm so}^{\alpha} A_{\rm c}^{1/3})/a_{\rm so}^{\alpha}]}{\left(1+\exp[(r - r_{\rm so}^{\alpha} A_{\rm c}^{1/3})/a_{\rm so}^{\alpha}]\right)^2} \tag{5}$$

$$V_{\rm coul}^{\alpha}(r) = \begin{cases} Z_{\alpha} Z_{\rm c} \frac{e^2}{2R_c} \left(3 - \frac{r^2}{R_c^2}\right), & (r < R_c = r_c^{\alpha} A_{\rm c}^{1/3}) \\ Z_{\alpha} Z_{\rm c} \frac{e^2}{r}, & (r \geq R_c) \end{cases} \tag{6}$$

where $A_c (= A-1)$ is the mass number of the core. The present version fixes

$$\left(\frac{\hbar}{m_{\pi}c}\right)^2 = 2.0 \ [\text{fm}^2] \tag{7}$$

5.2 Perey factor

The Perey factors[15] of the nucleon are expressed as

$$P^{\alpha}(r) = 1 - \frac{b^{\alpha}}{1+\exp[(r - r_{\rm eff}^{\alpha} A_{\rm c}^{1/3})/a_{\rm eff}^{\alpha}]} \tag{8}$$

where α =ich = 1-4. For Δ we set $P^{\Delta}(r) = 1$.

5.3 Spreading width

The spreading width of the nucleon is expressed by the phenomenological formula[10]

$$\frac{\gamma^{\alpha}(\varepsilon)}{2} = \alpha_{\rm spw}^{\alpha} \left[\frac{(\varepsilon^{\alpha})^2}{(\varepsilon^{\alpha})^2 + (\varepsilon_0^{\alpha})^2}\right] \left[\frac{(\varepsilon_1^{\alpha})^2}{(\varepsilon^{\alpha})^2 + (\varepsilon_1^{\alpha})^2}\right] \tag{9}$$

with

$$\varepsilon^\alpha = \epsilon - \epsilon_{\rm F}^\alpha \tag{10}$$

where α =ich = 1-4, and $\epsilon_{\rm F}^\alpha$ is the Fermi energy

$$\epsilon_{\rm F}^\alpha = \frac{1}{2}\,(\epsilon^\alpha \text{ of the highest occupied level} + \epsilon^\alpha \text{ of the lowest unoccupied level}) \tag{11}$$

We do not use γ for Δ.

The program has an option to determine the strength of the imaginary potential by

$$W_0^\alpha(\epsilon^\alpha) = \frac{\gamma(\varepsilon^\alpha)}{2} \tag{12}$$

It also has another option to add the imaginary part to the bound state energy as

$$\epsilon_h^\alpha \longrightarrow \tilde{\epsilon}_h^\alpha = \epsilon_h^\alpha - {\rm i}\frac{\gamma(\varepsilon_h^\alpha)}{2} \tag{13}$$

5.4 Effective *ph* interactions

The effective *ph* interaction cosists of the isovector spin-scalar interaction $V^{\rm ss}$ and the isovector spin-vector interaction $V^{\rm sv}$

$$V^{ph} = V^{\rm s} + V^{\rm v} \tag{14}$$

5.4.1 Isovector spin-scalar interaction

The program uses a contact interaction

$$V_{12}^{\rm s}(\boldsymbol{r}_1 - \boldsymbol{r}_2) = V_\tau(\boldsymbol{\tau_1}\cdot\boldsymbol{\tau_2})\delta(\boldsymbol{r}_1 - \boldsymbol{r}_2) \tag{15}$$

for the isovector spin-scalar modes.

5.4.2 Isovector spin-vector interaction

For the isovector spin-vector modes, the *ph* interaction is given in the momentum representation as

$$V_{12}^{\rm v}(\boldsymbol{q},\omega) = V_{12}^{\rm L}(\boldsymbol{q},\omega) + V_{12}^{\rm T}(\boldsymbol{q},\omega)\ , \tag{16}$$

where

$$\begin{aligned} V_{12}^{\rm L}(\boldsymbol{q},\omega) &= W_{\rm L}^{\rm NN}(q,\omega)\,(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)(\boldsymbol{\sigma}_1\cdot\hat{\boldsymbol{q}})(\boldsymbol{\sigma}_2\cdot\hat{\boldsymbol{q}}) \\ &+ W_{\rm L}^{\rm N\Delta}(q,\omega)\,[\{(\boldsymbol{\tau}_1\cdot\boldsymbol{T_2})(\boldsymbol{\sigma}_1\cdot\hat{\boldsymbol{q}})(\boldsymbol{S}_2\cdot\hat{\boldsymbol{q}}) + (1\leftrightarrow 2)\} + \text{h.c.}] \\ &+ W_{\rm L}^{\Delta\Delta}(q,\omega)\,\Big[\Big\{(\boldsymbol{T}_1\cdot\boldsymbol{T}_2)(\boldsymbol{S}_1\cdot\hat{\boldsymbol{q}})(\boldsymbol{S}_2\cdot\hat{\boldsymbol{q}}) + (\boldsymbol{T}_1\cdot\boldsymbol{T}_2^\dagger)(\boldsymbol{S}_1\cdot\hat{\boldsymbol{q}})(\boldsymbol{S}_2^\dagger\cdot\hat{\boldsymbol{q}})\Big\} + \text{ h.c.}\Big] \end{aligned} \tag{17a}$$

$$\begin{aligned} V_{12}^{\rm T}(\boldsymbol{q},\omega) &= W_{\rm T}^{\rm NN}(q,\omega)\,(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)(\boldsymbol{\sigma}_1\times\hat{\boldsymbol{q}})\cdot(\boldsymbol{\sigma}_2\times\hat{\boldsymbol{q}}) \\ &+ W_{\rm T}^{\rm N\Delta}(q,\omega)\,[\{(\boldsymbol{\tau}_1\cdot\boldsymbol{T_2})(\boldsymbol{\sigma}_1\times\hat{\boldsymbol{q}})\cdot(\boldsymbol{S}_2\times\hat{\boldsymbol{q}}) + (1\leftrightarrow 2)\} + \text{h.c.}] \\ &+ W_{\rm T}^{\Delta\Delta}(q,\omega)\,\Big[\Big\{(\boldsymbol{T}_1\cdot\boldsymbol{T}_2)(\boldsymbol{S}_1\times\hat{\boldsymbol{q}})\cdot(\boldsymbol{S}_2\times\hat{\boldsymbol{q}}) + (\boldsymbol{T}_1\cdot\boldsymbol{T}_2^\dagger)(\boldsymbol{S}_1\times\hat{\boldsymbol{q}})\cdot(\boldsymbol{S}_2^\dagger\times\hat{\boldsymbol{q}})\Big\} \\ &\qquad + \text{h.c.}] \end{aligned} \tag{17b}$$

where $\boldsymbol{S}$ and $\boldsymbol{T}$ are the spin and isospin transition operators from N to Δ, respectively.

5.4.3 $\pi + \rho + g' + h'$ model

The program adopts the $\pi + \rho + g' + h'$ model, in which

$$W_{\rm L}^{\rm NN}(q,\omega) = \frac{f_{\pi{\rm NN}}^2}{m_\pi^2}\left(g'_{\rm NN}\Gamma_g(t) + 2h'_{\rm NN}\Gamma_h(t) + a_{\rm OB}\frac{q^2}{t-m_\pi^2}\Gamma_{\pi{\rm NN}}^2(t)\right) \tag{18a}$$

$$W_{\rm L}^{\rm N\Delta}(q,\omega) = \frac{f_{\pi{\rm NN}}f_{\pi{\rm N}\Delta}}{m_\pi^2}\left(g'_{\rm N\Delta}\Gamma_g(t) + 2h'_{\rm N\Delta}\Gamma_h(t) + a_{\rm OB}\frac{q^2}{t-m_\pi^2}\Gamma_{\pi{\rm NN}}(t)\Gamma_{\pi{\rm N}\Delta}(t)\right) \tag{18b}$$

$$W_{\rm L}^{\Delta\Delta}(q,\omega) = \frac{f_{\pi{\rm N}\Delta}^2}{m_\pi^2}\left(g'_{\Delta\Delta}\Gamma_g(t) + 2h'_{\Delta\Delta}\Gamma_h(t) + a_{\rm OB}\frac{q^2}{t-m_\pi^2}\Gamma_{\pi{\rm N}\Delta}^2(t)\right) \tag{18c}$$

$$W_{\rm T}^{\rm NN}(q,\omega) = \frac{f_{\pi{\rm NN}}^2}{m_\pi^2}\left(g'_{\rm NN}\Gamma_g(t) - h'_{\rm NN}\Gamma_h(t) + a_{\rm OB}C_{\rho{\rm NN}}\frac{q^2}{t-m_\rho^2}\Gamma_{\rho{\rm NN}}^2(t)\right) \tag{19a}$$

$$W_{\rm T}^{\rm N\Delta}(q,\omega) = \frac{f_{\pi{\rm NN}}f_{\pi{\rm N}\Delta}}{m_\pi^2}\left(g'_{\rm N\Delta}\Gamma_g(t) - h'_{\rm N\Delta}\Gamma_h(t) + a_{\rm OB}C_{\rho{\rm N}\Delta}\frac{q^2}{t-m_\rho^2}\Gamma_{\rho{\rm NN}}(t)\Gamma_{\rho{\rm N}\Delta}(t)\right) \tag{19b}$$

$$W_{\rm T}^{\Delta\Delta}(q,\omega) = \frac{f_{\pi{\rm N}\Delta}^2}{m_\pi^2}\left(g'_{\Delta\Delta}\Gamma_g(t) - h'_{\Delta\Delta}\Gamma_h(t) + a_{\rm OB}C_{\rho\Delta\Delta}\frac{q^2}{t-m_\rho^2}\Gamma_{\rho{\rm N}\Delta}^2(t)\right) \tag{19c}$$

where

$$t = \omega^2 - q^2, \qquad C_{\rho ab} = \frac{f_{\rho ab}^2}{m_\rho^2}\left[\frac{f_{\pi ab}^2}{m_\pi^2}\right]^{-1} \tag{20}$$

$$\Gamma_{\pi ab}(t_\pi) = \left(\frac{\Lambda_{\pi ab}^2 - m_\pi^2}{\Lambda_{\pi ab}^2 - t}\right)^{n_{\pi ab}}, \quad \Gamma_{\rho ab}(t_\rho) = \left(\frac{\Lambda_{\rho ab}^2 - m_\rho^2}{\Lambda_{\rho ab}^2 - t}\right)^{n_{\rho ab}} \tag{21}$$

$$\Gamma_g(q,\omega) = \left[\frac{\Lambda_g^2}{\Lambda_g^2 - t}\right]^{n_g}, \quad \Gamma_h(q,\omega) = \frac{q^2}{m_\pi^2}\left[\frac{\Lambda_h^2}{\Lambda_h^2 - t}\right]^{n_h}, \tag{22}$$

with $a, b =$ N or Δ. The program assumes

$$C_\rho = C_{\rho{\rm NN}} = C_{\rho{\rm N}\Delta} = C_{\rho\Delta\Delta} \tag{23}$$

5.5 Convergence check

EPSJ: The criterion for the convergence check of the response functions with respect to the transferred J. It determines the smallest $J_{\rm conv}$ which satisfies

$$\frac{(2J_{\rm conv}+1)R_{J_{\rm conv}}(q,\omega)}{\sum^{J_{\rm conv}}(2J+1)R_J(q,\omega)} < {\rm EPSJ} \tag{24}$$

6 Examples

6.1 Quasi-elastic scattering (QES)

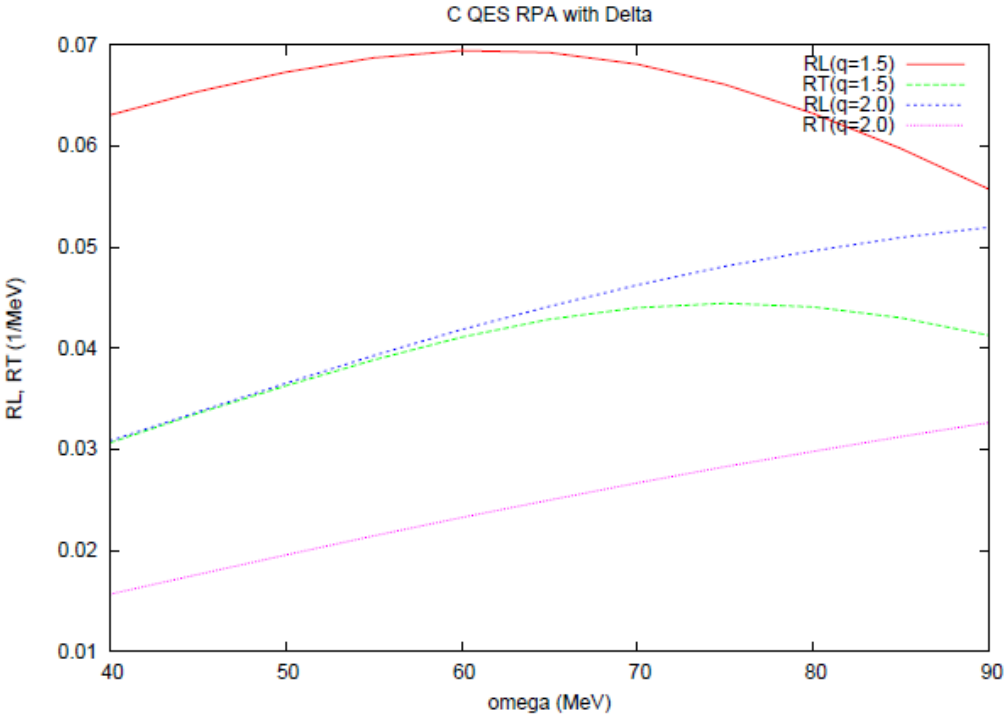
[Exsample] ^{12}C→^{12}N QES.
$q = 1.5, 2.0\text{fm}^{-1}$, ω =40-90MeV (step=5MeV).
Maximum transferred $J_{\text{tr}} = 10$. Convergence check is applied.
RPA with Δ
Mean field parameters for the nucleons
Shape parameters: from ref.[11]
Spin-orbit potential: from ref.[6]
The Perey factor parameters: from ref.[6]
Spreading widths: both for particles and holes by the emperical formula [10].
Mean field parameters for Delta: from ref.[6]
Effective *ph* interaction: V_t+LM+OBEP.
Spin-scalar parameters: from ref.[14]
OBEP parameters: from ref.[12]
LM parameters: $(g'_{\text{NN}}, g'_{\text{N}\Delta}, g'_{\Delta\Delta}) = (0.6, 0.3, 0.5)$ without form factor [6].

[Input] See the file /input/C/C-QES.dat

```
1110201701
C QES Ztr=1 q=1.5,2.0 w=40-90 Jtr< =10 Wp=spw Wh=spw RPA wDelta
21111
1.0     12.0    6.0     1.0
2.0     2.0     1.5     0.5     11.0    40.0    5.0
3.0     0.0     10.0    1.0     2.0     0.0     0.0
4.0     10.0    50.0    3.0     120.0
5.1     0.0     1.27    0.67    6.5     1.27    0.67    1.27
5.2     0.0     1.27    0.67    6.5     1.27    0.67    1.27
5.3     0.0     1.27    0.67    6.5     1.27    0.67    1.27
5.4     0.0     1.27    0.67    6.5     1.27    0.67    1.27
5.5     30.0    1.27    0.67    0.0     1.27    0.67    1.27
6.3     999.0   1.27    0.67    1.0     0.0     1.27    0.67
6.4     999.0   1.27    0.67    1.0     0.0     1.27    0.67
7.1     1.27    0.67    0.3     10.75   18.0    110.0   16.559
7.2     1.27    0.67    0.3     10.75   18.0    110.0   13.818
7.3     1.27    0.67    0.3     10.75   18.0    110.0   16.559
7.4     1.27    0.67    0.3     10.75   18.0    110.0   13.818
8.1     2.0
0       1       1.5     18.720
0       0       0.5
8.2     2.0
0       1       1.5     15.956
0       0       0.5
9.0     283.0
9.1     0.08    2.0     2.18
9.2     139.0   0.0     1300.0  1.0     1300.0  1.0
9.3     770.0   0.0     2000.0  1.0     2000.0  1.0
9.4     0.6     0.3     0.5
0.0
```

[**Output**] See the file /OUTPUT/C/C-QES.log

The calculated response functions R_{T} and R_{L} are shown in the figure below.



6.2 GT-spectrum

[Exsample 1] No correlations (0th)

^{90}Zr $\rightarrow^{90}$Nb (1^+) at $q = 0$ ω =0-20MeV (step=0.02MeV).
Mean field parameters
Shape parameters: from ref.[11]
Perey factor parameters: from ref.[6]
Imaginary potential depth of particles: 0.1MeV
Spreading width of holes: 0.0MeV

[Input] See input/Zr/Zr_GT-_0th.dat

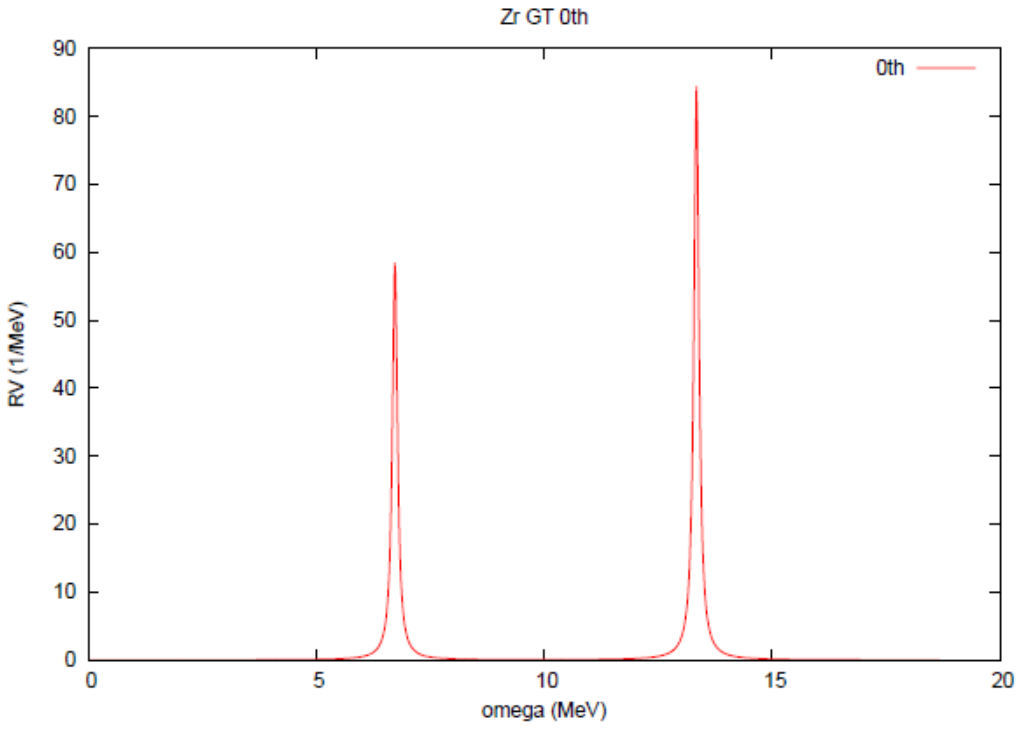
```
 1116201701
 Zr GT- q=0 w=0-20 m*=0.7 Wp=0.1 Wh=0 0th
 0000000
 1.0        90.0       40.0       1.0
 2.0        1.0        0.0        0.0        1001.0     0.0        0.02
 3.0        1.0        1.0        2.0        2.0
 4.0        16.0       80.0       3.0        120.0
 5.1        0.0        1.27       0.67       8.24       1.27       0.67       1.27
 5.2        0.0        1.27       0.67       8.24       1.27       0.67       1.27
 5.3        0.0        1.27       0.67       8.24       1.27       0.67       1.27
 5.4        0.0        1.27       0.67       8.24       1.27       0.67       1.27
 6.3        0.1        1.27       0.67       1.0        0.0        1.27       0.67
 6.4        0.1        1.27       0.67       1.0        0.0        1.27       0.67
 7.1        1.27       0.67       0.3
 7.2        1.27       0.67       0.3
 7.3        1.27       0.67       0.3
 7.4        1.27       0.67       0.3
 8.1        11.0
 0          4          4.5        11.969
 1          1          0.5
 1          1          1.5
 0          3          2.5
 0          3          3.5
 0          2          1.5
 1          0          0.5
 0          2          2.5
 0          1          0.5
 0          1          1.5
 0          0          0.5
 8.2        10.0
 1          1          0.5        8.357
 1          1          1.5
 0          3          2.5
 0          3          3.5
 0          2          1.5
 1          0          0.5
 0          2          2.5
```

(continued)

```
0          1          0.5
0          1          1.5
0          0          0.5
0.0
```

[Output] See the file /OUTPUT/Zr/Zr_GT-_0th.log

The calculated unperturbed GT response function are shown in the figure below.



[Exsample 2] RPA without Δ

^{90}Zr $\rightarrow^{90}$Nb (1^+) at $q = 0$ ω =0-30MeV (step=0.1MeV).

Mean field parameters

Shape parameters: from ref.[11]

Perey factor parameters: from ref.[6]

Imaginary potential depth of particles: calculated from the emperical formula [10]

Spreading width of holes: calculated from the emperical formula [10]

Fermi energy: experimental values

Effective *ph* interaction: LM + OBEP.

OBEP parameters: from ref.[13]

LM parameters: $g'_{\rm NN} = 0.6$ without form factor [7].

[Input] See input/Zr/Zr_GT-_RPA_noD.dat

```
 1215201701
 Zr GT- q=0 w=0-30 m*=0.7 Wp=spw Wh=spw RPA noD
 20011
 1.0        90.0       40.0       1.0
 2.0        1.0        0.0        0.0        301.0      0.0        0.1
 3.0        1.0        1.0        2.0        2.0
 4.0        16.0       80.0       3.0        120.0
 5.1        0.0        1.27       0.67       8.24       1.27       0.67       1.27
 5.2        0.0        1.27       0.67       8.24       1.27       0.67       1.27
 5.3        0.0        1.27       0.67       8.24       1.27       0.67       1.27
 5.4        0.0        1.27       0.67       8.24       1.27       0.67       1.27
 6.3        999.0      1.27       0.67       1.0        0.0        1.27       0.67
 6.4        999.0      1.27       0.67       1.0        0.0        1.27       0.67
 7.1        1.27       0.67       0.3        10.75      18.0       110.0      9.582
 7.2        1.27       0.67       0.3        10.75      18.0       110.0      6.758
 7.3        1.27       0.67       0.3        10.75      18.0       110.0      9.582
 7.4        1.27       0.67       0.3        10.75      18.0       110.0      6.758
 8.1        11.0
 0          4          4.5        11.97
 1          1          0.5
 1          1          1.5
 0          3          2.5
 0          3          3.5
 0          2          1.5
 1          0          0.5
 0          2          2.5
 0          1          0.5
 0          1          1.5
 0          0          0.5
 8.2        10.0
 1          1          0.5        8.35
 1          1          1.5
 0          3          2.5
 0          3          3.5
 0          2          1.5
```

(continued)

1	0	0.5				
0	2	2.5				
0	1	0.5				
0	1	1.5				
0	0	0.5				
9.1	0.0778	1.69706	2.94058			
9.2	138.03	0.0	1300.0	1.0	1200.0	1.0
9.3	769.0	0.0	1400.0	1.0	1400.0	2.0
9.4	0.6					
0.0						

[**Output**] See the file /OUTPUT/Zr/Zr_GT-_RPA_noD.log

[**Exsample 3**] RPA for N-space and TDA for N-Δ coupling

^{90}Zr $\rightarrow^{90}$Nb (1^+) at $q = 0$ ω =0-30MeV (step=0.1MeV).
RPA for nucleon space, while TDA for N-Δ coupling
Mean field parameters for the nucleons
Shape parameters: from ref.[11]
Perey factor parameters: from ref.[6]
Imaginary potential depth of particles: calculated from the emperical formula [10]
Spreading width of holes: calculated from the emperical formula [10]
Fermi energy: experimental values
Mean field parameters for Delta: from ref.[6]
Effective *ph* interaction: LM + OBEP.
OBEP parameters: from ref.[13]
LM parameters: $(g'_{\rm NN}, g'_{\rm N\Delta}, g'_{\Delta\Delta}) = (0.6, 0.35, 0.5)$ without form factor [7].

[**Input**] See input/Zr/Zr_GT-_RPA_wD.dat

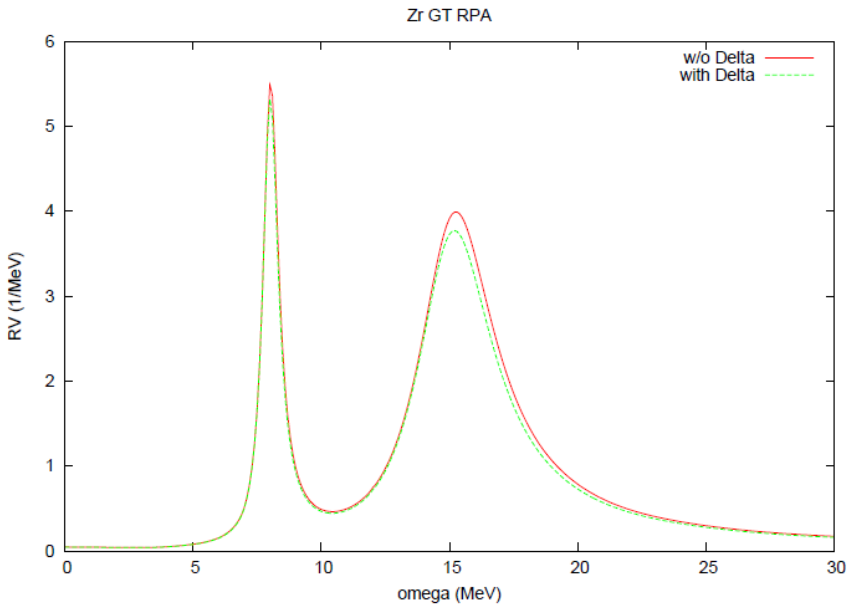
1215201701							
Zr GT- q=0 w=0-30 m*=0.7 Wp=spw Wh=spw RPA-TDA(D)							
31011							
1.0	90.0	40.0	1.0				
2.0	1.0	0.0	0.0	301.0	0.0	0.1	
3.0	1.0	1.0	2.0	2.0			
4.0	16.0	80.0	3.0	120.0			
5.1	0.0	1.27	0.67	8.24	1.27	0.67	1.27
5.2	0.0	1.27	0.67	8.24	1.27	0.67	1.27
5.3	0.0	1.27	0.67	8.24	1.27	0.67	1.27
5.4	0.0	1.27	0.67	8.24	1.27	0.67	1.27
5.5	30.0	1.27	0.67	0.0	1.27	0.67	1.27
6.3	999.0	1.27	0.67	1.0	0.0	1.27	0.67
6.4	999.9	1.27	0.67	1.0	0.0	1.27	0.67
7.1	1.27	0.67	0.3	10.75	18.0	110.0	9.582
7.2	1.27	0.67	0.3	10.75	18.0	110.0	6.758
7.3	1.27	0.67	0.3	10.75	18.0	110.0	9.582
7.4	1.27	0.67	0.3	10.75	18.0	110.0	6.758

(continued)

```
8.1         11.0
0           4           4.5         11.97
1           1           0.5
1           1           1.5
0           3           2.5
0           3           3.5
0           2           1.5
1           0           0.5
0           2           2.5
0           1           0.5
0           1           1.5
0           0           0.5
8.2         10.0
1           1           0.5         8.35
1           1           1.5
0           3           2.5
0           3           3.5
0           2           1.5
1           0           0.5
0           2           2.5
0           1           0.5
0           1           1.5
0           0           0.5
9.1         0.0778      1.69706     2.94058
9.2         138.03      0.0         1300.0      1.0         1200.0      1.0
9.3         769.0       0.0         1400.0      1.0         1400.0      2.0
9.4         0.6         0.35        0.5
0.0
```

[Output] See the file /OUTPUT/Zr/Zr_GT-_RPA_wD.log

The calculated GT response functions with and without Δ are shown in the figure below.



6.3 Spin dipole (SD) modes

[Exsample 1] ^{208}Pb $\rightarrow^{208}$Bi (1^-)

$q = 0.1\text{fm}^{-1}$, ω =10-50MeV (step=0.5MeV).
RPA in N-space while TDA for N-Δ coupling
Mean field parameters for Nucleons
Shape parameters: from ref.[11]
Spin-orbit potential: from ref.[11]
The Perey factor parameters: from ref.[14]
Spreading widths: both for particles and holes by the emperical formula [10].
Fermi energy: experimental values
Mean field parameters for Delta: from ref.[14]
Effective *ph* interaction: V_t+LM+OBEP.
Spin-scalar parameters: from ref.[14]
OBEP parameters: from ref.[13].
LM parameters: $(g'_{\text{NN}}, g'_{\text{N}\Delta}, g'_{\Delta\Delta}) = (0.64, 0.35, 0.5)$ without form factors [14].

[input] See the file /input/Pb/Pb_1-_wD.dat

```
1212201701
Pb 1- Ztr=1 q=0.1 w=10-50 m*=.7 Wp=spw Wh=spw RPA+TDA(D) Vt+LM+OBEP
31011
1.0     208.0   82.0    1.0
2.0     1.0     0.1     0.0     81.0    10.0    0.5
3.0     1.0     1.0     1.0     1.0
4.0     16.0    80.0    3.0     120.0
5.1     0.0     1.27    0.67    7.5     1.27    0.67    1.27
5.2     0.0     1.27    0.67    7.5     1.27    0.67    1.27
5.3     0.0     1.27    0.67    7.5     1.27    0.67    1.27
5.4     0.0     1.27    0.67    7.5     1.27    0.67    1.27
5.5     30.0    1.27    0.67    0.0     1.27    0.67    1.27
6.3     999.0   1.27    0.67    1.0     0.0     1.27    0.67
6.4     999.0   1.27    0.67    1.0     0.0     1.27    0.67
7.1     1.27    0.67    0.3     10.75   26.0    110.0   5.659
7.2     1.27    0.67    0.3     10.75   26.0    110.0   5.902
7.3     1.27    0.67    0.3     10.75   26.0    110.0   5.659
7.4     1.27    0.67    0.3     10.75   26.0    110.0   5.902
8.1     22.0
2       1       0.5     7.368
1       3       2.5
2       1       1.5
0       6       6.5
0       5       4.5
1       3       3.5
1       2       1.5
2       0       0.5
0       5       5.5
0       4       3.5
1       2       2.5
0       4       4.5
```

(continued)

1	1	0.5				
0	3	2.5				
1	1	1.5				
0	3	3.5				
0	2	1.5				
1	0	0.5				
0	2	2.5				
0	1	0.5				
0	1	1.5				
0	0	0.5				
8.2	16.0					
2	0	0.5	8.008			
1	2	1.5				
0	5	5.5				
0	4	3.5				
1	2	2.5				
0	4	4.5				
1	1	0.5				
0	3	2.5				
1	1	1.5				
0	3	3.5				
0	2	1.5				
1	0	0.5				
0	2	2.5				
0	1	0.5				
0	1	1.5				
0	0	0.5				
9.0	283.0					
9.1	0.0778	1.69706	2.94058			
9.2	138.03	0.0	1300.0	1.0	1200.0	1.0
9.3	769.0	0.0	1400.0	1.0	1400.0	2.0
9.4	0.64	0.35	0.5			
0.0						

[**Output**] See the file /OUTPUT/Pb/Pb_1-_wD.log

[**Exsample 2**] ^{208}Pb $\rightarrow^{208}$Bi (2^-)

$q = 0.1\text{fm}^{-1}$, ω =10-50MeV (step=0.5MeV).
RPA in N-space while TDA for N-Δ coupling
Mean field parameters for Nucleons
Shape parameters: from ref.[11]
Spin-orbit potential: from ref.[14]
The Perey factor parameters: from ref.[14]
Spreading widths: both for particles and holes by the emperical formula [10].
Fermi energy: experimental values
Mean field parameters for Delta: from ref.[14]
Effective *ph* interaction: LM + OBEP.

OBEP parameters: from ref.[13].
LM parameters: $(g'_{\mathrm{NN}}, g'_{\mathrm{N}\Delta}, g'_{\Delta\Delta}) = (0.64, 0.35, 0.5)$ without form factor [14].

[input] See the file /input/Pb/Pb_2-_wD.dat

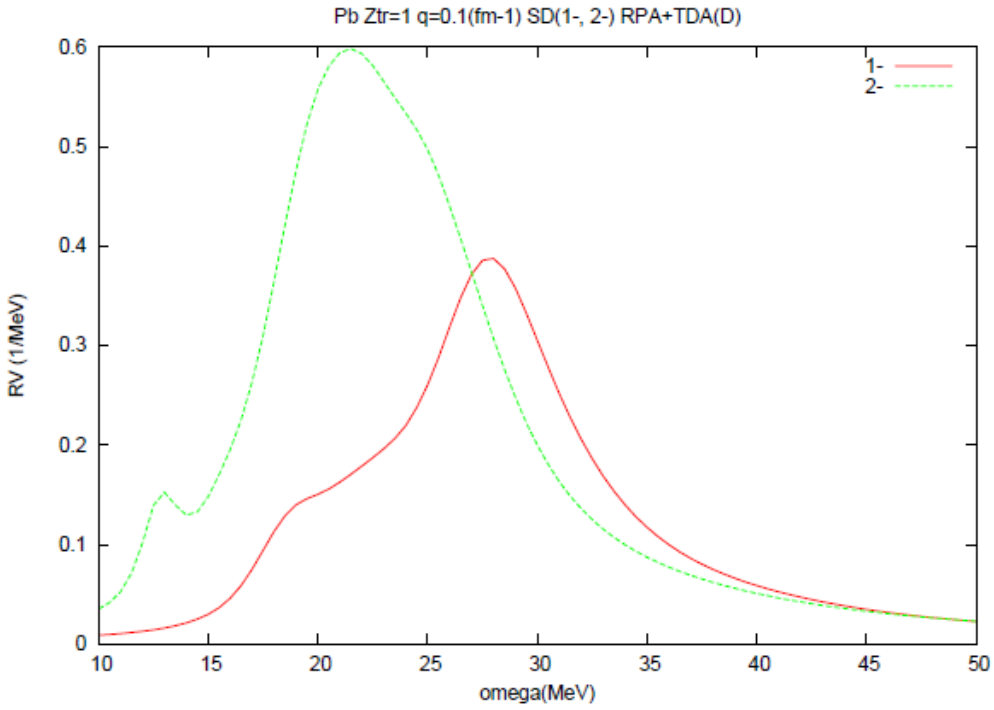
```
1212201701
Pb 2- Ztr=1 q=0.1 w=10-50 m*=.7 Wp=spw Wh=spw RPA+TDA(D) LM+OBEP
31011
1.0     208.0   82.0    1.0
2.0     1.0     0.1     0.0     81.0    10.0    0.5
3.0     2.0     2.0     2.0     2.0
4.0     16.0    80.0    3.0     120.0
5.1     0.0     1.27    0.67    7.5     1.27    0.67    1.27
5.2     0.0     1.27    0.67    7.5     1.27    0.67    1.27
5.3     0.0     1.27    0.67    7.5     1.27    0.67    1.27
5.4     0.0     1.27    0.67    7.5     1.27    0.67    1.27
5.5     30.0    1.27    0.67    0.0     1.27    0.67    1.27
6.3     999.0   1.27    0.67    1.0     0.0     1.27    0.67
6.4     999.0   1.27    0.67    1.0     0.0     1.27    0.67
7.1     1.27    0.67    0.3     10.75   26.0    110.0   5.659
7.2     1.27    0.67    0.3     10.75   26.0    110.0   5.902
7.3     1.27    0.67    0.3     10.75   26.0    110.0   5.659
7.4     1.27    0.67    0.3     10.75   26.0    110.0   5.902
8.1     22.0
2       1       0.5     7.368
1       3       2.5
2       1       1.5
0       6       6.5
0       5       4.5
1       3       3.5
1       2       1.5
2       0       0.5
0       5       5.5
0       4       3.5
1       2       2.5
0       4       4.5
1       1       0.5
0       3       2.5
1       1       1.5
0       3       3.5
0       2       1.5
1       0       0.5
0       2       2.5
0       1       0.5
0       1       1.5
0       0       0.5
```

(continued)

```
8.2         16.0
2           0           0.5         8.008
1           2           1.5
0           5           5.5
0           4           3.5
1           2           2.5
0           4           4.5
1           1           0.5
0           3           2.5
1           1           1.5
0           3           3.5
0           2           1.5
1           0           0.5
0           2           2.5
0           1           0.5
0           1           1.5
0           0           0.5
9.1         0.0778      1.69706     2.94058
9.2         138.03      0.0         1300.0      1.0         1200.0      1.0
9.3         769.0       0.0         1400.0      1.0         1400.0      2.0
9.4         0.64        0.35        0.5
0.0
```

[Output] See the file /OUTPUT/Pb/Pb_2-_wD.log

The calculated response functions for the 1^- and 2^- excitations in Ex. 1 and 2 are shown in the figure below.



7 Definitions of output quantities

This program treats only the isovector transitions.

7.1 Spin and isospin operators in the spherical tensor form

We use the spherical tensor representation for the nucleon spin operators $\boldsymbol{\sigma}$ as

$$\sigma_{\pm 1} = \mp \frac{\sigma_x \pm \mathrm{i}\sigma_y}{\sqrt{2}}, \qquad \sigma_0 = \sigma_z \tag{25}$$

and the spin transition operators $\boldsymbol{S}$ from nucleon(N) to Δ as

$$S_{\pm 1} = \mp \frac{S_x \pm \mathrm{i}S_y}{\sqrt{2}}, \qquad S_0 = S_z \tag{26}$$

Similarly the nucleon isospin operators $\boldsymbol{\tau}$ and the N-Δ transition isospin operators $\boldsymbol{T}$ are expressed as

$$\tau_{\pm 1} = \mp \frac{\tau_x \pm \mathrm{i}\tau_y}{\sqrt{2}}, \quad \tau_0 = \tau_z, \qquad T_{\pm 1} = \mp \frac{T_x \pm \mathrm{i}T_y}{\sqrt{2}}, \quad T_0 = T_z \tag{27}$$

We note that the commonly used $t_\pm$ are expressed as

$$t_\pm = t_x \pm \mathrm{i}t_y = \mp \frac{\tau_{\pm 1}}{\sqrt{2}} \tag{28}$$

We use the notations for the hermite conjugate operators as

$$S^\dagger_{\pm 1} = \left(S^\dagger\right)_{\pm 1} = \mp \frac{S^\dagger_x \pm \mathrm{i}S^\dagger_y}{\sqrt{2}} = \mp \left(\frac{S_x \mp \mathrm{i}S_y}{\sqrt{2}}\right)^\dagger = -\left(S_{\mp 1}\right)^\dagger \tag{29}$$

and the corresponding notations for the isospin transition oparators.

Note the relations

$$S^\dagger_\mu = (-1)^\mu \left(S_{-\mu}\right)^\dagger, \qquad T^\dagger_\nu = (-1)^\nu \left(T_{-\nu}\right)^\dagger \tag{30}$$

7.2 One-body transition operators

We introduce the following one-body transition operators from N to N and from N to Δ.

(1) **Isovector spin-scalar operators** [1]

$$O_{\nu,\mathrm{S}}(\boldsymbol{q}) \equiv \frac{1}{\sqrt{2}} \sum_k \tau_{\nu,k}\, \mathrm{e}^{-\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r}_k} \tag{31}$$

For this mode Δ does not contribute.

[1] The operators here have the factors $\mathrm{e}^{-\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r}_k}$ instead of $\mathrm{e}^{\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r}_k}$ often used, because the Fourier transform of the density operator $\rho(\boldsymbol{r}) = \sum_k \delta(\boldsymbol{r} - \boldsymbol{r}_k)$ is

$$\tilde{\rho}(\boldsymbol{q}) = \int \rho(\boldsymbol{r}) \mathrm{e}^{-\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r}} d\boldsymbol{r} = \sum_k \mathrm{e}^{-\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r}_k}$$

(2) **Isovector spin-vector operators**

$$O^{\rm N}_{\nu\mu}(\boldsymbol{q}) \equiv \frac{1}{\sqrt{2}} \sum_k \tau_{\nu,k}\, \sigma_{\mu,k}\, {\rm e}^{-{\rm i}\boldsymbol{q}\cdot\boldsymbol{r}_k} \tag{32}$$

$$O^{\Delta}_{\nu\mu}(\boldsymbol{q}) \equiv \frac{1}{\sqrt{2}} \sum_k \left(T_{\nu,k}\, S_{\mu,k} + T^{\dagger}_{\nu,k}\, S^{\dagger}_{\mu,k} \right) {\rm e}^{-{\rm i}\boldsymbol{q}\cdot\boldsymbol{r}_k} \tag{33}$$

(3) **Isovector spin-longitudinal operators**

$$O^{\rm N}_{\nu,{\rm L}}(\boldsymbol{q}) \equiv -\frac{1}{\sqrt{2}} \sum_k \tau_{\nu,k}\, (\boldsymbol{\sigma}_k \cdot \hat{\boldsymbol{q}})\, {\rm e}^{-{\rm i}\boldsymbol{q}\cdot\boldsymbol{r}_k} \tag{34}$$

$$O^{\Delta}_{\nu,{\rm L}}(\boldsymbol{q}) \equiv -\frac{1}{\sqrt{2}} \sum_k \left\{ T_{\nu,k}\, (\boldsymbol{S}_k \cdot \hat{\boldsymbol{q}}) + T^{\dagger}_{\nu,k}\, (\boldsymbol{S}^{\dagger}_k \cdot \hat{\boldsymbol{q}}) \right\} {\rm e}^{-{\rm i}\boldsymbol{q}\cdot\boldsymbol{r}_k} \tag{35}$$

(4) **Isovector spin-transverse operators**

$$O^{\rm N}_{\nu\mu,{\rm T}}(\boldsymbol{q}) \equiv -\frac{1}{2} \sum_k \tau_{\nu,k}\, \left[\boldsymbol{\sigma}_k \times \hat{\boldsymbol{q}}\right]_\mu {\rm e}^{-{\rm i}\boldsymbol{q}\cdot\boldsymbol{r}_k} \tag{36}$$

$$O^{\Delta}_{\nu\mu,{\rm T}}(\boldsymbol{q}) \equiv -\frac{1}{2} \sum_k \left\{ T_{\nu,k}\, \left[\boldsymbol{S}_k \times \hat{\boldsymbol{q}}\right]_\mu + T^{\dagger}_{\nu,k}\, \left[\boldsymbol{S}^{\dagger}_k \times \hat{\boldsymbol{q}}\right]_\mu \right\} {\rm e}^{-{\rm i}\boldsymbol{q}\cdot\boldsymbol{r}_k} \tag{37}$$

7.3 Response functions for the isovector transitions

7.3.1 Hamiltonian, state vector and energy

We consider the nuclear system A, whose intrinsic Hamiltonian is $H_{\rm A}$. We write the state vector and the energy of the n-th eigenstate as Φ_n and $\mathcal{E}_n$, respectively, where $n = 0$ denotes the ground state, whose spin is assumed to be $J_0 = 0$, e. g.

$$H_{\rm A}\Phi_n = \mathcal{E}_n \Phi_n \tag{38}$$

We write the excitation energy as

$$\omega_n = \mathcal{E}_n - \mathcal{E}_0 \tag{39}$$

Note that for charge exchange transitions, $\mathcal{E}_0$ is the ground state energy of the mother nucleus, but not of the daughter nucleus. Therefore, the excitation energy here means that with respect to the ground state of the mother nucleus.

7.3.2 Response function

Corresponding to the one-body operators in subsect. 7.2, the program calculates the following response functions in RPA, TDA or simply the mean field approximation (0-th).

(1) **Spin-scalar response function**

$$R_{\nu,{\rm S}}(q,\omega) \equiv \sum_{n\neq 0} |\langle \Phi_n | O_{\nu,{\rm S}}(\boldsymbol{q}) | \Phi_0 \rangle|^2 \delta(\omega - \omega_n) \tag{40}$$

(2) **Spin-vector response function**

We define the spin-vector-spin-vector response functions as

$$R^{ab}_{\nu,\mu\mu'}(\boldsymbol{q},\omega) \equiv \sum_{n\neq 0}\langle\Phi_0|\left[O^a_{\nu\mu}(\boldsymbol{q})\right]^\dagger|\Phi_n\rangle\langle\Phi_n|O^b_{\nu\mu'}(\boldsymbol{q})|\Phi_0\rangle\delta(\omega-\omega_n), \quad (a,b=\mathrm{N\ or\ }\Delta) \quad (41)$$

The program presents only its trace

$$R^{ab}_{\nu,\mathrm{V}}(q,\omega) \equiv \sum_{\mu} R^{ab}_{\nu,\mu\mu}(\boldsymbol{q},\omega) \quad (42)$$

which we call the spin-vector response function.

(3) **Spin-longitudinal response function**

$$R^{ab}_{\nu,\mathrm{L}}(q,\omega) \equiv \sum_{n\neq 0}\langle\Phi_0|\left[O^a_{\nu,\mathrm{L}}(\boldsymbol{q})\right]^\dagger|\Phi_n\rangle\langle\Phi_n|O^b_{\nu,\mathrm{L}}(\boldsymbol{q})|\Phi_0\rangle\delta(\omega-\omega_n), \quad (a,b=\mathrm{N\ or\ }\Delta) \quad (43)$$

(4) **Spin-transverse response function**

$$R^{ab}_{\nu,\mathrm{T}}(q,\omega) \equiv \sum_{n\neq 0}\langle\Phi_0|\left[O^a_{\nu,\mathrm{T}}(\boldsymbol{q})\right]^\dagger|\Phi_n\rangle\langle\Phi_n|O^b_{\nu,\mathrm{T}}(\boldsymbol{q})|\Phi_0\rangle\delta(\omega-\omega_n), \quad (a,b=\mathrm{N\ or\ }\Delta) \quad (44)$$

This is calculated by the relation

$$R^{ab}_{\nu,\mathrm{T}}(q,\omega) = \frac{1}{2}(R^{ab}_{\nu,\mathrm{V}}(q,\omega) - R^{ab}_{\nu,\mathrm{L}}(q,\omega)), \quad (a,b=\mathrm{N\ or\ }\Delta) \quad (45)$$

due to the vector relation $(\boldsymbol{A}\cdot\hat{\boldsymbol{q}})(\boldsymbol{B}\cdot\hat{\boldsymbol{q}}) + [\boldsymbol{A}\times\hat{\boldsymbol{q}}]\cdot[\boldsymbol{B}\times\hat{\boldsymbol{q}}] = (\boldsymbol{A}\cdot\boldsymbol{B})$.

Since $J_0 = 0$, all these response functions are independent of the direction of $\boldsymbol{q}$.

(5) **Response functions in N+Δ space**

To show the combined responses from N and Δ, we conventionally define the isovector spin-vector operators as

$$\tau_\nu\sigma_\mu + \frac{f_{\pi\mathrm{N}\Delta}}{f_{\pi\mathrm{NN}}}\left\{T_\nu S_\mu + T^\dagger_\nu S^\dagger_\mu\right\} \quad (46)$$

and present the response functions

$$R_{\nu,\mathrm{X}}(q,\omega) = R^{\mathrm{NN}}_{\nu,\mathrm{X}}(q,\omega) + \frac{f_{\pi\mathrm{N}\Delta}}{f_{\pi\mathrm{NN}}}\left\{R^{\mathrm{N}\Delta}_{\nu,\mathrm{X}}(q,\omega) + R^{\Delta\mathrm{N}}_{\nu,\mathrm{X}}(q,\omega)\right\} + \left(\frac{f_{\pi\mathrm{N}\Delta}}{f_{\pi\mathrm{NN}}}\right)^2 R^{\Delta\Delta}_{\nu,\mathrm{X}}(q,\omega) \quad (47)$$

with X = V, L or T.

7.4 Sum rule values

The program provides the energy non-weighted and weighted sum rule values of these response functions.

The energy non-weighted sums are defined as

$$S^{(0)}_{\nu,\mathrm{X}}(q) = \int_0^{\omega_{\max}} R_{\nu,\mathrm{X}}(q,\omega)\ d\omega \quad (48)$$

and the energy weighted sums are defined as

$$S^{(1)}_{\nu,\mathrm{X}}(q) = \int_0^{\omega_{\max}} \omega R_{\nu,\mathrm{X}}(q,\omega)\ d\omega \quad (49)$$

where X = S, V, L or T.

8 Formalism

We sketch the formalism used in the program based on refs. [1]-[7].

8.1 Response functions and polarization propagators

8.1.1 One-body transition density operator

We consider the one-body transition density operators

$$\rho_F(\boldsymbol{r}) = \sum_i^A F_i \delta(\boldsymbol{r} - \boldsymbol{r}_i) \tag{50}$$

where i is the particle number, and F represents a spin-isospin operator in the N+Δ space.

To unify formulas for N and Δ, we introduce the notations, $\sigma_\mu^{(s),ab}, \tau_\nu^{(t),ab}$, as

$$\sigma^{(0),\mathrm{NN}} = 1, \quad \sigma_\mu^{(1),\mathrm{NN}} = \sigma_\mu, \quad \sigma^{(1),\Delta\mathrm{N}} = S_\mu, \quad \sigma_\mu^{(1),\mathrm{N}\Delta} = S_\mu^\dagger \tag{51}$$

$$\tau_\nu^{(1),\mathrm{NN}} = \tau_\nu, \quad \tau^{(1),\Delta\mathrm{N}} = T_\nu, \quad \tau_\nu^{(1),\mathrm{N}\Delta} = T_\nu^\dagger \tag{52}$$

The operator F denotes one of

$$\tau_{\nu,i}^{(1),\mathrm{NN}} \sigma_{\mu,i}^{(s),\mathrm{NN}} \quad \text{and} \quad \tau_{\nu,i}^{(1),\Delta\mathrm{N}} \sigma_{\mu,i}^{(1),\Delta\mathrm{N}} + \tau_{\nu,i}^{(1),\mathrm{N}\Delta} \sigma_{\mu,i}^{(1),\mathrm{N}\Delta} \tag{53}$$

8.1.2 Response functions and polarization propagators

The response functions for those operators are defined as

$$R_{FF'}(\boldsymbol{r}, \boldsymbol{r}'; \omega) = \sum_n \langle \Phi_0 | \rho_F^\dagger(\boldsymbol{r}) | \Phi_n \rangle \langle \Phi_n | \rho_{F'}(\boldsymbol{r}') | \Phi_0 \rangle \delta(\omega - \omega_n) \tag{54}$$

They can be written as

$$R_{FF'}(\boldsymbol{r}, \boldsymbol{r}'; \omega) = -\frac{1}{\pi} \mathrm{Im} \Pi_{FF'}(\boldsymbol{r}, \boldsymbol{r}'; \omega), \qquad (\omega > 0) \tag{55}$$

with the polarization propagator

$$\begin{aligned} \Pi_{FF'}(\boldsymbol{r}, \boldsymbol{r}'; \omega) &= \langle \Phi_0 | \rho_F^\dagger(\boldsymbol{r}) \frac{1}{\omega - (H_\mathrm{A} - \mathcal{E}_0) + \mathrm{i}\,\delta} \rho_{F'}(\boldsymbol{r}') | \Phi_0 \rangle \\ &+ \langle \Phi_0 | \rho_{F'}(\boldsymbol{r}') \frac{1}{-\omega - (H_\mathrm{A} - \mathcal{E}_0) + \mathrm{i}\,\delta} \rho_F^\dagger(\boldsymbol{r}) | \Phi_0 \rangle \end{aligned} \tag{56}$$

8.2 Mean field approximation

8.2.1 Hamiltonian

As the 0-th order approximation (0-th), we start with the mean field approximation, in which the intrinsic Hamiltonian H_A is replaced by the mean field hamiltonian H_0

$$H_A \longrightarrow H_0 = \sum_k^A (\hat{h}_k^\mathrm{N} + \hat{h}_k^\Delta) - T_\mathrm{c.m.} \tag{57}$$

where $\hat{h}^{\rm N}$ and $\hat{h}^{\Delta}$ are the N and Δ single particle hamiltonians, respectively.

We write the ground state and its energy of H_0 by $|0\rangle$ and $\mathcal{E}_0^{(0)}$, namely

$$H_0|0\rangle = \mathcal{E}_0^{(0)}|0\rangle \tag{58}$$

We assume that the ground state does not include Δ and its spin $J_0 = 0$.

We write a single particle state of a nucleon in the occupied state as $|h\rangle$, that of the unoccupied state as $|p_{\rm N}\rangle$, and a single Δ state as $|p_\Delta\rangle$.

They obey the equations

$$h^{\rm N}|h\rangle = \epsilon_h|h\rangle, \quad h^{\rm N}|p_{\rm N}\rangle = \epsilon_p^{\rm N}|p_{\rm N}\rangle, \quad h^{\Delta}|p_\Delta\rangle = (\epsilon_p^{\Delta} + \Delta m)|p_\Delta\rangle \tag{59}$$

with

$$\Delta m = m_\Delta - m_{\rm N} \tag{60}$$

8.2.2 Free polarization propagator

The polarization propagator in this approximation is written as

$$\Pi^{(0)}_{FF'}(\boldsymbol{r},\boldsymbol{r}';\omega) = \Pi^{(0),{\rm FW}}_{FF'}(\boldsymbol{r},\boldsymbol{r}';\omega) + \Pi^{(0),{\rm BK}}_{FF'}(\boldsymbol{r},\boldsymbol{r}';\omega) \tag{61}$$

with the forward and bachward free polarization propagators

$$\Pi^{(0),{\rm FW}}_{FF'}(\boldsymbol{r},\boldsymbol{r}';\omega) = \langle 0|\rho_F^\dagger(\boldsymbol{r})\frac{1}{\omega - (H_0 - \mathcal{E}_0^{(0)}) + {\rm i}\,\delta}\rho_{F'}(\boldsymbol{r}')|0\rangle \tag{62}$$

$$\Pi^{(0),{\rm BK}}_{FF'}(\boldsymbol{r},\boldsymbol{r}';\omega) = \langle 0|\rho_{F'}(\boldsymbol{r}')\frac{1}{-\omega - (H_0 - \mathcal{E}_0^{(0)}) + {\rm i}\,\delta}\rho_F^\dagger(\boldsymbol{r})|0\rangle \tag{63}$$

We call $\Pi^{(0)}_{FF'}$ the free (unperturbed) polarization propagator.

Noting that $\rho_{F'}(\boldsymbol{r}')|0\rangle$ and $\rho_F^\dagger(\boldsymbol{r})|0\rangle$ are the sum of 1-N-particle-1-hole states $|h^{-1}p_{\rm N}\rangle$ and 1-Δ-1-hole states $|h^{-1}p_\Delta\rangle$, and the propagators $(\pm\omega - (H_0 - \mathcal{E}_0^{(0)}) + {\rm i}\,\delta)^{-1}$ are diagnal with respect to them, we decompose the polarization propagators (62) and (63) as

$$\Pi^{(0),{\rm FW}}_{FF'}(\boldsymbol{r},\boldsymbol{r}';\omega) = \Pi^{(0),{\rm FW,N}}_{FF'}(\boldsymbol{r},\boldsymbol{r}';\omega) + \Pi^{(0),{\rm FW},\Delta}_{FF'}(\boldsymbol{r},\boldsymbol{r}';\omega) \tag{64}$$

$$\Pi^{(0),{\rm BK}}_{FF'}(\boldsymbol{r},\boldsymbol{r}';\omega) = \Pi^{(0),{\rm BK,N}}_{FF'}(\boldsymbol{r},\boldsymbol{r}';\omega) + \Pi^{(0),{\rm BK},\Delta}_{FF'}(\boldsymbol{r},\boldsymbol{r}';\omega) \tag{65}$$

with

$$\Pi^{(0),{\rm FW},a}_{FF'}(\boldsymbol{r},\boldsymbol{r}';\omega) = \langle 0|\rho_F^\dagger(\boldsymbol{r})G^a_{ph}(\omega)\rho_{F'}(\boldsymbol{r}')|0\rangle \tag{66}$$

$$\Pi^{(0),{\rm BK},a}_{FF'}(\boldsymbol{r},\boldsymbol{r}';\omega) = \langle 0|\rho_{F'}(\boldsymbol{r}')G^a_{ph}(-\omega)\rho_F^\dagger(\boldsymbol{r})|0\rangle \tag{67}$$

where $a = {\rm N}$ or Δ, and $G^a_{ph}(\omega)$ is the ph Green's functions defined as

$$G^a_{ph}(\omega) = \sum_h\sum_{p_a}|h^{-1}p_a\rangle\frac{1}{\omega - (\epsilon_p^a - \epsilon_h) + {\rm i}\,\delta}\langle h^{-1}p_a| \tag{68}$$

8.3 RPA and TDA – Ring approximation

The program takes into account *ph* correlations by RPA or TDA without exchange terms, which are called "ring approximation".

The correlations are induced by the effective *ph* interaction V_{ph}, which can be expanded by the transition density operators as

$$V_{ph} = \sum_{FF'} \int d^3\boldsymbol{r} d^3\boldsymbol{r}' \rho_F(\boldsymbol{r}) W_{FF'}(\boldsymbol{r},\boldsymbol{r}') \rho^\dagger_{F'}(\boldsymbol{r}') \tag{69}$$

The polarization propagators in RPA are then given by the RPA equation

$$\begin{aligned} \Pi^{\rm RPA}_{FF'}(\boldsymbol{r},\boldsymbol{r}') &= \Pi^{(0)}_{FF'}(\boldsymbol{r},\boldsymbol{r}') \\ &+ \sum_{F''F'''} \int d^3\boldsymbol{r}'' d^3\boldsymbol{r}''' \Pi^{(0)}_{FF''}(\boldsymbol{r},\boldsymbol{r}'') W_{F''F'''}(\boldsymbol{r}'',\boldsymbol{r}''') \Pi^{\rm RPA}_{F'''F'}(\boldsymbol{r}''',\boldsymbol{r}') \end{aligned} \tag{70}$$

In TDA only the forward propagation is taken in eq.(70), namely
the polarization propagators in TDA are given by the TDA equation

$$\Pi^{\rm TDA} = \Pi^{(0),{\rm FW}} + \Pi^{(0),{\rm FW}} W \Pi^{\rm TDA} \tag{71}$$

8.4 Angular momentum representation

Calculations are carried out in the angular momentum representation.

8.4.1 Isovector transition densities

The transition density operators $\rho_F(\boldsymbol{r})$ defined by eqs. (50) and (53) are written as

$$\rho^a_F(\boldsymbol{r}) = \sum_{JM} \sum_{lm} (lms\mu|JM) \rho^{(\nu),a}_{lsJM}(r) \left({\rm i}^l Y_{lm}(\hat{\boldsymbol{r}}) \right)^*, \qquad (s=0,1,\ a={\rm N},\Delta) \tag{72}$$

where

$$\rho^{(\nu),{\rm N}}_{lsJM}(r) = \sum_i \tau^{(1),{\rm NN}}_{\nu,i} \left[{\rm i}^l Y_l(\hat{\boldsymbol{r}}_i) \times \sigma^{(s),{\rm NN}}_i \right]^J_M \frac{\delta(r-r_i)}{r r_i} \tag{73}$$

$$\begin{aligned} \rho^{(\nu),\Delta}_{ls=1JM}(r) &= \sum_i \left\{ \tau^{(1),\Delta{\rm N}}_{\nu,i} \left[{\rm i}^l Y_l(\hat{\boldsymbol{r}}_i) \times \sigma^{(1),\Delta{\rm N}}_i \right]^J_M + \tau^{(1),{\rm N}\Delta}_{\nu,i} \left[{\rm i}^l Y_l(\hat{\boldsymbol{r}}_i) \times \sigma^{(1),{\rm N}\Delta}_i \right]^J_M \right\} \\ &\times \frac{\delta(r-r_i)}{r r_i} \end{aligned} \tag{74}$$

8.4.2 Free polariization propagators

We consider only the spin-diagonal polarization propagators, thus we need only the free polarization propagators

$$\Pi^{(0),{\rm FW},a}_{ll'sJ\nu}(r,r';\omega) = \langle 0| \left[\rho^{(\nu),a}_{lsJM}(r) \right]^\dagger G^a_{ph}(\omega)\ \rho^{(\nu),a}_{l'sJM}(r')\ |0\rangle \tag{75}$$

$$\Pi^{(0),{\rm BK},a}_{ll'sJ\nu}(r,r';\omega) = \langle 0|\ \rho^{(\nu),a}_{lsJM}(r')\ G^a_{ph}(-\omega)\ \left[\rho^{(\nu),a}_{l'sJM}(r) \right]^\dagger |0\rangle \tag{76}$$

where $a = {\rm N},\Delta$. Since the ground state spin $J_0 = 0$, they are diagonal with respect to J and M and further independent of M. Thus we omit the subscript M of Π. Detailed formulas are presented in sect. 8.6.

8.4.3 Effective ph interaction

We employ the effective ph interaction of eq. (14)

$$V_{ph} = V^{\rm s} + V^{\rm v} \tag{77}$$

which can be expressed as

$$V^{\rm s} = \sum_{JM\nu} \int r^2 dr \int r'^2 dr' \rho^{(\nu),{\rm N}}_{J0JM}(r) W^{\rm NN}_{JJ0J}(r,r') \left[\rho^{(\nu),{\rm N}}_{J0JM}(r')\right]^\dagger \tag{78}$$

$$V^{\rm v} = \sum_{ab={\rm N},\Delta} \sum_{JM\nu} \sum_{ll'} \int r^2 dr \int r'^2 dr' \rho^{(\nu),a}_{l1JM}(r) W^{ab}_{ll'1J}(r,r') \left[\rho^{(\nu),a}_{l'1JM}(r')\right]^\dagger \tag{79}$$

Explicit expressions of $W^{ab}_{ll'sJ}(r,r')$ for the interactions (15)-(17) are given below.

A. Spin-scalar part

The contact interaction (15) can be rewritten as

$$V^{\rm s}_{12}(\boldsymbol{r}_1 - \boldsymbol{r}_2) = V_\tau\ (\boldsymbol{\tau_1}\cdot\boldsymbol{\tau_2})\delta(\boldsymbol{r}_1-\boldsymbol{r}_2) = V_\tau \sum_\nu \tau_{\nu,1}(\tau_{\nu,2})^\dagger \sum_{lm} Y_{lm}(\hat{\boldsymbol{r}}_1) Y^*_{lm}(\hat{\boldsymbol{r}}_2) \frac{\delta(r_1-r_2)}{r_1 r_2} \tag{80}$$

thus we get

$$W^{\rm NN}_{JJ0J}(r_1,r_2) = V_\tau \frac{\delta(r_1-r_2)}{r_1 r_2} \tag{81}$$

B. Spin-vector part

This parts are given in the momentum representation $V^{\rm sv}_{12}(\boldsymbol{q},\omega)$ in eqs. (16) and (17). Carrying out the Fourier transformation

$$V^{\rm v}_{12}(\boldsymbol{r}_1 - \boldsymbol{r}_2;\omega) = \int \frac{d\boldsymbol{q}^3}{(2\pi)^3} V^{\rm v}_{12}(\boldsymbol{q},\omega) {\rm e}^{{\rm i}\boldsymbol{q}\cdot(\boldsymbol{r}_1-\boldsymbol{r}_2)} \tag{82}$$

and using the relation

$$(\boldsymbol{\sigma}\cdot\hat{\boldsymbol{q}}){\rm e}^{{\rm i}\boldsymbol{q}\cdot\boldsymbol{r}} = -4\pi \sum_{lJM} a_{Jl} j_l(qr) Y^*_{JM}(\hat{\boldsymbol{q}}) \left[{\rm i}^l Y_l(\hat{\boldsymbol{r}}) \times \sigma^{(1)}\right]^J_M \tag{83}$$

with

$$a_{Jl} = (10J0|l0) \tag{84}$$

we get

$$W^{ab}_{l'l1J}(r',r;\omega) = \delta_{l'l}\delta_{lJ} W^{{\rm T},ab}_{JJJ}(r',r;\omega) + a_{Jl'}a_{Jl}\left(W^{{\rm L},ab}_{l'lJ}(r',r;\omega) - W^{{\rm T},ab}_{l'lJ}(r',r;\omega)\right) \tag{85}$$

with

$$W^{{\rm L},ab}_{l'lJ}(r',r;\omega) \equiv \frac{2}{\pi}\int q^2 dq j_{l'}(qr') W^{ab}_{\rm L}(q;\omega) j_l(qr) \tag{86}$$

$$W^{{\rm T},ab}_{l'lJ}(r',r;\omega) \equiv \frac{2}{\pi}\int q^2 dq j_{l'}(qr') W^{ab}_{\rm T}(q;\omega) j_l(qr) \tag{87}$$

The program uses the $\pi+\rho+g'+h'$ model for $V^{\rm v}_{12}(\boldsymbol{q},\omega)$ given in sect.5.4.

8.4.4 RPA and TDA

The polarization propagators in RPA for the isovector spin-scalar modes, $\Pi^{\rm RPA}_{J\nu}(r,r')$ are given by the RPA equations

$$\Pi^{\rm RPA}_{J\nu}(r,r') = \Pi^{(0),{\rm N}}_{JJ0J\nu}(r,r') + \int r_1^2 dr_1 \int r_2^2 dr_2 \Pi^{(0),{\rm N}}_{JJ0J\nu}(r,r_1) W^{\rm NN}_{JJ0J}(r_1,r_2) \Pi^{\rm RPA}_{J\nu}(r_2,r') \quad (88)$$

For the isovector spin-vector modes, $\Pi^{{\rm RPA},ab}_{ll'J\nu}(r,r')$ $(a,b={\rm N},\Delta)$, are given by

$$\begin{aligned}
&\begin{pmatrix} \Pi^{\rm RPA,NN}_{ll'J\nu}(r,r') & \Pi^{\rm RPA,N\Delta}_{ll'J\nu}(r,r') \\ \Pi^{\rm RPA,\Delta N}_{ll'J\nu}(r,r') & \Pi^{\rm RPA,\Delta\Delta}_{ll'J\nu}(r,r') \end{pmatrix} = \begin{pmatrix} \Pi^{(0),{\rm N}}_{ll'1J\nu}(r,r') & 0 \\ 0 & \Pi^{(0),\Delta}_{ll'1J\nu}(r,r') \end{pmatrix} \\
&\quad + \sum_{l_1 l_2} \int r_1^2 dr_1 \int r_2^2 dr_2 \begin{pmatrix} \Pi^{(0),{\rm N}}_{ll_1 1J\nu}(r,r_1) & 0 \\ 0 & \Pi^{(0),\Delta}_{ll_1 1J\nu}(r,r_1) \end{pmatrix} \\
&\quad \times \begin{pmatrix} W^{\rm NN}_{l_1 l_2 1J}(r_1,r_2) & W^{\rm N\Delta}_{l_1 l_2 1J}(r_1,r_2) \\ W^{\Delta{\rm N}}_{l_1 l_2 1J}(r_1,r_2) & W^{\Delta\Delta}_{l_1 l_2 1J}(r_1,r_2) \end{pmatrix} \begin{pmatrix} \Pi^{\rm RPA,NN}_{l_2 l' J\nu}(r_2,r') & \Pi^{\rm RPA,N\Delta}_{l_2 l' J\nu}(r_2,r') \\ \Pi^{\rm RPA,\Delta N}_{l_2 l' J\nu}(r_2,r') & \Pi^{\rm RPA,\Delta\Delta}_{l_2 l' J\nu}(r_2,r') \end{pmatrix} \quad (89)
\end{aligned}$$

The polarization propagators in TDA, $\Pi^{\rm TDA}_{J\nu}(r,r')$ and $\Pi^{{\rm TDA},ab}_{ll'J\nu}(r,r')$, are obtained by the replacements

$$\Pi^{(0),a}_{ll'sJ\nu}(r,r') \longrightarrow \Pi^{(0),{\rm FW},a}_{ll'sJ\nu}(r,r') \quad (90)$$

in the RPA equations (88) and (89).

8.5 Specific response functions

The program presents the specific response functions given in sect. 7.3, whose detailed expressions are given below.

8.5.1 Momentum representation

The transition density operators in the momentum representation are given from eq. (72) as

$$\begin{aligned}
\rho^a_F(\boldsymbol{q}) &= \int d^3\boldsymbol{r} \rho_F(\boldsymbol{r}) {\rm e}^{-{\rm i}\boldsymbol{q}\cdot\boldsymbol{r}} = \sum_{JM}\sum_{lm} (lms\mu|JM) 4\pi Y^*_{lm}(-\hat{\boldsymbol{q}}) \int r^2 dr 4\rho^{(\nu),a}_{lsJM}(r) j_l(qr) \\
&= \sum_{JM}\sum_{lm} (lms\mu|JM) Y^*_{lm}(\hat{\boldsymbol{q}})\ \rho^{(\nu),a}_{lsJM}(q) \quad (91)
\end{aligned}$$

where

$$\rho^{(\nu),a}_{lsJM}(q) = 4\pi(-1)^l \int r^2 dr \rho^{(\nu),a}_{lsJM}(r) j_l(qr), \qquad (a={\rm N},\Delta) \quad (92)$$

and $j_l(x)$ is the spherical Bessel function.

Correspondingly the polarization propagators in the momentum space are given by

$$\Pi_{J\nu}(q;\omega) = (4\pi)^2 \int r^2 dr \int r'^2 dr' j_J(qr) \Pi_{J\nu}(r,r';\omega) j_J(qr') \quad (93)$$

$$\Pi^{ab}_{ll'J\nu}(q;\omega) = (4\pi)^2 \int r^2 dr \int r'^2 dr' j_l(qr) \Pi^{ab}_{ll'J\nu}(r,r';\omega) j_{l'}(qr') \quad (94)$$

where we supressed the suffix RPA (TDA) on Π.

The response functions in the momentum representation are now given by

$$R_{J\nu}(q;\omega) = -\frac{1}{\pi}{\rm Im}\Pi_{J\nu}(q;\omega), \quad R^{ab}_{ll'J\nu}(q;\omega) = -\frac{1}{\pi}{\rm Im}\Pi^{ab}_{ll'J\nu}(q;\omega) \quad (95)$$

8.5.2 Isovector spin-scalar response functions

The isovector spin-scalar operators (31) can be written as

$$O_{\nu,\mathrm{S}}(\boldsymbol{q}) = \frac{1}{\sqrt{2}} \sum_{lm} \sum_{i} \tau_{\nu,i}\, \mathrm{e}^{-\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r}_i} = \frac{1}{\sqrt{2}} \sum_{lm} Y^*_{lm}(\hat{\boldsymbol{q}}) \rho^{(\nu),\mathrm{N}}_{l0lm}(q) \tag{96}$$

Then the isovector spin-scalar response function (40) is given by

$$R_{\nu,\mathrm{S}}(q,\omega) = \frac{1}{2} \sum_{lm} Y^*_{lm}(\hat{\boldsymbol{q}}) Y_{lm}(\hat{\boldsymbol{q}}) R_{l\nu}(q,\omega) = \frac{1}{8\pi} \sum_{l} (2l+1) R_{l\nu}(q,\omega) \tag{97}$$

where we used the relation

$$\sum_{m} Y^*_{lm}(\hat{\boldsymbol{q}}) Y_{lm}(\hat{\boldsymbol{q}}) = \frac{2l+1}{4\pi} \tag{98}$$

8.5.3 Isovector spin-vector response function

The isovector spin-vector operators (32) and (33) can be written as

$$O^{\mathrm{N}}_{\nu\mu}(\boldsymbol{q}) = \frac{1}{\sqrt{2}} \sum_{i} \tau_{\nu,i}\, \sigma_{\mu,i}\, \mathrm{e}^{-\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r}_i} = \frac{1}{\sqrt{2}} \sum_{lm} \sum_{JM} (lm1\mu|JM) Y^*_{lm}(\hat{\boldsymbol{q}})\, \rho^{(\nu),\mathrm{N}}_{l1JM}(q)$$

and similarly

$$O^{\Delta}_{\nu\mu}(\boldsymbol{q}) = \frac{1}{\sqrt{2}} \sum_{lm} \sum_{JM} (lm1\mu|JM) Y^*_{lm}(\hat{\boldsymbol{q}})\, \rho^{(\nu),\Delta}_{l1JM}(q) \tag{99}$$

We define the isovector spin-vector response functions (42) as

$$\begin{aligned} R^{ab}_{\nu,\mathrm{V}}(q,\omega) &= \frac{1}{2} \sum_{lm} \sum_{l'm'} \sum_{JM} \sum_{\mu} Y^*_{lm}(\hat{\boldsymbol{q}}) Y_{l'm'}(\hat{\boldsymbol{q}}) (1\mu lm|JM)(1\mu l'm'|JM) R^{ab}_{ll'J\nu}(q,\omega) \\ &= \frac{1}{8\pi} \sum_{J} (2J+1) \sum_{l=J-1}^{J+1} R^{ab}_{llJ\nu}(q,\omega), \qquad (a,b = \mathrm{N} \text{ or } \Delta) \end{aligned} \tag{100}$$

8.5.4 Isovector spin-longitudinal response function

The isovector spin-longitudinal operators (34) can be written as

$$\begin{aligned} O^{\mathrm{N}}_{\nu,\mathrm{L}}(\boldsymbol{q}) &= \frac{1}{\sqrt{2}} \sum_{i} \tau_{\nu,i} (\boldsymbol{\sigma}_i \cdot \hat{\boldsymbol{q}})\, \mathrm{e}^{-\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r}_i} = -\frac{4\pi}{\sqrt{2}} \sum_{lJM} a_{Jl} Y^*_{JM}(-\hat{\boldsymbol{q}}) \sum_{i} j_l(qr_i) \left[\mathrm{i}^l Y_l(\hat{\boldsymbol{r}}_i) \times \sigma_i\right]^J_M \\ &= -\frac{1}{\sqrt{2}} \sum_{lJM} a_{Jl} Y^*_{JM}(\hat{\boldsymbol{q}}) \rho^{(\nu),\mathrm{N}}_{l1JM}(q) \end{aligned} \tag{101}$$

where a_{Jl} is defined in eq. (84) and we used the relation (83).

Similarly the operator (35) can be written as

$$O^{\Delta}_{\nu,\mathrm{L}}(\boldsymbol{q}) = -\frac{1}{\sqrt{2}} \sum_{lJM} a_{Jl} Y^*_{JM}(\hat{\boldsymbol{q}}) \rho^{(\nu),\Delta}_{1lJM}(q) \tag{102}$$

We define the isovector spin-longitudinal response functions (43) as

$$\begin{aligned} R^{ab}_{\nu,\mathrm{L}}(q,\omega) &= \frac{1}{2} \sum_{ll'} \sum_{JM} a_{Jl} a_{Jl'} Y^*_{JM}(\hat{\boldsymbol{q}}) Y_{JM}(\hat{\boldsymbol{q}}) R^{ab}_{ll'J\nu}(q,\omega) \\ &= \frac{1}{8\pi} \sum_{J} (2J+1) \sum_{ll'} a_{Jl} a_{Jl'} R^{ab}_{ll'J\nu}(q,\omega), \quad (a,b = \mathrm{N} \text{ or } \Delta) \end{aligned} \tag{103}$$

8.5.5 Isovector spin-transverse response function

From eq. (45), the isovector spin-transverse response function (44) is given by

$$R^{ab}_{\nu,\mathrm{T}} = \frac{1}{2}\left(R^{ab}_{\nu,\mathrm{V}} - R^{ab}_{\nu,\mathrm{L}}\right) \tag{104}$$

The full response functions in the N+Δ space are given by eq. (47).

8.6 Calculation of the free polarization propagator

Before solving the RPA equations (88), (89), we have to calculate the free polarization propagators (75), (76). Here we present details of its calculation.

8.6.1 Single particle wave functions

First we prepare the single particle wave functions of the nucleon occupied states $|h\rangle$, unooccupied states $|p_{\mathrm{N}}\rangle$, and the single Δ states $|p_\Delta\rangle$ as

$$\langle \boldsymbol{r}|h\rangle = \langle \boldsymbol{r}|n\ l_h s_h j_h m_h\ \alpha_h\rangle = \frac{u^{\alpha_h}_{nl_hs_hj_h}(r)}{r}\mathcal{Y}_{l_hs_hj_hm_h}(\hat{\boldsymbol{r}}) \tag{105}$$

$$\langle \boldsymbol{r}|p_{\mathrm{N}}\rangle = \langle \boldsymbol{r}|\epsilon\ l_p s_p j_p m_p\ \alpha_{\mathrm{N}}\rangle = \frac{u^{\alpha_{\mathrm{N}}}_{l_ps_pj_p}(r;\epsilon)}{r}\mathcal{Y}_{l_ps_pj_pm_p}(\hat{\boldsymbol{r}}) \tag{106}$$

$$\langle \boldsymbol{r}|p_\Delta\rangle = \langle \boldsymbol{r}|\epsilon\ l_p s_p j_p m_p\ \alpha_\Delta\rangle = \frac{u^{\alpha_\Delta}_{l_ps_pj_p}(r;\epsilon)}{r}\mathcal{Y}_{l_ps_pj_pm_p}(\hat{\boldsymbol{r}}) \tag{107}$$

where $\alpha_h = \mathrm{n},\mathrm{p}$, $\alpha_{\mathrm{N}} = \mathrm{n},\mathrm{p}$ and $\alpha_\Delta = \Delta^-, \Delta^0, \Delta^+, \Delta^{++}$, and

$$\mathcal{Y}_{lsjm}(\hat{\boldsymbol{r}}) = \left[\mathrm{i}^l Y_l(\hat{\boldsymbol{r}}) \times \chi_s\right]^j_m \tag{108}$$

and χ_s is the spin wave function with $s = 1/2$ for N and $s = 3/2$ for Δ.

These states obey the equations

$$h^{\alpha_h}|n\, l_h s_h j_h m_h\ \alpha_h\rangle = \epsilon^{\alpha_h}_{n\, l_h s_h j_h}|n\, l_h s_h j_h m_h\ \alpha_h\rangle \tag{109}$$

$$h^{\alpha_{\mathrm{N}}}|\epsilon\, l_p s_p j_p m_p\ \alpha_{\mathrm{N}}\rangle = \epsilon\ |\epsilon\, l_p s_p j_p m_p\ \alpha_{\mathrm{N}}\rangle \tag{110}$$

$$h^{\alpha_\Delta}|\epsilon\, l_p s_p j_p m_p\ \alpha_\Delta\rangle = (\epsilon + \Delta m)\ |\epsilon\, l_p s_p j_p m_p\ \alpha_\Delta\rangle \tag{111}$$

where $\Delta m = m_\Delta - m_{\mathrm{N}}$.

Nucleon wave functions The mean field potential of the nucleon is usually non-local and often complex and energy dependent. To treat the non-locality we use the effective mass approximation[15]. In this method we write the wave functions as

$$u^{\alpha_h}_{nl_hs_hj_h}(r) = \sqrt{P^{\alpha_h}(r)}v^{\alpha_h}_{n_hl_hs_hj_h}(r), \qquad u^{\alpha_{\mathrm{N}}}_{l_ps_pj_p}(r;\epsilon) = \sqrt{P^{\alpha_{\mathrm{N}}}(r)}v^{\alpha_{\mathrm{N}}}_{l_ps_pj_p}(r;\epsilon) \tag{112}$$

where $P^\alpha(r)$ is the Perey factor. Then $v^{\alpha_h}_{nl_hs_hj_h}(r)$ and $v^{\alpha_{\mathrm{N}}}_{l_ps_pj_p}(r;\epsilon)$ obey the Schödinger equations with the local potentials $U^{\alpha_h}_{l_hs_hj_h}(r)$ and $U^{\alpha_{\mathrm{N}}}_{l_ps_pj_p}(r;\epsilon)$

$$\left[-\frac{1}{2\mu_{\mathrm{N}}}\frac{d^2}{dr^2} + \frac{1}{2\mu_{\mathrm{N}}}\frac{l_h(l_h+1)}{r^2} + U^{\alpha_h}_{l_hs_hj_h}(r)\right]v^{\alpha_h}_{nl_hs_hj_h}(r) = \epsilon^{\alpha_h}_{nl_hs_hj_h}v^{\alpha_h}_{nl_hs_hj_h}(r) \tag{113}$$

$$\left[-\frac{1}{2\mu_{\mathrm{N}}}\frac{d^2}{dr^2} + \frac{1}{2\mu_{\mathrm{N}}}\frac{l_p(l_p+1)}{r^2} + U^{\alpha_{\mathrm{N}}}_{l_ps_pj_p}(r,\epsilon)\right]v^{\alpha_{\mathrm{N}}}_{l_ps_pj_p}(r;\epsilon) = \epsilon\ v^{\alpha_{\mathrm{N}}}_{l_ps_pj_p}(r;\epsilon) \tag{114}$$

respectively[2], where the reduced masses are given by

$$\mu_{\rm N} = \frac{A-1}{A} m_{\rm N} \tag{115}$$

For the occupied states we require the normalization condition

$$\int |u^{\alpha_h}_{nl_hs_hj_h}(r)|^2 dr = \int P^{\alpha_h}(r) |v^{\alpha_h}_{nl_hs_hj_h}(r)|^2 dr = 1 \tag{116}$$

For the unoccupied states, we write the regular and singular radial wave functions as

$$f^{\alpha_{\rm N}}_{l_ps_pj_p}(r,\epsilon) = u^{\alpha_{\rm N},{\rm reg}}_{l_ps_pj_p}(r;\epsilon) = \sqrt{P^{\alpha_{\rm N}}(r)}\; v^{\alpha_{\rm N},{\rm reg}}_{l_ps_pj_p}(r;\epsilon) \tag{117}$$

$$h^{\alpha_{\rm N}}_{l_ps_pj_p}(r,\epsilon) = u^{\alpha_{\rm N},{\rm sing}}_{l_ps_pj_p}(r;\epsilon) = \sqrt{P^{\alpha_{\rm N}}(r)}\; v^{\alpha_{\rm N},{\rm sing}}_{l_ps_pj_p}(r;\epsilon) \tag{118}$$

which require the boundary conditions

$$f^{\alpha}_{lsj}(r;\epsilon) \to N r^{l+1} \qquad (r \to 0) \tag{119}$$

$$h^{\alpha}_{lsj}(r;\epsilon) \sim N' \exp\left[{\rm i} \left(kr - \eta {\rm ln} 2kr - \frac{l}{2}\pi + \sigma_l \right) \right], \quad (r \to \infty) \tag{120}$$

where

$$k = \sqrt{2\mu_{\rm N}\epsilon}, \quad \eta = \frac{Z_p Z_c e^2}{|k|/\mu_{\rm N}} \tag{121}$$

Single Δ wave function We assume that the mean field potential for Δ is local, and thus we do not consider a Perey factor for Δ. Writing the Hamiltonian h^{α_Δ} as

$$h^{\alpha_\Delta} = (T^{\alpha_\Delta} + U^{\alpha_\Delta}(r) + \Delta m) \tag{122}$$

the radial parts of the single particle wave fuctions obey the Schödinger equations

$$\left[-\frac{1}{2\mu_\Delta}\frac{d^2}{dr^2} + \frac{1}{2\mu_\Delta}\frac{l_p(l_p+1)}{r^2} + U^{\alpha_\Delta}_{l_ps_pj_p}(r) \right] u^{\alpha_\Delta}_{l_ps_pj_p}(r;\epsilon) = \epsilon\; u^{\alpha_\Delta}_{l_ps_pj_p}(r;\epsilon) \tag{123}$$

with

$$\mu_\Delta = \frac{(A-1)m_{\rm N} m_\Delta}{(A-1)m_{\rm N} + m_\Delta} \tag{124}$$

Their regular and singular radial wave functions are written as $f^{\alpha_\Delta}_{l_ps_pj_p}(r,\epsilon)$ and $f^{\alpha_\Delta}_{l_ps_pj_p}(r,\epsilon)$, respectively, and obey the similar boundary conditions (119) and (120).

8.6.2 Single particle Green's function

Secondly we prepare single particle Green's functions of the particle α

$$g^{\alpha}_{\rm sp}(\epsilon) \equiv \sum_c \frac{|c\rangle\langle c|}{\epsilon - \epsilon^\alpha_c + {\rm i}\,\delta} \tag{125}$$

where c runs over the complete set of the particle $\alpha (= {\rm n}, {\rm p}, \Delta^-, \Delta^0, \Delta^+, \Delta^{++})$.

[2] lsj dependence of U comes from the spin-orbit force

Writting their coordinate representation as

$$g^{\alpha}_{\rm sp}(\boldsymbol{r},\boldsymbol{r}';\epsilon) = \langle \boldsymbol{r}| \frac{1}{\epsilon - h^{\alpha}(\epsilon) + {\rm i}\delta} |\boldsymbol{r}'\rangle = \sum_{ljm} \mathcal{Y}_{lsjm}(\Omega_r) \frac{g^{{\rm sp},\alpha}_{lsj}(r,r';\epsilon)}{rr'} \mathcal{Y}^{\dagger}_{lsjm}(\Omega_{r'}), \tag{126}$$

we obtain their radial parts as

$$g^{{\rm sp},\alpha}_{lsj}(r,r';\epsilon) = \frac{2\mu_{\alpha}}{W(f^{\alpha}_{lsj}, h^{\alpha}_{lsj})} f^{\alpha}_{lsj}(r_<;\epsilon) h^{\alpha}_{lsj}(r_>;\epsilon) \tag{127}$$

where $r_< = \min(r,r')$, $r_> = \max(r,r')$, and $W(f,h)$ is the Wronskian

$$W(f,h) = \left| \begin{array}{cc} f & h \\ f' & h' \end{array} \right| \tag{128}$$

8.6.3 Particle Green's functions

The free ph Green's function, $G^{a}_{ph}(\omega)$ of eq. (68), has the sum $\sum_{p_\alpha}$, which is restricted to the unoccupied states and runs over the particle states in continuum up to infinity.

To handle these problems we introduce the particle Green's functions

$$g^{\alpha}_{p}(\epsilon) \equiv \sum_{p_\alpha \in {\rm unocc}} \frac{|p_\alpha\rangle\langle p_\alpha|}{\epsilon - \epsilon^{\alpha}_{p} + {\rm i}\,\delta} \tag{129}$$

and write their coordinate representation as

$$g^{\alpha}_{p}(\boldsymbol{r},\boldsymbol{r}';\epsilon) = \langle \boldsymbol{r}| g^{\alpha}_{p}(\epsilon) |\boldsymbol{r}'\rangle = \sum_{ljm} \mathcal{Y}_{lsjm}(\Omega_r) \frac{g^{\alpha}_{lsj}(r,r';\epsilon)}{rr'} \mathcal{Y}^{\dagger}_{lsjm}(\Omega_r) \tag{130}$$

By use of them, the free ph Green's functions $G^{a}_{ph}(\omega)$ are expressed as

$$G^{\rm N}_{ph}(\omega) = \sum_{\alpha_{\rm N}} \sum_{h} |h^{-1}\rangle\, g^{\alpha_{\rm N}}_{p}(r,r';\omega + \tilde{\epsilon}_h)\, \langle h^{-1}| \tag{131}$$

$$G^{\Delta}_{ph}(\omega) = \sum_{\alpha_{\Delta}} \sum_{h} |h^{-1}\rangle\, g^{\alpha_{\Delta}}_{p}(r,r';\omega + \tilde{\epsilon}_h - \Delta m)\, \langle h^{-1}| \tag{132}$$

where $\alpha_{\rm N}$ =n, p and $\alpha_{\Delta} = \Delta^-, \Delta^0, \Delta^+, \Delta^{++}$, and $\tilde{\epsilon}_h$ is the complex occupied state energy given by eq.(13)

$$\epsilon^{\alpha}_{h} \longrightarrow \tilde{\epsilon}^{\alpha}_{h} = \epsilon^{\alpha}_{h} - {\rm i}\frac{\gamma(\varepsilon^{\alpha}_{h})}{2} \tag{133}$$

For Δ's, we can identify

$$g^{\alpha_\Delta}_{p}(\epsilon) = g^{\alpha_\Delta}_{\rm sp}(\epsilon) \tag{134}$$

because there are no occupied states.

In practice, we choose that the local potentials for the nucleon unoccupied states $U^{\alpha_{\rm N}}(r,\epsilon)$ are complex and energy dependent, while those for the occupied states $U^{\alpha_h}(r)$ are real and energy independent[3]. Consequently, the orthogonality between the nucleon unoccupied and occupied states, $|p_{\rm N}\rangle$ and $|h\rangle$, is violated, which we have to take care of to calculate $g^{\alpha_{\rm N}}_{p}(\epsilon)$.

[3]Their explicit forms are given in eqs. (1)-(6).

8.6.4 Orthogonality condition model

To cope with the problem, we adopt the orthogonality condition model[16]. Since $U^{\alpha_h}(r)$ is hermitian and energy independent, the occupied single particle states $|h\rangle$'s are orthonormal to each other. So we can define the projection operator Γ to the occupied states as

$$\Gamma \equiv \sum_h |h\rangle\langle h|. \tag{135}$$

and we can express $g_p^{\alpha_{\rm N}}$ by the known $g_{\rm sp}^{\alpha_{\rm N}}$ as

$$g_p^{\alpha_{\rm N}} = g_{\rm sp}^{\alpha_{\rm N}} - g_{\rm sp}^{\alpha_{\rm N}}\Gamma\left(\Gamma g_{\rm sp}^{\alpha_{\rm N}}\Gamma\right)^{-1}\Gamma g_{\rm sp}^{\alpha_{\rm N}} \tag{136}$$

This method is called the *orthogonality condition model*.

8.6.5 Matrix elements of the transition density operators

Thirdly we calculate the matrix elements of the transition density operators between the ground state and the *ph* states.

Noting the formulas given in eq. (3B-25) of [11],

$$\langle (j_h^{-1}j_p)J||O^{(J)}||0\rangle = (-1)^{j_h+j_p-J}\langle j_p||O^{(J)}||j_h\rangle \tag{137}$$
$$\langle 0||O^{(J)}||(j_h^{-1}j_p)J\rangle = -\langle j_h||O^{(J)}||j_p\rangle \tag{138}$$

we obtain the expression

$$\langle h^{-1}p;JM|\rho^{(\nu),a}_{lsJM}(r)|0\rangle = \langle\alpha_p|\tau_\nu^{(1),a{\rm N}}|\alpha_h\rangle(-1)^{j_p+j_h-J}B^{a{\rm N}}_{lsJ}(p,h)\frac{u_h^{\alpha_h}(r)}{r}, \tag{139}$$
$$\langle 0|\rho^{(\nu),a}_{lsJM}(r)|h^{-1}p;J-M\rangle = -\langle\alpha_h|\tau_\nu^{(1),{\rm N}a}|\alpha_p\rangle(-1)^{J-M}B^{{\rm N}a}_{lsJ}(h,p)\frac{u_h^{\alpha_h}(r)}{r} \tag{140}$$

with

$$B^{ab}_{lsJ}(x,y) \equiv \sqrt{(2j_x+1)(2j_y+1)}\left\{\begin{array}{ccc} l_x & s_x & j_x \\ l_y & s_y & j_y \\ l & s & J\end{array}\right\}\langle l_x||{\rm i}^l Y_l(\hat{\boldsymbol{r}})||l_y\rangle\langle s_x||\sigma^{(s),ab}||s_y\rangle \tag{141}$$
$$= {\rm i}^{l_y+l_x-l}\sqrt{\frac{(2j_x+1)(2j_y+1)(2l_x+1)(2l_y+1)}{4\pi}}(l_y0l_x0|l0)$$
$$\times \left\{\begin{array}{ccc} l_x & s_x & j_x \\ l_y & s_y & j_y \\ l & s & J\end{array}\right\}\langle s_x||\sigma^{(s),ab}||s_y\rangle \tag{142}$$

where $a,b = {\rm N}$ or Δ, $h = (n_h l_h s_h j_h, \alpha_h)$, $p = (l_p s_p j_p, \alpha_p)$, $\alpha_h =$ p or n, and $\alpha_p =$ one of p, n, $\Delta^-, \Delta^0, \Delta^+, \Delta^{++}$, and $s_h = \frac{1}{2}$.

Here we used the formula

$$\langle l_x s_x j_x||\left[{\rm i}^l Y_l(\hat{\boldsymbol{r}})\times\sigma^{(s),xy}\right]^{(J)}||l_y s_y j_y\rangle$$
$$= \sqrt{(2J+1)(2j_x+1)(2j_y+1)}\left\{\begin{array}{ccc} l_x & s_x & j_x \\ l_y & s_y & j_y \\ l & s & J\end{array}\right\}\langle l_x||{\rm i}^l Y_l(\hat{\boldsymbol{r}})||l_y\rangle\langle s_x||\sigma^{(s),xy}||s_y\rangle \tag{143}$$

and

$$\langle l_x||{\rm i}^l Y_l(\hat{\boldsymbol{r}})||l_y\rangle = {\rm i}^{l_y-l_x+l}\sqrt{\frac{(2l+1)(2l_y+1)}{4\pi}}(l_y0l0|l_x0). \tag{144}$$

The double bar matrix elements in the spin space are given by

$$\langle\frac{1}{2}||\sigma^{(0),|\mathrm{NN}}||\frac{1}{2}\rangle = \sqrt{2}, \quad \langle\frac{1}{2}||\sigma^{(1),|\mathrm{NN}}||\frac{1}{2}\rangle = \sqrt{6}, \tag{145}$$

$$\langle\frac{3}{2}||\sigma^{(1),\Delta\mathrm{N}}||\frac{1}{2}\rangle = 2, \qquad \langle\frac{1}{2}||\sigma^{(1),\mathrm{N}\Delta}||\frac{3}{2}\rangle = -2 \tag{146}$$

We note that $B^{ab}_{lsJ}(x,y)$ is real because we treat $u_h(r)$ as real and $l_y + l_x + l$ =even.

The matrix elements of the isospin operators are given by

$$\langle \mathrm{n}|\tau^{(1),\mathrm{NN}}_{+1}|\mathrm{p}\rangle = -\sqrt{2}, \quad \langle \mathrm{p}|\tau^{(1),\mathrm{NN}}_{0}|\mathrm{p}\rangle = -1, \tag{147}$$

$$\langle \mathrm{n}|\tau^{(1),\mathrm{NN}}_{0}|\mathrm{n}\rangle = 1, \quad \langle \mathrm{p}|\tau^{(1),\mathrm{NN}}_{-1}|\mathrm{n}\rangle = \sqrt{2}, \tag{148}$$

$$\langle \Delta^0|\tau^{(1),\Delta\mathrm{N}}_{+1}|\mathrm{p}\rangle = \sqrt{\frac{1}{3}}, \quad \langle \Delta^+|\tau^{(1),\Delta\mathrm{N}}_{0}|\mathrm{p}\rangle = \sqrt{\frac{2}{3}}, \quad \langle \Delta^{++}|\tau^{(1),\Delta\mathrm{N}}_{-1} = 1|\mathrm{p}\rangle, \tag{149}$$

$$\langle \Delta^-|\tau^{(1),\Delta\mathrm{N}}_{+1}|\mathrm{n}\rangle = 1, \quad \langle \Delta^0|\tau^{(1),\Delta\mathrm{N}}_{0}|\mathrm{n}\rangle = \sqrt{\frac{2}{3}}, \quad \langle \Delta^+|\tau^{(1),\Delta\mathrm{N}}_{-1}|\mathrm{n}\rangle = \sqrt{\frac{1}{3}} \tag{150}$$

8.6.6 Free polalization propagators

From eqs. (130), (131), (132), (139) and (140), we finally get the expressions of the free polarization propagators (75), (76) as

$$\begin{aligned}
&\Pi^{(0),\mathrm{FW},a}_{ll'sJ\nu}(r,r';\omega) \\
&= \sum_{\alpha_h\alpha_p}\sum_{h,p} |\langle\alpha_p|\tau^{(1),a\mathrm{N}}_{\nu}|\alpha_h\rangle|^2 B^{a\mathrm{N}}_{lsJ}(p,h) B^{a\mathrm{N}}_{l'sJ}(p,h) \frac{u^{\alpha_h}_h(r)}{r} \frac{g^{\alpha_p}_p(r,r';\omega+\tilde{\epsilon}^{\alpha_h}_h)}{rr'} \frac{u^{\alpha_h}_h(r')}{r'}
\end{aligned} \tag{151}$$

$$\begin{aligned}
&\Pi^{(0),\mathrm{BK},a}_{ll'sJ\nu}(r,r';\omega) \\
&= \sum_{\alpha_h\alpha_p}\sum_{h,p} |\langle\alpha_h|\tau^{(1),\mathrm{N}a}_{\nu}|\alpha_p\rangle|^2 B^{\mathrm{N}a}_{l'sJ}(h,p) B^{\mathrm{N}a}_{lsJ}(h,p) \frac{u^{\alpha_h}_h(r')}{r'} \frac{g^{\alpha_p}_p(r',r;-\omega+\tilde{\epsilon}^{\alpha_h}_h)}{r'r} \frac{u^{\alpha_h}_h(r)}{r}
\end{aligned} \tag{152}$$

where $a = \mathrm{N}$ or Δ, $\alpha_h = \mathrm{p}$ or n, $\alpha_p =$ one of p, n, $\Delta^-, \Delta^0, \Delta^+, \Delta^{++}$, and $p = (l_p s_p j_p)$, $h = (n_h l_h s_h j_h)$, and $\tilde{\epsilon}^{\alpha_h}_h$ is given by eq. (13).

References

[1] T. Izumoto, M. Ichimura, C.M. Ko and P.J. Siemens, Phys. Letts **112B**, 315(1982).

[2] M. Ichimura, K. Kawahigashi, T.S. Jorgensen and C. Gaarde, Phys. Rev. **C39**, 1446(1989).

[3] K. Nishida, and M. Ichimura, Phys. Rev. **C51**(1995), 269.

[4] Y. Nakaoka, M. Ichimura, Prog. Theret. Phys. **102**, 599(1999).

[5] T. Wakasa et al. , Phys. Rev. **C59**, 3177(1999).

[6] K. Kawahigashi, K. Nishida, A. Itabashi, and M. Ichimura, Phys. Rev. **C63**, 044609(2001).

[7] M. Ichimura, H. Sakai , and T. Wakasa, Prog. Part. Nucl. Phys. **56**, 446(2006).

[8] M. Ichimura and K. Kawahigashi, Phys. Rev. **C45**, 1822(1992).

[9] O. Mikoshiba, T. Terasawa and M. Tanifuji, Nucl. Phys. **A168**, 417(1971).

[10] R. D Smith and J. Wambach, Phys. Rev. C **38**, 100(1988).

[11] A. Bohr and B. Mottelson, Nuclear Structure vol.1, W.A. Benjamin, Inc. , 1989.

[12] W.M. Alberico, M. Ericson, and A. Molinari, Nucl. Phys. **A379**, 429(1982).

[13] R. Machleidt, K. Holinde and Ch. Elster, Phys. Rep. **149**, 1(1987).

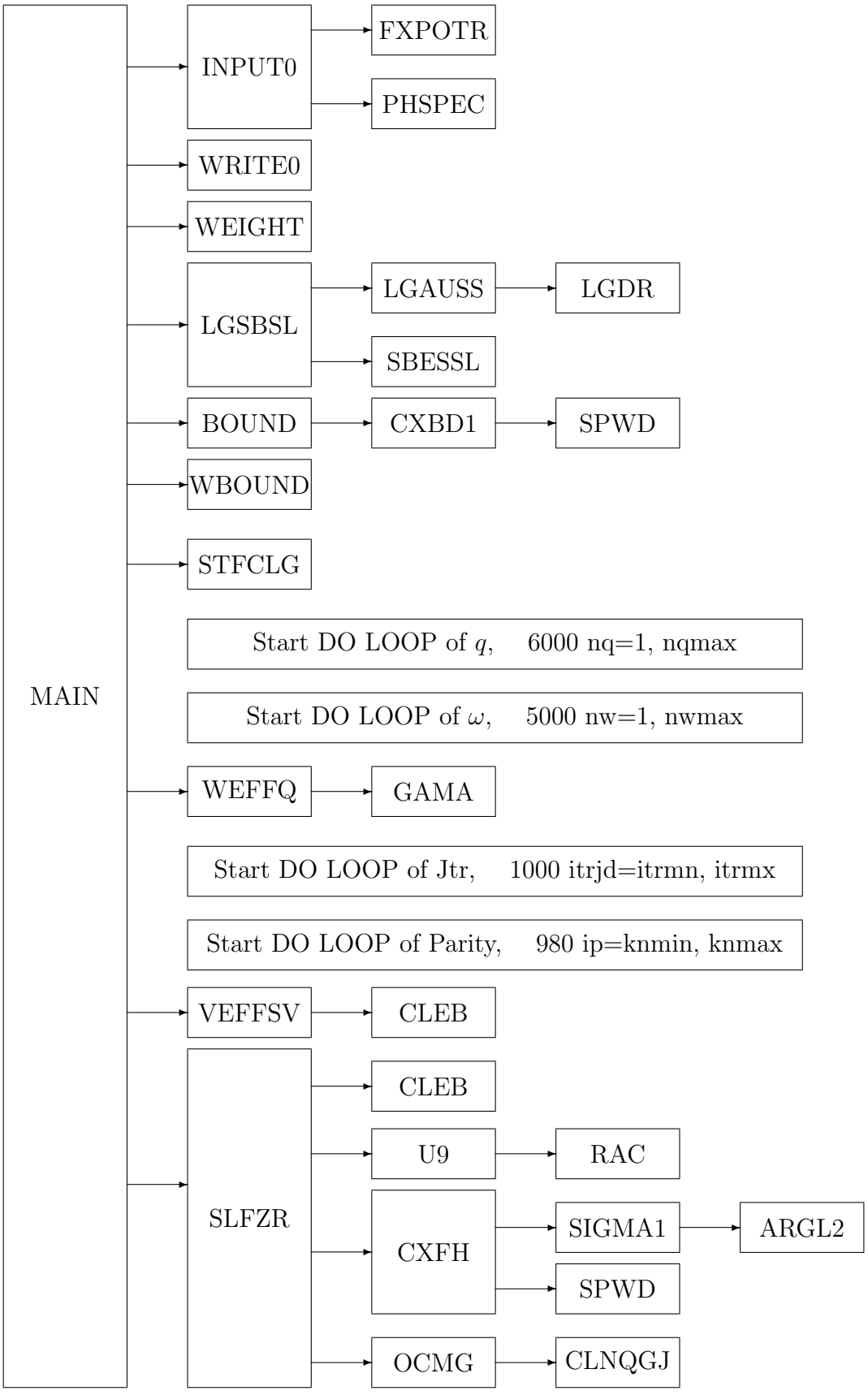
[14] T. Wakasa et al, Phys. Rev. **C85**, 064606 (2012).

[15] G. R. Satchler, Direct Nuclear Reactions, Clarendon, Oxford, 1983

[16] T. Izumoto and A. Mori, Phys. Lett. 82B, 163 (1979).

A Subroutines and functions

A.1 Structure of subroutines and functions



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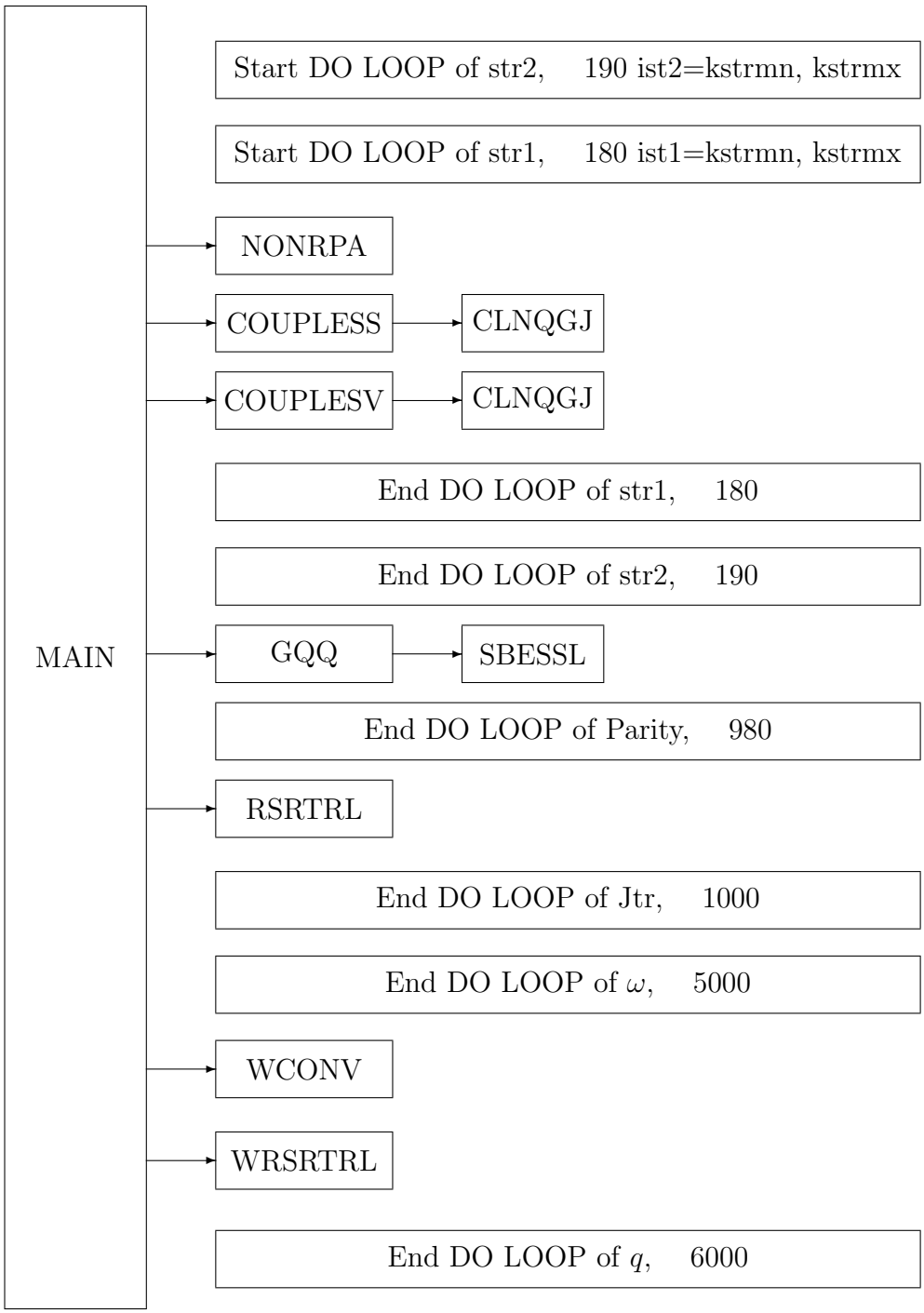
MAIN
Start DO LOOP of str2, 190 ist2=kstrmn, kstrmx
Start DO LOOP of str1, 180 ist1=kstrmn, kstrmx
NONRPA
COUPLESS
CLNQGJ
COUPLESV
CLNQGJ
End DO LOOP of str1, 180
End DO LOOP of str2, 190
GQQ
SBESSL
End DO LOOP of Parity, 980
RSRTRL
End DO LOOP of Jtr, 1000
End DO LOOP of ω, 5000
WCONV
WRSRTRL
End DO LOOP of q, 6000

A.2 List of subroutines

List of the subprograms in the order of the subroutine structure figures (A.1)

main	main program
input0	Read input data
fxpotR	Prefix some potential parameters in 'input0'
phspec	Set ph/Δh state specification in 'input0'
write0.	Output the input data
weight	Make table of weights for Sympson integration
lgsbsl	Make table of Legendre-Gauss Integral mesh points, weights and the spherical Bessel function at the mesh points
lgauss	Generate Legendre-Gauss integral mesh points and weights
lgdr	Legendre function
sbessl	Spherical bessel function
stfclg	make table of log of factorials
bound	Set data table of propeties and wave functions of the occupied states
cxbd1	Calculate bound state wave functions
spwd	Calculate spreading width of the single particle states
wbound	Output bound state parameters and wave functions
weffq	Calculate isovector spin vector ph effective interaction $W_{\rm T}(q,\omega), W_{\rm L}(q,\omega)$
gama	Calculate vertex form factor, $\Gamma(t,m,\Lambda,n)=[(\Lambda^2-m^2)/(\Lambda^2-t)]^n$
veffsv	Calculate isovector spin vector ph effective interaction in the r-space
cleb	Clebsh-Gordan coefficient
slfzr	Calculate free ph(Δ h) Green's function $G^{(0)}_{l's'lsJ}(r,r')$
u9	9j-symbol
rac	Racah coefficient
cxfh	Calculate the regular and singular single particle waves, $f_{lj}(r), h_{lj}(r)$
sigma1	Coulomb phase shift in 'cxfh' $\sigma(l,\eta)$
ARGL2	Function used in 'sigma1'
ocmg	Apply the orthogonality condition to the particle wave functions
clnqgj	Matrix inversion $A^{-1}B$ for complex martrices A and B
nonrpa	Calculate Free $ph(\Delta h)$ polarization propagator
coupless	Calculate the isovector spin scalar polarization propagator in the r-space.
couplesv	Calculate the isovector spin vector polarization propagators in the r-space.
gqq	Calculate $\Pi_{ll'sJ}(q,\omega)$ from $\Pi_{ll'sJ}(r,r')$
rsrtrl	Calculate spin scalar and spin vector response functions
wconv	Output conversion check data (Jconv)
wrsrtrl	Output $R_{\rm S}(q,\omega), R_{\rm S}(q,\omega), R_{\rm T}(q,\omega), R_{\rm L}(q,\omega), R_{\rm V}(\theta,\omega)$. Calculate $S^{(i)}_{\rm S}(q), R^{(i)}_{\rm S}(q), S^{(i)}_{\rm T}(q), S^{(i)}_{\rm L}(q), S^{(i)}_{\rm V}(q), \quad (i=0,1)$, and output them